Appendix A

A.1 Material parameters for MBBA and I 52

Table A.1. Material parameters for MBBA at 25°C [147, 148], and for I 52 at 30°C – 60°C [32] as used in Ref. [42]

Parameter		MBBA	I 52
Orientational elasticities in units of 10^{-12} N	K_{11}	6.66	18.4 *
	K_{22}	4.2	12.65 *
	K_{33}	8.61	23
Conductivities in units of $10^{-8}(\Omega\text{m})^{-1}$	$\sigma_\perp$	1.0	$0.28 \cdots 1.41$ $^\&$
	σ_a	0.5	$0.073 \cdots 0.63$ $^{\&\ *}$
Dielectric permittivities in units of $\epsilon_0 = 8.8542 \times 10^{-12} \frac{As}{Vm}$	$\epsilon_\perp$	5.25	$3.01 \cdots 2.90$ $^\&$
	ϵ_a	-0.53	$0.056(T - 63°\text{C})/(38°\text{C})$
Viscosities in units of $10^{-3} \frac{Ns}{m^2}$ **	α_1	-18.1	$0.1\gamma_1$ *
	α_2	-110.4	$-0.9\gamma_1$ †
	α_3	-1.1	$0.1\gamma_1$ *
	α_4	82.6	2η
	α_5	77.9	$0.65\gamma_1$ ††
	α_6	-33.6	$-0.15\gamma_1$ *
mobilities in units of $10^{-10}\text{m}^2/(\text{Vs})$	$\sqrt{\mu_\perp^+ \mu_\perp^-}$	1.6	$0.4 + 0.07(T - 30°\text{C})/(30°\text{C})$
recombination rate (1/sec)	τ_{rec}^{-1}	$\approx 0.2^{\&\&}$	$\approx 0.1^{\&\&}$

* Value fitted to the experimental threshold and roll-angle curves
** If not given in terms of γ_1 or η
† $\alpha_2 = \alpha_3 - \gamma_1$ $\quad ^{\dagger\dagger}$ Onsager relation $\alpha_5 = \alpha_6 - \alpha_2 - \alpha_3$
$^\&$ Left value for 30°C; right value for 60°C, see Table A.2.
$^{\&\&}$ Fits of the WEM predictions to experiments

As decribed in detail in Chapter 5, the mobility parameter is fitted to the measured Hopf frequencies as function of the external frequency (in I 52 for each temperature). The recombination parameter was estimated in Chapter 6.

The rotational viscosity γ_1, the bulk viscosity $\eta = \alpha_4/2$, and $\epsilon_\perp$ where measured for I52 in Ref. [149] as function of the temperature. Interpolations to the temperatures relevant in this work are given in Table A.2 below. The conductivity as function of the temperature was measured in Ref. [30]. The anisotropy of the conductivity was fitted, for each temperature, to the measured values of $\overline{V}_c$ in the limit of small external frequencies. All viscosities with the exception of the isotropic $\alpha_4 = 2\eta$ are assumed to have the same temperature dependence as γ_1. The prefactors, in units of γ_1, were determined to fit the threshold and roll angles.

Table A.2 Temperature dependence of the material parameters of I 52

Temperature (°C)	30	35	40	45	50	60
$\epsilon_\perp(\epsilon_0)$	3.01	2.99	2.98	2.96	2.94	2.90
$\gamma_1(10^{-3}\frac{Ns}{m^2})$	207	163	130	105	87	65
$\eta = \alpha_4/2(10^{-3}\frac{Ns}{m^2})$	19.8	15.8	12.8	10.7	9.2	7.7
$\sigma_\perp(10^{-8}(\Omega\text{m})^{-1})$	0.28	0.37	0.49	0.65	0.85	1.41
$\sigma_a/\sigma_\perp$	0.26	0.3	0.34	0.38	0.42	0.45

A.2 Linearization of the WEM equations for nonzero diffusivities and with respect to a nontrivial basic state

The WEM equations (3.18) and (3.19) are expressed in terms of α and D instead of $\tilde{\alpha}$ and $\tilde{D}$, where α and D are defined in the Eqs. (4.4) and (4.5). The resulting equations are linearized around a nontrivial basic state of the functional form given by Eq. (4.1). Denoting the z derivatives of the fields of the basic state with a prime ($\delta\rho_0 = \delta\phi''$), the result for the Fourier modes with wavevector $\boldsymbol{q} = (q, p)$ is given by

$$\begin{aligned}
P_1\partial_t(\hat{\epsilon}_q\overline{\phi} \;&+\; \epsilon_a E_0 iq\overline{n}_z) \\
&= \left[-\sigma_0\hat{\sigma}_q + \sigma_0'\partial_z - 2Ds_1\hat{\sigma}_q\hat{\epsilon}_q\right]\overline{\phi} - \left[E_0\partial_z + \delta\rho_0 + \alpha^{-1}Dd_1\hat{\sigma}_q\right]\overline{\sigma} \qquad \text{(A.1)} \\
&+ \left\{-\sigma_a\sigma_0E_0 + D\left[\alpha^{-1}d_1\sigma_a\sigma_0' + 2s_1(\sigma_a\delta\rho_0' - \hat{\sigma}_q\epsilon_aE_0)\right]\right\} iq\overline{n}_z - P_1\delta\rho_0'\overline{v}_z, \\
P_1\partial_t\overline{\sigma} = \;&\alpha\left\{-\alpha s_1\left[(E_0\hat{\epsilon}_q - \delta\rho_0')\partial_z + \delta\rho_0(\hat{\sigma}_q + \hat{\epsilon}_q + 2r\hat{\epsilon}_q)\right]\right. \\
&- d_1\left[\sigma_0\hat{\sigma}_q - \sigma_0'\partial_z + Ds_1\hat{\sigma}_q\hat{\epsilon}_q - r\sigma_0\hat{\epsilon}_q\right]\left.\right\}\overline{\phi} \\
&- \left[\alpha d_1(E_0\partial_z + \delta\rho_0(1-r)) + Ds_2\hat{\sigma}_q + 2r\sigma_0\right]\overline{\sigma} \\
&+ \left\{\alpha d_1\left[-\sigma_0\sigma_aE_0 + Ds_1(\sigma_a\delta\rho_0' - \hat{\sigma}_q\epsilon_aE_0) + r\sigma_0\epsilon_aE_0\right]\right. \qquad \text{(A.2)} \\
&+ \alpha^2s_1E_0\left[-\delta\rho_0\sigma_a - \epsilon_a(2\delta\rho_0(1+r) + E_0\partial_z)\right] + Ds_2\sigma_a\sigma_0'\left.\right\} iq\overline{n}_z - P_1\sigma_0'\overline{v}_z,
\end{aligned}$$

with the (homogeneous) BCs

$$[E_0 - \alpha^{-1} D d_1 \partial_z]\overline{\sigma} + [\sigma_0 - 2Ds_1\partial_z^2]\partial_z\overline{\phi} \;=\; 0, \tag{A.3}$$
$$[D\partial_z - \alpha E_0 d_1]\overline{\sigma} - \alpha[d_1\sigma_0 + 2\alpha s_1(\delta\rho_0 + E_0\partial_z)]\partial_z\overline{\phi} \;=\; 0. \tag{A.4}$$

The operators $\hat{\epsilon}_q$ and $\hat{\sigma}_q$ are given by Eq. (5.10). The linearized director and momentum-balance equations are given by the Eqs. (5.5) – (5.8) with the volume force $E_0\hat{\epsilon}_q\overline{\phi}$ in Eq. (5.8) replaced by $(\sqrt{2R}\cos\omega_0 t - \delta\phi_0')\hat{\epsilon}_q\overline{\phi} - \delta\rho_0\phi_0'$. The electric field is given by $E_0 = \sqrt{2R}\cos\omega_0 t - \delta\phi_0'$.

A.3 The 3 × 3 eigenvalue system of the one-mode approximation of the linearized WEM equations

Inserting the adiabatically eliminated charge density (5.20), and the velocities (5.18) and (5.19), into the Galerkin projection of the linearized WEM equations (5.4), (5.5), and (5.6) with v_x replaced by $(i\partial_z v_z - pv_y)/q$, leads to following eigenvalue system for modes $\propto e^{\lambda t}$,

$$(\lambda - \lambda_\sigma)\sigma^{(0)} + \tilde{\alpha}^2 R\sigma_a^{(\text{eff})}\, iqn_z^{(0)} \;=\; 0, \tag{A.5}$$
$$-\frac{RC_z^2}{\sigma_a^{(\text{eff})}(1+\omega'^2)}\sigma^{(0)} + (\lambda - \lambda_z)iqn_z^{(0)} + \frac{p}{q}\left(m_{zy}\partial_t - \lambda_{0z}\frac{k_{zy}}{K_{yy}}\right)qn_y^{(0)} \;=\; 0, \tag{A.6}$$
$$\frac{p}{q}\left[\frac{RC_y^2}{\sigma_a^{(\text{eff})}(1+\omega'^2)}\right]\sigma^{(0)} + \frac{p}{q}\left[m_{yz}\partial_t - \lambda_{0y}\left(\frac{k_{zy}}{K_{yy}} + \frac{R}{R_{0y}}\right)\right] iqn_z^{(0)} + (\lambda - \lambda_{0y})qn_y^{(0)} \;=\; 0, \tag{A.7}$$

where

$$m_{zy} = -\frac{\lambda_{0z}}{K_{zz}}\left(\frac{a_2 a_2' \eta_{zy}}{\eta_{yy}\eta_z^{(\text{eff})}} + \frac{\alpha_2\alpha_3 I_2}{\eta_{yy} q}\right), \quad m_{yz} = \frac{\lambda_0 y}{\lambda_{0z}}\frac{K_{zz}}{K_{yy}} m_{zy}. \tag{A.8}$$

The growth rate λ_z of the SM for normal rolls is given by

$$\lambda_z = \lambda_{0z}\left(1 - \frac{R}{R_{0z}}\right), \tag{A.9}$$

where R_{0z} is given by Eq. (5.21) for $p = 0$.

The parameter R_{0y} is given by

$$R_{0y} = \frac{K_{yy}\eta_{yy}\eta_z^{(\text{eff})}}{a_2'\sigma_a^{(\text{eff})}\eta_{zy}}. \tag{A.10}$$