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Examination for the Master's Course Methods of Econometrics winter semester 2019/20

Problem 1 (30 points)

Consider a national economy with the sectors

- 1: Car manufacturers, total output in a given time: $x_1 = 1$ (e.g., in units of 1 million €),
- 2: Suppliers of the car manufacturers: $x_2 = 0.8$,
- 3: All the rest of industries and services: $x_3 = 10$.

Furthermore, the IO coefficient matrix of the input-output model of Leontief is given by

$$\mathsf{A} = \begin{pmatrix} 0.05 & 0.10 & 0.01 \\ 0.70 & 0.0625 & ? \\ 0.10 & 0.40 & 0.30 \end{pmatrix}.$$

(a) Explain briefly the variables of and the assumptions behind Leontief's model $\boldsymbol{x} = \boldsymbol{y} + \mathsf{A}\,\boldsymbol{x}$ *in the context of this problem.* Particularly, argue that the high value $A_{21} = 0.7$ is plausible.

(b) The suppliers of the car manufacturers deliver their products mainly to the car manufacturers and also some to the rest of the industry, but nothing to the end consumer, $y_2 = 0$. Calculate the missing coefficient A_{23} from this condition.

(c) Calculate the flows of products/services $\boldsymbol{y}$ to the end consumers assuming $A_{23} = 0.005$.

(d) Suddenly, the consumer's demand for new vehicles drops by 10 %. Give the new total outputs $\boldsymbol{x}$ of all sectors and the percentaged changes $\Delta x_i / x_i$ once a new steady state has been reached.

Hint: Use the "old" demand vector $\boldsymbol{y} = (0.77,\, 0,\, 6.58)^{\mathrm{T}}$ and the already calculated matrix of the final demand

$$\mathsf{B} = (\mathsf{I} - \mathsf{A})^{-1} = \begin{pmatrix} 1.15 & 0.13 & 0.0174 \\ 0.862 & 1.17 & 0.0207 \\ 0.657 & 0.686 & 1.44 \end{pmatrix}.$$

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Problem 2 (45 points)

In order to assess the decision making at a busy pedestrians crossing, a traffic analyst records the number of pedestrians accepting or rejecting a certain gap between two consecutive vehicles. "Accepting" means that the pedestrian in question uses the gap to cross the street while "rejecting" the gap means that he or she waits for the next opportunity. Besides the time gap, the speed and the type of the approaching vehicle have been recorded

Gap index n	1	2	3	4	5	6	7	8	9	10	11	12
Time gap T [s]	7	5	9	5	4	1	5	4	5	2	6	8
speed v [km/h]	50	15	30	40	15	50	60	20	30	40	60	45
Vehicle type	truck	bike	car	car	bike	car	car	bike	car	car	car	car
Gap accepted ($i=1$)	1	3	3	1	1	0	0	1	1	0	1	2
Gap rejected ($i=2$)	2	0	1	2	1	1	2	2	1	2	2	0

(a) Give reasons why all factors in the deterministic utilities must be formulated in an alternative-specific way. Why can we set the utility for rejected gaps ("waiting") to $V_2 = 0$?

(b) The decisions are now analyzed by a binary Logit model with following utility functions for accepted ($i = 1$) and rejected ($i = 2$) gaps:

$$V_1 = \beta_1 + \beta_2 T + \beta_3 v + \beta_4 \begin{cases} 1 & \text{bike approaching} \\ 0 & \text{otherwise} \end{cases} + \beta_5 \begin{cases} 1 & \text{truck approaching} \\ 0 & \text{otherwise,} \end{cases}$$

$$V_2 = 0.$$

Explain the meaning of parameters β_2 to β_5 and the expected signs.

(c) Give the realized property sums $X_m = \sum_n \sum_i x_{mni} y_{ni}$ for the parameters β_1 ($m = 1$), β_4, β_5, and the corresponding expected property sums for the model with $\hat{\boldsymbol{\beta}} = \mathbf{0}$. (Do not calculate X_2 and X_3!) Why $\hat{\boldsymbol{\beta}} = \mathbf{0}$ is not a ML estimator for the given data?

(d) The ML estimates parameters are now given as (value $\pm$ standard deviation $\sqrt{V_{jj}}$)

$$\hat{\beta}_1 = -3.20 \pm 3.42, \quad \hat{\beta}_2 = 0.747 \pm 0.369, \quad \hat{\beta}_3 = -0.0369 \pm 0.0508,$$
$$\hat{\beta}_4 = 1.09 \pm 1.86, \quad \hat{\beta}_5 = -0.88 \pm 1.46.$$

Calculate the modelled probability that a person accepts the first gap and compare with the actual percentage.

(e) Show that the time gap, but not the speed, is a relevant factor (test the associated parameter against the null hypothesis of zero at an error probability $\alpha = 5\,\%$).

(f) Argue that the utility difference for gap acceptance between an approaching bike and a truck is given by $\hat{\beta}_4 - \hat{\beta}_5$. Test if this is significant.

Hints: you need to introduce a new random variable $\hat{\gamma} = \hat{\beta}_4 - \hat{\beta}_5$ and evaluate the test statistics $T = \hat{\gamma}/\sqrt{V(\hat{\gamma})} \sim N(0,1)$ (asymptotically) if H_0. To calculate $V(\hat{\gamma})$, apply the variance formula for a sum of correlated random variables using the covariance $\text{Cov}(\hat{\beta}_4, \hat{\beta}_5) = -0.355$ of the estimation errors.

(g) Most parameter estimates have no significance although the associated factors obviously matter. Explain why.

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Problem 3 (45 points)

Many cyclists nowadays track their rides using sports watches or smartphone apps. They record, for each section i, the speed v_i, the acceleration a_i, and the gradient α_i (one percent uphill equals $\alpha_i = 0.01$). Additionally, the user has to tell the device his or her total weight m_i (including that of the bike). Among others, these devices calculate the mechanical power y_i in Watts [W]. In order to find out how the calculation works, a cyclist tests the model

$$y_i = \beta_0 + \beta_1 v_i^3 + \beta_2 m_i v_i + \beta_3 m_i \alpha_i v_i + \beta_4 m_i a_i v_i + \epsilon_i, \quad \epsilon_i \sim \text{ i.i.d. } N(0, \sigma^2)$$

using following data:

Person/section	1	2	3	4	5	6	7	8	9	10
Speed [m/s]	8	9	2.8	11.5	6	4.5	2.5	6	12.5	11
Acceleration [m/s^2]	0	0	0	0.2	0.4	0	-0.3	0	0	0
Gradient [%]	0	0	5	-3	0	2	6	0	-3	0
Mass [kg]	78	78	78	69	69	69	92	92	92	92
Power [W]	134	166	135	212	240	100	89	87	35	280

(a) Does the factor $\beta_1 v_i^3$ result from the wind drag, the rolling resistance, the uphill gradient, or the acceleration? *Hint:* Consider the variables of each of the contributions.

(b) Consider the compact formulation

$$\boldsymbol{y} = \mathbf{X}\,\boldsymbol{\beta} + \boldsymbol{\epsilon}, \qquad \text{Cov}(\boldsymbol{\epsilon}) = \sigma^2 \mathbf{1}\,.$$

Give the dimensions (rows $\times$ columns) of the objects $\boldsymbol{y}$, $\mathbf{X}$, $\boldsymbol{\beta}$, $\boldsymbol{\epsilon}$, and $\text{Cov}(\boldsymbol{\epsilon})$.

(c) Give the first line (row) of the system matrix $\mathbf{X}$ (relating to the first person/section).

(d) A standing cyclists does not produce any mechanical power, $y = 0$ if $v_i = 0$. Furthermore, elementary physics tells us that the power needed for going uphill is given by $P_i^{\text{up}} = m_i g \alpha_i v_i$ with $g = 9.81\,\text{m/s}^2$, and the power to overcome inertia when accelerating is given by $P_i^{\text{acc}} = m_i a_i v_i$. What does this imply for the expected values of β_0, β_3, and β_4?

(e) The OLS estimation gives the following results (e $-$ 05 means $*10^{-5}$ and so on):

$$\hat{\boldsymbol{\beta}} = \begin{pmatrix} 8.14 \\ 0.136 \\ 0.0876 \\ 9.78 \\ 1.02 \end{pmatrix}, \quad \hat{\mathbf{V}}(\hat{\boldsymbol{\beta}}) = \begin{pmatrix} 21.5 & 0.0131 & -0.0476 & -0.321 & -0.0368 \\ 0.0131 & 5.7\text{e}-05 & -7.9\text{e}-05 & 0.0010 & -9.7\text{e}-07 \\ -0.0476 & -7.9\text{e}-05 & 0.00016 & -0.00043 & 5.3\text{e}-05 \\ -0.321 & 0.0010 & -0.00043 & 0.0447 & 0.00165 \\ -0.0368 & -9.7\text{e}-07 & 5.3\text{e}-05 & 0.0016 & 0.000396 \end{pmatrix}$$

Give the expected power for the first person/section. Which power this person would need on the same section when driving at 10 m/s (36 km/h) instead of 8 m/s (28.8 km/h)?

(f) Give the estimated standard deviations of $\hat{\beta}_1$ and $\hat{\beta}_2$ and test whether these parameters are significant at $\alpha = 5\,\%$. Also test whether the null hypotheses $\beta_0 = 0$, $\beta_3 = g$ and $\beta_4 = 1$ can be rejected at the same error probability. *Hint:* The indices of the parameter vector and the variance-covariance matrix start at zero!

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(g) According to a physical model, the power to overcome the rolling resistance is given by $P_i^{\text{roll}} = \mu m_i g v_i$ with the dimensionless friction coefficient μ. Give the friction coefficient that is implied by the parameter estimate.

(h) A reduced model with only two parameters is given by

$$y_i = \beta_1 v_i^3 + \beta_2 m_i v_i + m_i(g\alpha_i + a_i)v_i.$$

How would you test the null hypothesis that this model describes the data equally well as the full model? (you do not need to perform the actual test). Which outcome would you expect?

Tables

Quantiles $z_p = \Phi^{-1}(p)$ of the standardnormal distribution $\Phi(z)$

$p = 0.60$	0.70	0.80	0.90	0.95	0.975	0.990	0.995	0.999	0.9995
0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Quantiles $t_{n,p}$ of the Student t distribution with n degrees of freedom

n	$p = 0.60$	0.70	0.80	0.90	0.95	0.975	0.990	0.995	0.999	0.9995
1	0.325	0.727	1.376	3.078	6.315	12.706	31.821	63.657	318.31	636.62
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925	22.327	31.598
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.553	0.906	1.440	1.943	2.447	3.153	3.707	5.208	5.959
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169	4.154	4.587
∞	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291