Methods in Transportation Econometrics and Statistics (Master)

Winter semester 2023/24, Tutorial No. 11

Problem 11.1: Combined destination and mode choice: Nested-Logit Model

Following table gives the combined destination and mode choices for the last shopping trips of ten interviewed persons. There are two feasible destinations, a smaller corner store "Aunt Emma" ($l = 1$) and a discount shop ($l = 2$), and two feasible transport modes, public transport (PT, $m = 1$) and the own car ($m = 2$). Attributes influencing the decision are the travel times T_{lm} for the destination-mode combinations, and the "fill level" F of the fridge ($F = 0$: completely empty, $F = 1$: full).

Person	T [min] Emma, PT	T [min] Emma, car	T [min] discount PT	T [min] discount car	Fill-level F	y_{11}	y_{12}	y_{21}	y_{22}
1	25	15	25	20	0.9	1	2	0	0
2	25	30	40	30	0.8	3	0	0	1
3	20	20	30	30	0.7	2	1	1	1
4	25	10	25	10	0.6	0	3	0	2
5	15	5	30	20	0.5	1	2	0	2
6	15	15	25	20	0.4	1	1	0	1
7	15	20	45	45	0.3	3	1	0	1
8	15	15	15	15	0.2	1	0	2	3
9	25	15	40	30	0.1	1	1	0	1
10	25	10	25	20	0.0	0	1	1	3

(a) How many trips and (combined) decisions have been recorded for the last person? Also give the total number N of all (combined) decisions.

(b) Draw the decision tree for a suitable nested logit (NL) model with the destination as top-level and the mode as lower-level decisions. Which exogenous factors are related to which level?

(c) Within the nests $l = 1$ (Emma) und $l = 2$ (discount), the scaled deterministic utilities (the scale factors λ_l are unknown at this stage) are formally identical:

$$\tilde{V}_{1m}/\lambda_1 = \beta_1 T_{1m} + \beta_2 \delta_{m1}, \qquad (1)$$
$$\tilde{V}_{2m}/\lambda_2 = \beta_3 T_{2m} + \beta_4 \delta_{m1}. \qquad (2)$$

Give the meaning and expected signs (if applicable) of the four parameters β_1 to β_4. Also indicate the kind of modelling of travel times (generic or alternative-specific) and the reference alternative.

(d) The maximum-likelihood estimation (see the contour plots in the lecture set of slides) results in

$$\hat{\beta}_1 = -0.18, \quad \hat{\beta}_2 = +0.88, \quad \hat{\beta}_3 = -0.29, \quad \hat{\beta}_4 = -0.42.$$

Calculate, for the last person, the scaled utility functions and the conditional choice probabilities $P_m|l$. Compare the latter with the realized choices.

(e) Calculate, for the last person, the inclusion values I_l for the top-level decision. What do they mean?

(f) The deterministic top-level utility is now specified as

$$W_l = \beta_5 F \delta_{l1} + \beta_6 \delta_{l1} + \lambda_1 I_l \delta_{l1} + \lambda_2 I_l \delta_{l2}. \tag{3}$$

A maximum-likeliood calibration gives the estimators

$$\hat{\beta}_5 = 2.9, \quad \hat{\beta}_6 = -2.0, \quad \hat{\lambda}_1 = 0.17, \quad \hat{\lambda}_2 = 0.21.$$

Indicate the meaning of β_5 and β_6 and check their signs. Are the scale (or correlation) parameters λ_i within their valid range? Based on their values, make statements about the degree of correlation of the random utility of the combined decision within the respective nest. Is it possible to restrict the model to a single correlation parameter λ? If so, specify the corresponding model.

(g) Calculate, for the last person, the choice probabilities $P_l P_m|l$ for all 2×2 combined choices? Compare these with the realized percentages frequencies.

(h) Instead of an NL model, one could also use a “normal” MNL. Defining a mapping from the double index (l, m) to a single index i such that $i = 1, ..., 4$ denote the combinations $(l, m) = (1, 1)$, (1,2), (2,1), and (2,2), the determinstic MNL utilities read

$$V_i = \beta_1 T_i(\delta_{i1} + \delta_{i2}) + \beta_2 \delta_{i1} + \beta_3 T_i(\delta_{i3} + \delta_{i4}) + \beta_4 \delta_{i3} + \beta_5 F(\delta_{i1} + \delta_{i2}) + \beta_6(\delta_{i1} + \delta_{i2}). \tag{4}$$

Show that all parameters have the same meaning as in the NL model and that, for $\lambda_l = 1$, the NL reverts to this MNL. Why might it still be better to use the NL? Give a test for making a decision between the models (no actual calculations required).

(i) In view of a fridge full to the brim ($F = 1$), it does make sense to not go shopping at all. Generalize the original NL (3) by adding a third trivial (one-alternative) nest “stay at home”. Discuss the sign of the new sensitivity parameter regarding the fridge fill level.