Chapter 2

The standard (Helfrich-Carr) description of electroconvection

The Helfrich-Carr model of electroconvection combines the static [53] and dynamic [26, 27] macroscopic description of NLCs with the quasi-static Maxwell equations under the assumption of an ohmic conductivity. Since nearly all past theoretical investigations are based on this model (see Ref. [50] and the references therein), it is referred to as the Standard Model (SM).

In this chapter I give the essential steps of the derivation of the SM. The main purpose is to show the various approximations and to discuss their possible relevance for a mechanism leading to travelling rolls.

In the first section, I discuss the choice of the macroscopic fields. One has to make sure that they contain all slow processes involved in the instability mechanism. In the following section, I give the canonical derivation in the framework of generalized hydrodynamics [54, 55, 56] (for more details see [57, 58]). Impatient readers may skip these two sections and go directly to Chapter 2.3 which gives a self-contained description of the SM in the form used throughout the rest of this work.

2.1 Macroscopic variables

In generalized hydrodynamics, one distinguishes three types of slow fields [55].

- Conserved quantities. They cannot be created or destroyed locally, so their dynamics is of the form $\partial_t X + \boldsymbol{\nabla} \cdot \boldsymbol{J}_X = 0$ where $\boldsymbol{J}_x$ is the current associated to X. In EHC as in other hydrodynamic systems, the components of the momentum density $g_i = \rho_m v_i$ [1] are such variables. Its currents define the tensor T_{ij} of the momentum flux, which is equal to the negative stress tensor.

- Broken-symmetry variables. They break a continuous symmetry but are not conserved, so their dynamical equations are of the form $\partial_t X + Y_X = 0$ where Y_X is sometimes called a "quasi current" [58]. Since, according to the Noether theorem, outer symmetries are related to conservation laws, this type of field can exist only in complex fluids with some inner symmetries. In NLCs without external fields, the director breaking the local rotational symmetry is such a field.

- Slowly relaxing variables. In contrast to the first two classes they are not truly hydrodynamic in that they relax in the homogeneous limit in a finite time. Nevertheless they cannot be neglected in EHC (and other confined systems), if their relaxation time is comparable to that of hydrodynamic fields with *nonzero* wavenumbers varying at length scales of the order of the distance d between the two electrodes. The dynamical equation is of the same form as that for broken-symmetry variables but, in contrast to the former, the static contribution to the energy density does not vanish in the homogeneous limit. The component of the director parallel to an electric (or magnetic) field is such a variable.

In both the SM and the WEM, the quasi-static Maxwell equations $\boldsymbol{\nabla}\times\boldsymbol{E} = \boldsymbol{\nabla}\times\boldsymbol{H} = 0$, $\partial_t\rho + \boldsymbol{\nabla}\cdot\tilde{\boldsymbol{J}} = 0$ are used, where $\tilde{\boldsymbol{J}} = \rho\boldsymbol{v} + \boldsymbol{J}$ and $\rho = \boldsymbol{\nabla}\cdot\boldsymbol{D}$ (Poisson equation). This means that there is only one independent slow electric quantity for which the charge density ρ can be taken. [2] With the normalization condition $\boldsymbol{n}^2 = 1$ and the further assumption of incompressibility, $\boldsymbol{\nabla}\cdot\boldsymbol{v} = 0$, the SM contains five independent fields, namely the charge density ρ, two director components, and two momentum densities g_i with the equations [3]

$$\partial_t\rho + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{v} + \boldsymbol{J}) = 0, \qquad (2.1)$$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})n_i + Y_i = 0, \qquad (2.2)$$

$$\partial_t g_i + \partial_j(g_i v_j + T_{ij}) = 0, \qquad (2.3)$$

[1] In general (e.g. for nonzero magnetic fields), $\boldsymbol{g}$ is not equal to $\rho_m\boldsymbol{v}$ [59]. Within the quasi-static approximation for the Maxwell equations, we have always $\boldsymbol{g} = \rho_m\boldsymbol{v}$.

[2] Later on, the potential ϕ of the induced field inhomogeneity will be used as independent electric variable.

[3] Throughout this work, summation over doubly occurring indices is assumed; the notation $\partial_j = \partial/\partial x_j$, $n_{i,j} = \partial_j n_i$ will be used freely.

where the convective currents proportional to $\boldsymbol{v}$ are shown explicitly and ρ as well as the (quasi-) currents $\boldsymbol{J}, \boldsymbol{Y}$ and $\underline{\underline{T}}$, are yet to be determined.

One has to watch for other slow processes. The mere existence of a slow field is, of course, not dangerous. To become relevant for EHC, the slow field must be excited by the SM variables and must couple back. A possible candidate is the order parameter S which becomes a slowly relaxing field near the clearing point [58]. It couples to the other fields via the S dependent relative anisotropies of the material parameters [25] and was suggested as a possible explanation for travelling waves [52]. The temperature (or internal energy) is slow as well, which is exploited in RBC of NLCs [60]. With planar boundary conditions (BC), the director enhances the buoyancy mechanism of isotropic RBC by a factor of the order of $\tau_d/\tau_{therm} \approx 1000$ (!) via heat focussing caused by the anisotropic thermal conductivity [61, 24]. Without an external temperature gradient, temperature inhomogenities are produced only in nonlinear order by the dissipative heat production R (Chapter 2.2). A simple order-of-magnitude estimation shows, that this contribution is negligible even in weakly-nonlinear calcuations [62]. With an external electric field, however, generalized hydrodynamics allows for a linear mechanism driven by thermoelectric effects [an electric field drives a thermal current and a temperature gradient drives an electric current, see Eq. (2.16) below], which may become important for thick cells.

Even the classic Maxwell equations have some subtleties if applied to polarizable media [59]. Only rather recently, the treatment of the fields $\boldsymbol{D}$ and $\boldsymbol{B}$ on equal footing as the other hydrodynamical variables has been carried through [63] leading in NLCs to new "dissipative" parts of the electromagnetic fields [57], which are not *a priori* small. Furthermore, such fields can induce a coupling between, e.g., a velocity gradient and the electric current, which clearly is relevant for EHC. There exist, however, no measurements of the material parametes involved, or even an experimental evidence of these effects; they are just allowed by symmetries. This concept will not be pursued in this work.

Finally, in this framework, the WEM is based on the assumption that the local conductivity becomes slowly relaxing. It is excited by charge-carrier migration effects and couples back to the other equations via the change of the conductivity in the charge conservation.

2.2 Derivation of the Standard Model

The framework set by Eqations (2.1) to (2.3) is valid for many systems. Now we specify them to NLCs by determining ρ and the (quasi) currents $\boldsymbol{J}, \boldsymbol{Y}$ and $\underline{\underline{T}}$. The functional dependence on the NLC material parameters is completely determined by symmetries [55, 58] and its derivation will be sketched in the following.

2.2.1 Statics

The thermodynamical potential suitable for equilibrium at a given temperature and for a given electric field is the free electric enthalpy $G = \int d^3r \{\epsilon - Ts - \boldsymbol{E}\cdot\boldsymbol{D}\}$ where ϵ and s are the densities of the energy and the entropy, respectively.[4] Near equilibrium, G is a quadratic form of the thermodynamic variables $\boldsymbol{n}$, $\rho_m\boldsymbol{v}$, and $\boldsymbol{E}$. Furthermore, it is extensive, $G = \int d^3r g$, and the scalar density g is invariant under rotations. Since $\boldsymbol{n}$ is a symmetry variable without an electric field, the elastic contribution of G depends only on gradients of $\boldsymbol{n}$. Respecting the uniaxiality of the state, the $\boldsymbol{n} \leftrightarrow -\boldsymbol{n}$ symmetry and the inversion symmetry in space and time, and restricting to the lowest-order expansion in the gradients, one obtains

$$G = \int d^3r \left\{ \frac{1}{2}\rho_m \boldsymbol{v}^2 + \frac{1}{2}K_{ijkl}n_{i,j}n_{k,l} - \frac{1}{2}\epsilon_{ij}E_iE_j - e_{ijk}n_{i,j}E_k \right\} \tag{2.4}$$

with

$$K_{ijkl}n_{i,j}n_{k,l} = K_{11}(\boldsymbol{\nabla}\cdot\boldsymbol{n})^2 + K_{22}[\boldsymbol{n}\times(\boldsymbol{\nabla}\times\boldsymbol{n})]^2 + K_{33}[\boldsymbol{n}\cdot(\boldsymbol{\nabla}\times\boldsymbol{n})]^2, \tag{2.5}$$

$$\epsilon_{ij} = \epsilon_\perp\delta_{ij} + \epsilon_a n_i n_j, \tag{2.6}$$

$$e_{ijk} = e_1\delta_{ij}n_k + e_3\delta_{ik}n_j. \tag{2.7}$$

Comparing Equation (2.4) with the general form of the free electric enthalpy gives the constitutive equation for $\boldsymbol{D}$ (and thus for ρ) and defines the "molecular field" $\boldsymbol{h}$ [25] as the thermodynamic conjugate of $\boldsymbol{n}$,

$$D_i = -\frac{\delta G}{\delta E_i} = \epsilon_{ij}E_j + e_{jki}n_{j,k} := \epsilon_{ij}E_j + P_i^{flexo}, \tag{2.8}$$

$$h_i = \frac{\delta G}{\delta n_i} = \frac{\partial g}{\partial n_i} - \partial_j\left(\frac{\partial g}{\partial n_{i,j}}\right). \tag{2.9}$$

The term (2.5) is the orientational-elastic Frank energy [53] due to splay (K_{11}), twist (K_{22}), and bend (K_{33}) deformations of the director; $\underline{\epsilon}$ is the uniaxial tensor of the

[4] In the Chapters 5-7, the control parameter is denoted by ϵ as well. A confusion should not arise.

dielectric permittivity, and e_{ijk} describes the flexoelectric effect. The flexoelectric polarization $\boldsymbol{P}^{flexo}$ leads to an additional charge density $\boldsymbol{\nabla}\cdot\boldsymbol{P}^{flexo}$ in Eq. (2.1)) that is not *a priori* small. In fact, the flexoeffect has been investigated rather extensively as a possible candidate to explain the travelling waves [64, 49, 65]. It cancels out, however, for AC driving in the "lowest order time expansion" (Chapter 5.2) which will be considered exclusively in this thesis. In addition, the flexoelectric coefficients are hard to measure [66].

2.2.2 Dynamics

The suitable potential to derive the dynamics is the energy

$$dE = \int d^3r \left\{ T\delta s + \tilde{\phi}\delta\rho + h\delta n + v_i \delta g_i \right\} := \int d^3 r \epsilon, \tag{2.10}$$

since its natural variables are those of the conservation laws and balance equations (2.1) - (2.3).

The central relation is the entropy balance

$$\partial_t s + \boldsymbol{\nabla}\cdot(s\boldsymbol{v} + \boldsymbol{J}_s) = \frac{R}{T}, \tag{2.11}$$

were the dissipation function R is the local heat production per volume from dissipative processes. With Eq. (2.10), $\partial_t s$ can be substituted by $T^{-1}(\partial_t \epsilon - \boldsymbol{v}\cdot\rho_m\partial_t\boldsymbol{v} - \boldsymbol{h}\cdot\partial_t\boldsymbol{n} - \tilde{\phi}\partial_t\rho)$. After eliminating the time derivatives with the energy balance $\partial_t\epsilon + \boldsymbol{\nabla}(\epsilon\boldsymbol{v} + \boldsymbol{J}_\epsilon) = 0$ and with the Eqs (2.1) - (2.3), we arrive (with the constitutive equation $g_i = \rho_m v_i$) at

$$\begin{aligned} R &= T\boldsymbol{\nabla}(s\boldsymbol{v} + \boldsymbol{J}_s) - \boldsymbol{\nabla}(\epsilon\boldsymbol{v} + \boldsymbol{J}_\epsilon) + \tilde{\phi}\boldsymbol{\nabla}(\rho\boldsymbol{v} + \boldsymbol{J}) \\ &+ \boldsymbol{h}(\boldsymbol{v}.\boldsymbol{\nabla}\boldsymbol{n} + \boldsymbol{Y}) + v_i(\rho_m v_j \partial_j v_i + \partial_j T_{ij}). \end{aligned} \tag{2.12}$$

The (quasi-) currents [5] on the right-hand side of Eq. (2.12) can be separated into independent dissipative parts (the superscript D will be used) and reversible parts, and the latter can be separated into transport parts shown explicitly in (2.12) and parts existing also in the frame of reference comoving with the local velocity [dashed in Eq. (2.21) below]. Now I determine the three parts separately.

[5]Henceforth, I will not distinguish explicitly betwee currents and quasi-currents

Dissipative parts

The dissipative currents make up the entropy production. Near or in local equilibrium, the entropy production is a quadratic form of the generalized forces driving the system out of (global) equilibrium. *In* equilibrium, the conjugates of conserved variables are constant and that of symmetry breaking or slowly relaxing variables are zero, so *near* local equilibrium, the Onsager forces of the conserved variables ρ, $g_i = \rho_m v_i$, and s are the gradients of the conjugate fields, $-\boldsymbol{\nabla}\tilde{\phi}$, $-\partial_i v_j$ and $-\boldsymbol{\nabla}T$, and the force of the director is the conjugate h itself. If one writes the forces as

$$F_\alpha := (E_i, h_i, -\partial_i v_j, -\partial_i T), \tag{2.13}$$

the dissipation function (the entropy production multiplied by T) takes the form

$$R = F_\alpha M_{\alpha\beta} F_\beta := F_\alpha J_\alpha \tag{2.14}$$

which defines the Onsager fluxes J_α. The matrix $M_{\alpha\beta}$ has to fulfil the Onsager relations [67, 68, 69]

$$M_{\alpha\beta} = t_\alpha t_\beta M_{\beta\alpha}, \tag{2.15}$$

where $t_\alpha = 1$ (-1) for forces of variables that are symmetric (antisymmetric) under time reversal. The signs (and possible prefactors) of F_α are defined such that the Onsager fluxes J_α are just the curents $\boldsymbol{J}, \boldsymbol{Y}, T_{ij}$ and $\boldsymbol{J}_{therm}$ as will be shown below. Applying the symmetry restrictions and Eq. (2.15), we obtain analogously to Eq. (2.4) in lowest order

$$\begin{aligned} R \; &= \tfrac{1}{2}\sigma_{ij} E_i E_j + \tfrac{1}{2\gamma_1} h_i \delta_{ij}^{\perp} h_j + \tfrac{1}{2}\eta_{ij,kl} v_{i,j} v_{k,l} \\ &+ \tfrac{1}{2}\kappa_{ij}(\partial_i T)(\partial_j T) + \kappa_{ij}^{el} E_i \partial_j T. \end{aligned} \tag{2.16}$$

The first term with the usual uniaxial form for the conductivity tensor $\sigma_{ij} = \sigma_\perp \delta_{ij} + \sigma_a n_i n_j$ is due to ohmic heating. The second term with the rotational viscosity γ_1 describes the rotational friction of the director relative to the moving fluid. To satisfy $\boldsymbol{n}^2 = 1$, the variational derivative in the definition of the molecular field must be restricted to variations perpendicular to the director itself. This means $\boldsymbol{h} \perp \boldsymbol{n}$ and is guaranteed by applying to $\boldsymbol{h}$ the tensor $\delta_{ij}^{\perp} = \delta_{ij} - n_i n_j$ projecting onto the plane perpendicular to the director. The third term with three viscosities (see below) describes the viscous heating. The two temperature-gradient terms are neglected, although, for a nonzero external electric field, the thermoelectric coupling $\propto \kappa_{ij}^{el}$ induces a thermal current in linear order (Chapter 2.1). [6]

[6] Sometimes, a further term $\tilde{e}_{ijk} h_i \partial_j E_k$ describing the dynamic analog of the flexoelectric effect, is introduced [58]. It is of higher order if there are no field gradients in the basic state; the corresponding material parameter has not been measured.

The dissipative currents are determined by rearranging Eq. (2.12) without the convective terms in a gradient and a sum of Onsager forces multiplied by fluxes,

$$\begin{aligned} R &= \partial_j \left(T J^D_{sj} + \tilde{\phi} J^D_j + v_i T^D_{ij} - \frac{\partial \epsilon}{\partial n_{i,j}} Y_i - J^D_{\epsilon j} \right) \\ &+ \boldsymbol{J}^D \cdot \boldsymbol{E}. + \boldsymbol{h} \cdot \boldsymbol{Y}^D - T^D_{ij} \partial_j v_i - \boldsymbol{J}^D_s \cdot \boldsymbol{\nabla} T, \end{aligned} \tag{2.17}$$

and comparing the result with Eq. (2.16). The gradient parts are balanced by $\boldsymbol{J}_\epsilon$, and the dissipative currents are (in the approximations of the SM)

$$J^D_i = \frac{\partial R}{\partial E_i} = \sigma_{ij} E_j, \tag{2.18}$$

$$Y^D_i = \frac{\partial R}{\partial h_i} = \gamma_1^{-1} \delta^\perp_{ij} h_j, \tag{2.19}$$

$$\begin{aligned} -T^D_{ij} = \frac{\partial R}{\partial v_{i,j}} &= \eta_{ijkl} v_{k,l} = \alpha_4 A_{ij} + (\alpha_1 + \gamma_1 \lambda^2) n_i n_j n_k n_l A_{kl} \\ &+ (\alpha_5 - \frac{\gamma_1 \lambda (1+\lambda)}{2})(n_i A_{jk} n_k + n_j A_{ik} n_k), \end{aligned} \tag{2.20}$$

where $A_{ij} = (\partial_i v_j + \partial_j v_i)/2$. As a result of the symmetry restrictions, one gets in Eq. (2.20) three general shear-viscosity coefficients. They have been expressed in terms of the more familiar Erickson-Leslie coefficients α_1, α_4 and α_5 [26, 27], and by γ_1 and a reversible parameter λ to be defined below.

Reversible parts for zero transport

The Onsager symmetries (2.15) are valid also for the reactive Onsager fluxes $J'_\alpha = M'_{\alpha\beta} F_\beta$, but now, of course, the heat production Eq. (2.14) has to vanish. This implies that the reactive Onsager matrix $M'_{\alpha\beta}$ has no diagonal terms and only those cross terms that couple variables with opposite time-reversal symmetry. This leads to $\boldsymbol{J}'_s = \boldsymbol{J}' = 0$, and to

$$\begin{pmatrix} Y'_i \\ T'_{ij} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2}\lambda_{ijk} \\ -\frac{1}{2}\lambda_{kji} & 0 \end{pmatrix} \begin{pmatrix} h_k \\ -\partial_j v_k \end{pmatrix}. \tag{2.21}$$

The symmetries lead to $\lambda_{ijk} = \lambda_1 \delta^\perp_{ij} n_k + \lambda_2 \delta^\perp_{ik} n_j$ and the condition of vanishing relative motion of the director in the case of a rigidly rotating fluid, $\partial_t \boldsymbol{n} = \boldsymbol{\omega} \times \boldsymbol{n}$ for $\partial_i v_j = \partial_j v_i$ and $\boldsymbol{\nabla} \times \boldsymbol{v} = 2\boldsymbol{\omega}$, leads to $\lambda_2 - \lambda_1 = 2$.[7] Thus $\underline{\underline{\lambda}}$ can be written as

$$\lambda_{ijk} = (\lambda - 1)\delta^\perp_{ij} n_k + (\lambda + 1)\delta^\perp_{ik} n_j. \tag{2.22}$$

[7] A more formal derivation using conservation of the angular momentum can be found in [58].

Often, the sum of the reactive and dissipative parts of the momentum-flux tensor is expressed in terms of the Erickson-Leslie coefficients $\alpha_1, ..., \alpha_6$ [26, 27], given in Eq. (2.28) below. Although this formulation is less systematic, it will be used throughout this thesis, mainly to enable an easy comparison with existing work. An intuitive picture of the various viscosities is given e.g. in [70, 31].

Transport parts

The transport or convective currents are related to Galilean invariance and therefore reversible as well. The condition that the dissipation function of the convective currents, Eq. (2.12) with $\boldsymbol{J}_s = \boldsymbol{J} = \boldsymbol{Y} = 0$, vanishes for any $\boldsymbol{v}$, can only be satisfied by an extra part T^t_{ij} of the momentum-flux tensor. The momentum-flux tensor (including the isotropic pressure) is the only non-convective current which gives contributions $\propto \boldsymbol{v}$ in (2.12) and thus can balance all other advective contributions. This leads to

$$T^t_{ij} = p\delta_{ij} + \pi_{ij} - E_i D_j, \qquad \pi_{ij} = \frac{\partial \epsilon}{\partial n_{k,j}} n_{k,i}, \tag{2.23}$$

where the pressure is given by the Gibbs relation [71, 59] $p = -\epsilon + Ts + \rho_m v^2 + \rho\tilde{\phi}$, and some transport parts were expressed by $\partial_i p = D_j \partial_i E_j - h_j \partial_i n_j + \rho_m v_j \partial_i v_j + s \partial_i T$.

The pressure will be eliminated later. The second term in Eq. (2.23), the Erickson stress [25], is the (nonlinear) counter term of the director advection term $\boldsymbol{v} \cdot \boldsymbol{\nabla n}$. In physical terms, velocity gradients change the elastic energy by changing the local director distortions which must be balanced by the mechanical power (i.e. velocity times a force) $v_i \partial_j \pi_{ij}$. The third term $E_i D_j := -T^{el}_{ij}$ is the balance to the charge advection. Its gradient, the electric volume force, is the main driving force of EHC.

By redefining the pressure as $\tilde{p} = p - \frac{1}{2}\epsilon_0 \epsilon_\perp E^2$, the volume force can be written as $-\partial_j T^{el}_{ij} = \rho E_i + P_j \partial_j E_i$, where the "polarization" $P_i = \epsilon_a n_i n_j E_j$ contains the inhomogeneity of the dielectric displacements. The polarization part is nonlinear and will be neglected in the following.

2.3 Basic equations

In summary, the equations of the SM are

$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla})\rho = -\boldsymbol{\nabla} \cdot (\underline{\underline{\sigma}} \boldsymbol{E}), \tag{2.24}$$

$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{n} = \boldsymbol{\omega} \times \boldsymbol{n} + \underline{\underline{\delta}}^\perp (\lambda \underline{\underline{A}} \boldsymbol{n} - \frac{1}{\gamma_1} \boldsymbol{h}), \tag{2.25}$$

$$\rho_m (\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) v_i = -\partial_i p - \partial_j (\pi_{ij} + T^{visc}_{ij}) + \rho E_i, \tag{2.26}$$

with the static conditions $\rho = \partial_i \epsilon_{ij} E_j$ (Poisson equation), $E_i = E_0(t)\delta_{i3} - \partial_i \phi$ (exploiting $\boldsymbol{\nabla} \times \boldsymbol{E} = 0$ and separating $\boldsymbol{E}$ into an external field and a field inhomogeneity), $\boldsymbol{n}^2 = 1$ (constant order parameter), and $\boldsymbol{\nabla} \cdot \boldsymbol{v} = 0$ (incompressibility).

Eq. (2.24) is just the charge balance for a weak anisotropic ohmic conductor where both the charge density and the current are relevant. Without forces, the director in Eq. (2.25) would move with the surrounding fluid like a rod in a river, $\boldsymbol{N} \equiv (\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{n} - \omega \times \boldsymbol{n} = 0$, where $\boldsymbol{\omega} = (\boldsymbol{\nabla} \times \boldsymbol{v})/2$ is the local fluid rotation. The forces onto the director in Eq. (2.25) come from the orientational elasticity described by the molecular field $\boldsymbol{h}$, and from a coupling of the director to the fluid shear $A_{ij} = (\partial_i v_j + \partial_j v_i)/2$ ("flow alignment"). The projection tensor $\delta^{\perp}_{ij} = \delta_{ij} - n_i n_j$ guarantees $n^2 = 1$. The molecular field is given by

$$h_i = \frac{\delta}{\delta n_i} \left(K_{mnkl} n_{m,n} n_{k,l} \right) - \epsilon_a (\boldsymbol{n} \cdot \boldsymbol{E}) E_i, \tag{2.27}$$

with $K_{ijkl} n_{i,j} n_{k,l}$ from Eq. (2.5). Sometimes, Eq. (2.25) is written as $\boldsymbol{n} \times (\boldsymbol{h} + \gamma_1 \boldsymbol{N} + \gamma_2 \underline{\underline{A}}\boldsymbol{n}) = 0$ with $\gamma_2 = -\lambda \gamma_1$ [72, 25, 73]. Both forms of the director equation can be expressed in terms of the Erickson-Leslie coefficients with the relations $\gamma_1 = \alpha_3 - \alpha_2$ and $\lambda = (\alpha_2 + \alpha_3)/(\alpha_2 - \alpha_3)$ obtained with the help of angular momentum conservation.

The negative viscous stress tensor (momentum-flux tensor) T^{visc}_{ij} has a reactive part T'_{ij} and a dissipative part T^{D}_{ij} given by the Eqs. (2.21) and (2.20), respectively. Often, the molecular field in T'_{ij} is expressed with the help of Eq. (2.25) in terms of $\boldsymbol{N}$ and $\underline{\underline{A}}\boldsymbol{n}$ and the two parts are written together in terms of the Leslie coefficients,

$$\begin{aligned} -T^{visc}_{ij} &= \alpha_1 n_i n_j n_k n_l A_{kl} + \alpha_2 n_j N_i + \alpha_3 n_i N_j \\ &\quad + \alpha_4 A_{ij} + \alpha_5 n_j n_k A_{ki} + \alpha_6 n_i n_k A_{kj}. \end{aligned} \tag{2.28}$$

In this formulation, the Onsager symmetries have to be condidered separately leading to the so-called Parodi relation [74] $\alpha_2 + \alpha_3 = \alpha_6 - \alpha_5$. At last, the nonlinear Erickson stress π_{ij} in Eq. (2.26) is given by Eq. (2.23). In the Eqs. (2.24)–(2.26), some small contributions have been neglected, e.g., the flexoeffect and the polarization part of the electric volume force (Chapter 2.2.2).

Equations (2.24) to (2.26) represent five independent equations for the potential ϕ of the electric field inhomogeneity, two director components (n_y and n_z for the planar geometry), and two velocity fields or a suitable representation for them, e.g. the toroidal and a poloidal potential g and f for divergence-free fluids[75]

$$\boldsymbol{v} = \boldsymbol{\nabla} \times \hat{\mathbf{z}} g + \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \hat{\mathbf{z}} f) \equiv \boldsymbol{\epsilon} g + \boldsymbol{\delta} f, \tag{2.29}$$

$$\boldsymbol{\epsilon} = (\partial_y, -\partial_x, 0), \quad \boldsymbol{\delta} = (\partial^2_{xy}, \partial^2_{yz}, -\partial^2_{xx} - \partial^2_{yy}). \tag{2.30}$$

class of response	effect	given by		in EHC leading to
static	orientational elasticity	K_{ijkl} = (2.5)	K_{11} K_{22} K_{33}	restoring torque from director distortions
	dielectric permittivity	$\epsilon_{ij} = \epsilon_\perp \delta_{ij} + \epsilon_a n_i n_j$	$\epsilon_\perp$ ϵ_a	dielectric torque
dynamic, reactive	flow alignment	λ_{ijk} =(2.22)	λ †	torque onto the director due to fluid shear
dynamic, dissipative	rotational viscosity		γ_1 †	torque onto the director due to relative rotation
	shear viscosity	η_{ijkl} = (2.20)	α_1 α_4 α_5	Damping of the fluid motion
	conductivity	$\sigma_{ij} = \sigma_\perp \delta_{ij} + \sigma_a n_i n_j$	$\sigma_\perp$ σ_a	charge focussing; electric volume force
† $\lambda = (\alpha_2 + \alpha_3)/(\alpha_2 - \alpha_3)$, $\gamma_1 = \alpha_3 - \alpha_2$				

To eliminate the pressure, one takes the y and z components of the curl of (2.26) [48] or applies the Hermitean conjugate of the operators $\boldsymbol{\delta}$ and $\boldsymbol{\epsilon}$ [76, 77, 62].

The planar-homogeneous "rigid" BC, used exclusively in this work, are

$$\begin{aligned} \phi(z = \pm d/2) &= 0 && \text{(ideally conducting plates)}, \\ \boldsymbol{n}(z = \pm d/2) &= (1,0,0) && \text{(rigid anchoring)}, \\ \boldsymbol{v}(z = \pm d/2) &= 0 && \text{(finite viscosity)}. \end{aligned} \tag{2.31}$$

In the horizontal x and y directions, I assume the system to be infinite (translationally invariant). To avoid destruction of the NLC by electrolytic effects, the cell is driven with an AC voltage,

$$E_0(t) = \frac{\overline{V}\sqrt{2}}{d} \cos \omega_0 t, \tag{2.32}$$

which conveniently introduces the external frequency as a second control parameter (typically, $\omega_0/2\pi = 10...1000$ Hz). The instability mechanism, however, is active also for DC.

In the table, the material parameters of the NLC are summarized that are contained in the SM.