Macroscopic Simulation of Open Systems and Micro-Macro Link

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Abstract. In this article we discuss the formulation of boundary conditions for a nonlocal macroscopic traffic model. Furthermore, we present a general scheme to derive nonlocal macroscopic models from a given microscopic car-following model. We show that there are models where the microscopic and the macroscopic version display very similar dynamics for identical (macroscopic) initial and boundary conditions. This enables to connect microscopic and macroscopic road sections by simple algorithms.

1 Boundary Conditions for a Nonlocal Macroscopic Traffic Model

As an example of a nonlocal macroscopic traffic model with well known properties we choose the Gaskinetic Traffic Model (GKT) [1]. It simulates freeway traffic by a set of two equations, the continuity equation $\partial\rho/\partial t + \partial(\rho V)/\partial x = 0$ for the traffic density $\rho(x,t)$, and a nonlocal equation for the macroscopic (locally averaged) velocity $V(x,t)$,

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)V + \frac{1}{\rho}\frac{\partial(\rho\theta)}{\partial x} = \frac{V_0 - V}{\tau} - \frac{C(\rho_a)\rho_a(\theta+\theta_a)}{2}B(\delta_V). \quad (1)$$

Here, $\theta = A(\rho)V^2$ is an approximation for the velocity-variance, $C(\rho)$ is an dimensionless effective cross-section that reproduces the finite-space requirement, $f_a(x,t) = f(x + \gamma(1/\rho_{\max} + TV))$ is the definition for the anticipated fields ($f = \rho, V$, or Θ), and $B(\delta_V)$ is the Boltzman interaction term. The Boltzman interaction term $B(\delta_V)$ is an increasing function of the scaled velocity-difference $\delta_V = \frac{(V-V_a)}{\sqrt{\theta+\theta_a}}$. It results from the gaskinetic ansatz and reproduces anticipatory behaviour similar to microscopic models.

The left-hand sides of the continuity and velocity equations constitute a hyperbolic set of partial differential equations. The direction of information flow due to convective and dispersive processes is given by the characteristic lines [2]. Specifically the velocities of information flow are given by:

$$\lambda_{1,2} = V - \frac{1}{2\rho}\frac{\partial P}{\partial V} \pm \sqrt{\frac{1}{4\rho^2}\left(\frac{\partial P}{\partial V}\right)^2 + \frac{\partial P}{\partial \rho}}, \quad \text{with} \quad P = \rho\Theta.$$

For all physically reasonable parameter sets they are always positive. Information Flow *against* the direction of motion of the simulated vehicles is caused by the nonlocal interaction term.

The consequences are: (i)There is always a transport of information in both directions, upstream and downstream, (ii) the best fitted differencing scheme for the numerical integration is the upwind scheme [3]. By chosing the upwind scheme, the downstream boundary conditions are only relevant for the non-local interaction term. This is in accordance with the boundary conditions for microscopic traffic simulations, where the first car at the downstream boundary always needs a "phantom front car" for its update process. In particular, the microscopic downstream boundary conditions determine only the *acceleration* of this first car but do not prescribe the flow of exiting vehicles. At the same time there always has to be a defined inflow of new vehicles at the upstream boundary. The same is true for simulating nonlocal macroscopic models with the upwind scheme. Therefore it is no problem to formulate downstream boundary conditions which are correct in each situation.

At the upstream boundary unphysical situations can emerge when a backward moving jam approaches the boundary and the prescribed inflow is higher than the flow inside the traffic jam. Then the equations are no longer well posed in a mathematical sense. Such situations may occur, whenever there is a slight offset in the arrival times of simulated and real jams at the boundary. This means the boundary conditions have to be implemented in a way that, on the one hand, they "swallow" unexpected or offset density waves and, on the other hand, return to the exact solution as soon as possible. This can be done by an algorithm that implements Dirichlet boundary conditions whenever the local group velocity at the upstream boundary ($x = 0$) is positive,

$$v_g = \left. \frac{\Delta(\rho V)}{\Delta\rho} \right|_{x=0} \geq 0,$$

and homogeneous Von Neumann boundary conditions otherwise. Here, $\Delta f = f(\delta) - f(0)$ with some $\delta > 0$.

This algorithm can be tested as follows. In order to obtain Dirichlet boundary conditions which belong to an exact solution of the model equations, we simulated a 20 km long circular road. The simulated scenario contains a local dipole-like perturbation in the initial conditions which develops to a backward moving cluster (Fig.1). We chose the fields at $x = 5$ and $x = 15$ km as values for the upstream and downstream boundaries, respectively, for the simulation of a new 10 km long *open* system. Figure 2 shows the comparison of the density profile of the two systems at two times t_1 and t_2. At $t_1 = 8$ min the cluster is not yet fully developed, and at $t_2 = 80$ min it has propagated once around the circle. The agreement is nearly perfect. Without the slight artificial offset between the profiles, introduced for better visualisation, both profiles would lay one upon the other.

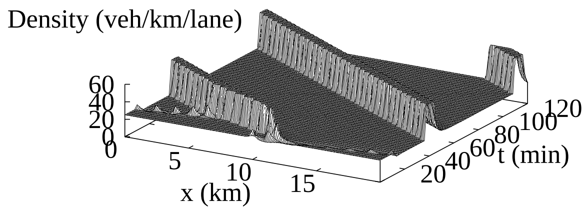

Fig. 1. Closed reference system simulated in order to obtain boundary conditions which belong to exact solutions of the model equations.

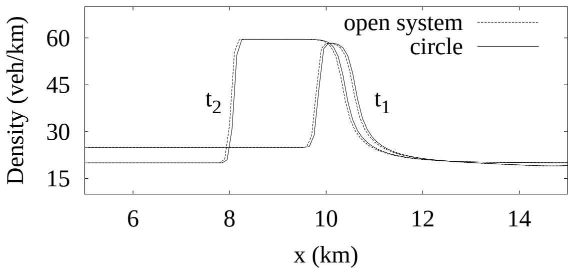

Fig. 2. Comparison of density profiles at two times $t_1 = 8$ min and $t_2 = 80$ min, obtained from the closed reference system (solid) and from the open system (dashed). For better visualisation one of the profiles is shifted by 100 m.

2 Micro-Macro Link

One of the biggest problems in comparing microscopic and macroscopic models via coarse graining is that a separation between the microscopic scale (single vehicles) and the macroscopic scale (where traffic densities and average velocities change) is hardly possible. Either the averaging intervals are too small and aggregate quantities like the density cannot be defined consistently, or the averaging intervals are too large and the dynamics of the model is smoothed out.

Generalizing an idea sketched in Ref. [4], we will now propose a way of obtaining amcroscopic from microscopic traffic models, which is different from the gas-kinetic approach and (at least) applicable to the case of identical driver-vehicle units. For this, we define an average velocity by a linear interpolation between the velocities of the single vehicles α:

$$V(x,t) = \frac{v_{\alpha+1}(t)[x_\alpha(t) - x] + v_\alpha(t)[x - x_{\alpha+1}(t)]}{x_\alpha(t) - x_{\alpha+1}(t)}, \tag{2}$$

where $x_\alpha \geq x \geq x_{\alpha+1}$, and the number α of the driver-vehicle units increases against the driving direction. The derivative with respect to time gives the *exact* equation:

$$\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right) V = \overline{a}(x,t), \tag{3}$$

where

$$\bar{a}(x,t) = \frac{a_{\alpha+1}(t)[x_\alpha(t) - x] + a_\alpha(t)[x - x_{\alpha+1}(t)]}{x_\alpha(t) - x_{\alpha+1}(t)} \tag{4}$$

is the linear interpolation of the single-vehicle accelerations a_α characterizing the microscopic model. For most car-following models, the acceleration function can be written in the form $a_\alpha = a_{\rm mic}(v_\alpha, \Delta v_\alpha, s_\alpha)$, where $\Delta v_\alpha = v_{\alpha-1} - v_\alpha$ is the approaching rate, and s_α the gap to the front vehicle. In order to obtain a macroscopic system of partial differential equations for the average velocity and density, the arguments v_α, Δv_α and s_α of the single-vehicle acceleration have to be expressed in terms of macroscopic fields as well. Speficially we make the following approximations: $\bar{a}(x,t) \approx a_{mic}(V(x), \Delta V(x), S(x))$, with $\Delta V(x) = V(x_a) - V(x)$, $S(x) = 1/2(\rho(x) + \rho(x_a))^{-1}$, and the nonlocality given by $x_a = x + 1/\rho(x)$. Because no averaging takes place there is no pressure term like the one in the GKT-Model described above. By this way a non-local macroscopic model naturally emerges from a given microscopic model. As Fig. 3 shows for the IDM [5], there are microscopic models where this procedure leads to a remarkable agreement. Figure 3(a) shows the spatio-temporal development of the density in a simulation of microscopic (bright grey) and macroscopic (dark grey) sections which have the same downstream or upstream boundary conditions, respectively. The boundary conditions have been obtained by a separate macroscopic simulation of the whole system. The density profile at the interface is shown in Fig. 3(b). Figure 3(c) shows density profiles 1 km upstream of the interface obtained from the microscopic simulation, compared with that obtained from the separate macropscopic simulation of the whole system.

The micro-macro link of traffic models is only of practical relevance if it includes (i) common initial and boundary conditions and (ii) *dynamic* interface conditions that do not require a separate (macroscopic) simulation of the whole system. Because information must flow through the interface in both directions, the formulation of dynamic interface conditions is a particularly tricky task. Figures 4(a) and 4(b) show the spacio-temporal evolution of the density on a circular road. One half is simulated with the microscopic version of the IDM (bright) and the other half is simulated with the macroscopic version of the same model (dark). As can be seen, it is possible to connect both sections in a way that both small perturbations propagating in *forward* direction and developed density clusters propagating *backwards* can pass the interfaces without any significant changes in shape or propagation velocity.

Simulations with single-lane microscopic models are fast and simple, but implementing on- or off-ramps requires more complicated multi-lane models

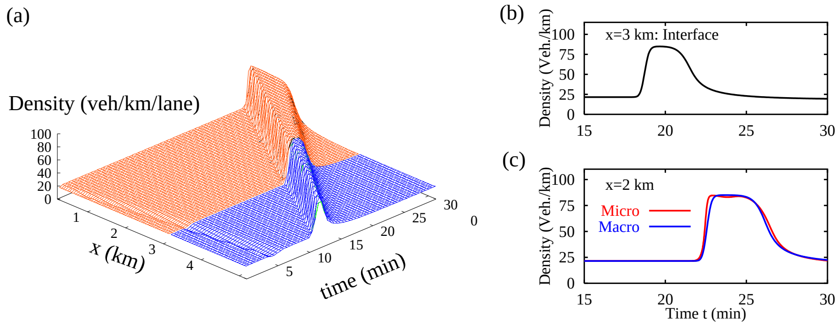

Fig. 3. Results of the simulation of microscopic (bright) and maroscopic (dark) sections simulated with the IDM. Both sections have the same upstream or downstream boundary conditions, respectively.

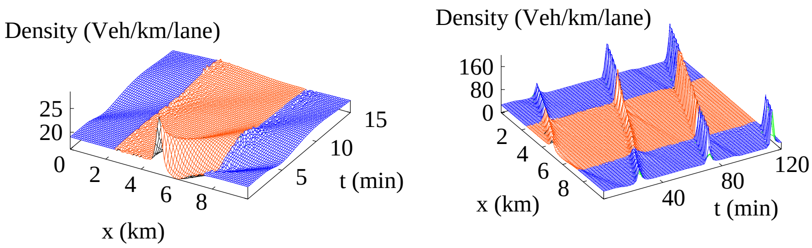

Fig. 4. Simulation of a circular road that consists by one half of a microscopic section (bright) and a macroscopic section (dark). In the simulation shwon on the left hand side the average density is low and the dipole-like perturbation in the initial condition propagates in forward direction. In the simulation on the right hand side the density is high and the perturbation develops to a backward moving cluster.

while they can be modelled in macroscopic models in a natural way by source terms in the continuity equation. The micro-macro link can be applied to combine both advantages. Figure 5 shows an open system that is simulated essentially by a microscopic single-lane model (bright sections). Only the

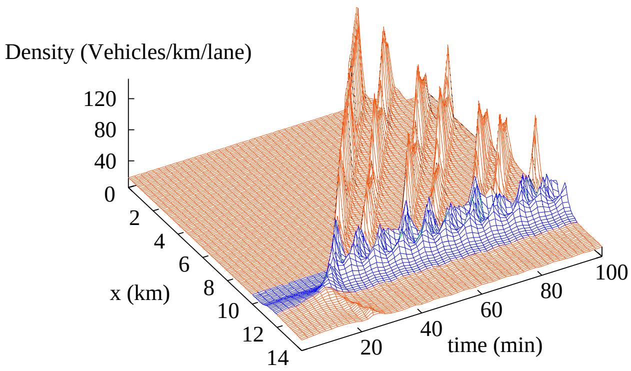

Fig. 5. Microscopic simulation of a TSG state caused by a high inflow from the ramp. The ramp section is realised by a macroscopic intersection between the two microscopic sections.

ramp section (dark) is simulated macroscopically. The on-ramp flow triggers a traffic breakdown to a TSG state, i.e. a cascade of jams propagating in the upstream microscopic section. The qualitative features of this complicated dynamics is identical to a purely macroscopic simulation of the same system.

References

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