Family name:	Forename:	Matrikel number:

Examination for the Master's Course Methods of Econometrics winter semester 2023/24

Problem 1 (45 points)

In a survey, the interviewees [die Befragten] are asked under which circumstances they would chose public transport (PT, Alternative 1) or the available car (Alternative 2) for a longer trip where walking or cycling is not feasible, given following conditions

Person	1	2	3	4	5	6	7	8	9	10
Cost PT [€]	2.00	2.00	2.00	0.00	0.00	2.00	2.00	2.00	2.00	0.00
Cost car [€]	2.00	2.00	2.00	2.00	2.00	3.00	2.00	2.00	3.00	2.00
Time PT [min]	30	40	50	40	50	60	30	40	50	40
Time car [min]	30	30	30	30	30	30	30	30	30	25
Daytime	day	night	day	night	day	night	day	night	night	night
Gender	m	m	m	m	m	m	f	f	f	f
Decision	PT	PT	car	PT	PT	car	PT	car	PT	car

(a) State and justify if this is a stated or revealed preference survey.

(b) Classify the exogeneous variables of the data into characteristics, socioeconomic variables, and external variables.

(c) The data is analyzed with a binomial Logit model with following specification:

$$V_i = \beta_1 \delta_{i1} + \beta_2 C_i + \beta_3 T_i + \beta_4 D \delta_{i1} + \beta_5 G D \delta_{i1},$$

where $D = 0$ ($D = 1$) stands for day (night), and $G = 0$ ($G = 1$) for men (women). Are the characteristics modelled in a generic or alternative-specific way?

(d) Give the meaning and the expected sign (if applicable) of the four parameters β_2 to β_5.

(e) The parameters are estimated as follows (expectation $\pm$ standard deviation):

$\hat{\beta}_1 = 3.12 \pm 2.49$, $\hat{\beta}_2 = -1.08 \pm 1.10$, $\hat{\beta}_3 = -0.199 \pm 0.134$, $\hat{\beta}_4 = 0.39 \pm 2.33$, $\hat{\beta}_5 = -2.42 \pm 2.41$.

Give, for the first person, the modelled probability for chosing public transport.

(f) Calculate the realized propery sums for β_1, β_2, β_4, and β_5 (NOT β_3!) and give their values for the model with the parameter vector $\hat{\boldsymbol{\beta}} = \mathbf{0}$.

(g) Give the value of time [€/h] as implied from the estimated parameters. How many more Euros are women willing to pay to drive a car instead of using public transport in the night, compared to men?

(h) A simplified model is specified as $V_i = \beta_1 \delta_{i1} + \beta_2 C_i + \beta_3 T_i$ and, after calibration, has a maximum log-likelihood $\tilde{L}_{\text{restr}} = -6.27$ while the original model has $\tilde{L} = -4.26$. Test the null hypothesis H_0 that the simplified model explains the data as well as the full model (Likelihood-ratio test, $\alpha = 5\,\%$). What could be the reason that H_0 cannot be rejected although the factor of parameter β_5 has a high effect strength?

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Problem 2 (45 points)

In order to develop a car navigation for the most eco-friendly route minimizing the energy (or, equivalently the CO_2 emissions), following model estimates the energy y needed to traverse a road segment of length L as a function of the elevation difference h (positive if uphill), travel time T (including standing times), average speed v, and speed variance ϑ:

$$y = \beta_1 L + \beta_2 T + \beta_3 h + \beta_4 L v^2 + \beta_5 L\vartheta + \epsilon, \quad \epsilon \sim \text{ i.i.d. } N(0, \sigma^2).$$

Notice that this model assumes that the car also needs energy/emits CO_2 if it is standing.

(a) Argue in the context that there is no need for an intercept β_0.

(b) Give the meaning and the expected sign of the parameters.

Hints: According to basic physics, the energy is equal to power times T = resistance force times $L+$ gravitational force times h. Furthermore, assume that the resistance force is composed of a constant friction force and the wind drag $\propto v^2$.

(c) If lengths are measured in m, speeds in m/s, and times in s, an ordinary least-squares calibration on 25 data points gives the estimates (expectation $\pm$ estimated standard deviation)

$$\hat\beta_1 = 210 \pm 50, \quad \hat\beta_2 = 2\,100 \pm 700, \quad \hat\beta_3 = 16\,000 \pm 2\,000, \quad \hat\beta_4 = 0.4 \pm 0.1, \quad \hat\beta_5 = 0.5 \pm 0.3.$$

Give the needed energy to traverse a road segment of 1 km length going 30 m uphill (gradient 3 %) in 40 s at steady driving ($\vartheta = 0$).

hint: The speed is given by the length and the traversing time; the energy is given in Ws (Watt-seconds) but you do not need to care, here.

(d) Argue that this model would break down (at least for gasoline and Diesel cars) for the situation in (c) with a downhill slope of -6 % instead of 3 % uphill.

hint: Just calculate the energy.

(e) The speed is now given in km/h instead of m/s (the time still in s and the road length still in m). Which parameter values β_j will change? by which factor?

(f) Why it is a bad idea to add an additional factor $\beta_6 v T$?

(g) Test the null hypotheses $H_{01} : \beta_2 = 0$ and $H_{02} : \beta_2 < 1\,000$ for $\alpha = 5\,\%$.

(h) The speed variance ϑ depends on the traffic situation which, of course, is not known for an offline navigation device. Therefore, one aggregates the last two factors to a single new one,

$$\beta_4 L v^2 + \beta_5 L\vartheta = \beta_4' L v^2 \quad \Rightarrow \beta_4' = \beta_4 + \beta_5 \frac{\vartheta}{v^2} = \beta_4 + \gamma\beta_5$$

where the average squared speed variation coefficient γ is estimated from the data to be $\gamma = 0.4$ (no errors assumed). Give the expectation and standard deviation of the parameter β_4' of the simplified offline model for a covariance $\hat V_{45} = -0.006$.

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Problem 3 (30 points)

Consider the CO_2 footprint of an e-bike with following main components:

- Iron/steel (motor, chain, gear shift, etc): 7 kg (CO_2-neutral recycling: 30 %)
- Aluminum (frame, wheels): 12 kg (recycling 50 %)
- Battery: 5 kg (recycling 0 %)
- Other materials (rubber, plastic etc): 1 kg (recycling 20 %).

The bike is used for 10 years at 3 000 km/Year before it is replaced. After 5 years, it needs a new battery. Other repairs/maintenance only give insignificant contributions. When driving, the e-bike needs 1 kWh electricity per 15 km which is taken from the German grid at a carbon intensity of 400 g/kWh.

(a) Calculate the combined material and energy repository y^{s} for all three life phases.

(b) The line of the emission factor matrix (iron, aluminum, battery, others, energy) for the pollutant CO_2 reads

$$\boldsymbol{C}' = (2, 25, 30, 1, 400\,\mathrm{g/kWh}).$$

Calculate the total CO_2 emissions $e_{\mathrm{CO_2}} = e_1$ for making, driving, and recycling the e-bike.

(c) How does the overall carbon footprint change for following scenarios (only change what is indicated keeping the rest as in the standard scenario):

(i) Riding the e-bike in France at a carbon intensity of only 50 g/kWh instead of 400 g/kWh,

(ii) using a smaller battery with half the range and half the weight or using novel technologies where the (5 kg) battery lasts for the whole lifetime,

(iii) using a smaller fraction of electrical support (you must always tread the pedals providing a minimum power by yourself) such that 0.1 kWh last for 30 km instead of 15 km.

(d) Calculate the CO_2 footprint when using a conventional bicycle (2 kg steel, 8 kg aluminum, 1 kg rubber/plastic, same recycling rates as above). Because of the increased physical activity, you additionally burn 4 g carbohydrates of food per kilometer. Typically, making food containing 1 kg of carbohydrates emits 3.5 kg CO_2 and burning it inside the body an additional 2.5 kg.

Quantiles $z_p = \Phi^{-1}(p)$ of the standardnormal distribution $\Phi(z)$

$p = 0.60$	0.70	0.80	0.90	0.95	0.975	0.990	0.995	0.999	0.9995
0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Quantiles $t_{n,p}$ of the Student t distribution with n degrees of freedom

n	$p = 0.60$	0.70	0.80	0.90	0.95	0.975	0.990	0.995	0.999	0.9995
1	0.325	0.727	1.376	3.078	6.315	12.706	31.821	63.657	318.31	636.62
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.153	3.707	5.208	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.154	4.587
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850
30	0.256	0.530	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Quantiles $\chi^2_{n,p}$ of the χ^2 distribution with n degrees of freedom

n	$p = 0.9900$	0.9750	0.9500	0.9000	0.8000	0.5000	0.2000	0.1000	0.05000
1	6.635	5.034	3.821	2.706	1.656	0.4589	0.06540	0.01638	0.004230
2	9.210	7.378	5.991	4.605	3.219	1.386	0.4463	0.2107	0.1026
3	11.34	9.348	7.815	6.251	4.642	2.366	1.005	0.5843	0.3518
4	13.28	11.15	9.488	7.779	5.989	3.357	1.649	1.064	0.7106
5	15.09	12.83	11.07	9.236	7.289	4.351	2.343	1.610	1.155