Exam to the Lecture
Traffic Dynamics and Simulation
SS 2023
Solutions

Total 120 points

Problem 1 (40 points)

(a) In microscopic models, the main dynamical elements are the vehicles and drivers (driver-vehicle unit) while, in macroscopic models, this is aggregated to locally averaged, mostly continuous spatiotemporal quantities.

- Variables micro: speed v_i, gap to the leader s_i, acceleration $\dot{v}_i$ of vehicle i
- Macro: Local density $\rho(x,t)$, speed $V(x,t)$, flow $Q(x,t)$.

(b) Which class is better suited?

(i) *Determine if it is likely that construction work at a freeway will lead to traffic jams in the vacation season:* Macroscopic model since congestions and travel times are relevant which can well be described using macroscopic models.

(ii) *will more assisted or autonomous vehicles lead to more or less congestions?* Microscopic models are more suitable since they allow to explicitly model a varying penetration rate of autonomous vehicle among human-driven ones.

(iii) *creating responsive surrounding traffic in driving simulators*: Only micro because you need individual vehicles.

(iv) *influence of dyamic routing on congestions and traffic flow quality*: Macroscopic models are enough since both the cause is macroscopic (congestions, difference in travel times), and also the effect (a varying percentage of vehicle, i.e., a partial flow of vehicles uses the deviation).

(v) *will traffic jams lead to more or less fuel consumption? Does it depend on the kind of jam (homogeneous or traffic waves)?* Here, a modal fuel consumption model of individual cars (micro-simulation) is more suitable since only this class of consumption/emission models depend on the detailled driving mode (speed, acceleration profile).

(c) Model formulated as ...

- set of coupled ordinary differential equations: any time continuous car-following model such as the OVM or the IDM.
- iterated map, i.e., a mapping of the state at time t to a later time $t+\Delta t$. Either a time continuous car-following model together with explicit update rules, e.g., ballistic update for the speeds and positions, $v_i(t+\Delta t) = v_i(t) + f(s_i, v_i, \Delta v_i)\Delta t$ and $x_i(t+\Delta t) = \frac{1}{2}[x_i(t) + (v_i(t) + v_i(t+\Delta t)]\,\Delta t$, or Newell's car-following model which is directly formulated as an iterated map
- partial differential equation: Macroscopic flow model, e.g. the LWR model
- cellular automaton: Nagel-Schreckenberg model.

(d) To model different driving styles in microscopic models, one sets the parameters of suitable models accordingly (I gave both the general answer and the specific IDM parameters; either of both will give full marks).

- fast vs slow: change the desired speed parameter (nearly any micromodel has such a parameter); IDM: v_0,
- agile/responsive vs sluggish/unresponsive: Depending on the model, change the reaction time, the desired acceleration/braking deceleration, or both; IDM: a,
- aggressive vs relaxed: change the desired time gap or, if the model does not contain such a parameter, change the parameters affecting the fundamental diagram (gap vs speed) such that the time gap=space gap divided by speed changes; IDM: T, possibly s_0,
- anticipative vs not experienced: Change the response to the relative speed (approaching rate) and/or, if present, the multi-anticipation parameters (looking at the next-nearest vehicle etc); IDM: b.

Problem 2 (20 points)

For a parking duration (in hours) obeying a uniform (0,2) distribution, we have for the parking occupancy number x_n at the end of the one-hour interval n ending at n hours as a function of the inflow I_n during this hour and the inflow I_{n-1} in the previous interval:

$$X_n = \frac{3}{4} I_n + \frac{1}{4} I_{n-1}.$$

Motivation: Drivers arriving just at n hours are still here with 100 % probability, those arriving one hour earlier with 50 %probability, so on average 75 %. The drivers arriving in the previous period may also still be present, with 50 % for those arriving at the end of the previoius interval, and zero for those having arrived at the beginning, i.e., 2 hours ago., so, on average, 25 %.
Application assuming no occupation at 9:00 h:

Perios	9:00-10:00	10:00-11:00	11:00-12:00	12:00-13:00
Hourly arrival number	500	1 000	1 000	800
Occupancy	375	875	1 000	850

Occupancy at 11:30h: Since, during each one-hour interval, we have constant inflows and constant ouflows (this follows from the uniform parking duration), we just at 11:30h have the arithmetic average:

$$X(11:30\,\mathrm{h}) = (X_{11} + X_{12})/2 = 938\,\text{vehicles}.$$

Problem 3 (20 points)

(a) The driving resistance when driving at constant speed is given by

$$F(v) = \mu m g + \frac{1}{2} c_d \rho A v^2.$$

At $50\,\mathrm{km/h} = 50/3.6\,\mathrm{m/s}$ and $130\,\mathrm{km/h} = 130/3.6\,\mathrm{m/s}$, we obtain

$$F(50/3.6\,\mathrm{m/s}) = 286.5\,\mathrm{N}, \quad F(130/3.6\,\mathrm{m/s}) = 748.5\,\mathrm{N}.$$

The fuel consumption per distance is given by

$$C_x = c_{\mathrm{spec}} \left(\frac{W_{\mathrm{mech}} + P_0 t}{x} \right) = c_{\mathrm{spec}} \left(\frac{F x + P_0 t}{x} \right) = c_{\mathrm{spec}} \left(F + \frac{P_0}{v} \right).$$

Here, we do not need any maximum function since the driving resistance at constant speed and on level roads is always positive. Furthermore, the consumption per 100 km is just given by $C_{100} = C_x 100\,000\,\mathrm{m}$.

For the two speeds, we have

$$C_{100}(50/3.6\,\mathrm{m/s}) = 3.35\,\mathrm{l}/100\,\mathrm{km}, \quad C_{100}(130/3.6\,\mathrm{m/s}) = 6.25\,\mathrm{l}/100\,\mathrm{km}.$$

(b) As in Part (a), the needed output energy from the motor is given by

$$W_{100} = \left(F + \frac{P_0}{v} \right) \; 100\,000\,\mathrm{m}$$

With a motor efficiency $\eta_m = 0.9$ and a battery charging/discharging efficiency of $\eta_b = 0.9$, we need following energy stored in the battery to drive 100 km:[1]

$$W_{100,\mathrm{batt}} = \frac{W_{100}}{\eta_b \eta_m}$$

For the two speeds, we need

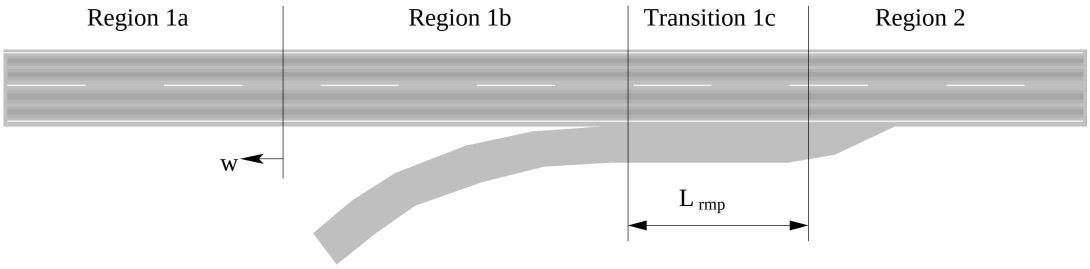
$$W_{100,\mathrm{batt}}(50/3.6\,\mathrm{m/s}) = 51.5e6\,\mathrm{Ws}, \quad W_{100,\mathrm{batt}}(130/3.6\,\mathrm{m/s}) = 103.1e6\,\mathrm{Ws},$$

or in kWh (division by 3.6e6):

$$W_{100,\mathrm{batt}}(50/3.6\,\mathrm{m/s}) = 14.3\,\mathrm{kWh}, \quad W_{100,\mathrm{batt}}(130/3.6\,\mathrm{m/s}) = 28.6\,\mathrm{kWh}.$$

Problem 4 (40 points)

Given is a two-lane freeway section with an onramp:



[1] At charging, another η_b will be in the denominator for the amount of electricity drawn from the charging station.

It is to be modelled with the LWR model using the fundamental diagram

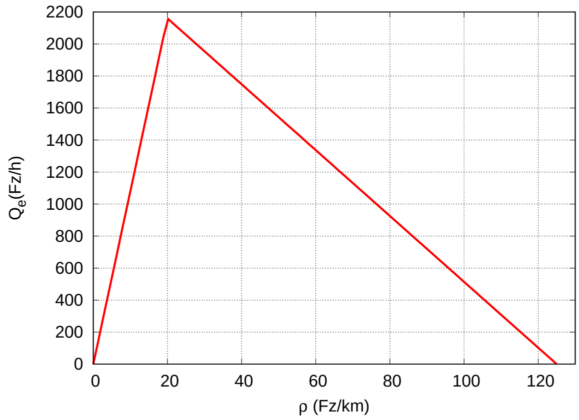
$$Q_e(\rho) = \min\left[V_0\rho, \frac{1}{T}\left(1 - l_{\text{eff}}\rho\right)\right].$$

with $l_{\text{eff}} = 8\,\text{m}$, $T = 1.4\,\text{s}$, and $v_0 = 108\,\text{km/h}$.

(a) The fundamental diagram is tridiagonal with the points (0,0), $(\rho_c, Q_{\max})$, and $\rho_{\max}, 0)$ with

$$\rho_c = \frac{1}{V_0 T + l_{\text{eff}}} = 20\,\text{km}, \quad Q_{\max} = V_0\rho_c = 2\,160\,\text{veh/h}, \quad \rho_{\max} = \frac{1}{l_{\text{eff}}} = 125\,\text{veh/km}$$

note: Watch out for the unit conversions!



(b) With $Q_{\max} = 2\,000\,\text{veh/h}$, the capacity of the main road is $2Q_{\max} = 4\,000\,\text{veh/h}$ which is greater than the mainroad and onramp demand combined. Thus, no breakdown will occur and the main road will be in the free part of the fundamental diagram (FD) with speed $V = 30\,\text{m/s}$ in all regions.

– Regions 1a and 1b:

$$\begin{aligned} Q_1^{\text{tot}} = Q_{\text{in}} &= 3\,000\,\text{veh/h}, \\ Q_1 = Q_{\text{in}}/2 &= 1\,500\,\text{veh/h}, \\ \rho_1^{\text{tot}} = Q_1^{\text{tot}}/V &= 27.78\,\text{veh/km}, \\ \rho_1 = \rho_1^{\text{tot}}/2 &= 13.89\,\text{veh/km} \end{aligned}$$

– Region 2:

$$\begin{aligned} Q_2^{\text{tot}} = Q_{\text{in}} + Q_{\text{on}} &= 3\,500\,\text{veh/h}, \\ Q_2 = Q_{\text{in}}/2 &= 1\,750\,\text{veh/h}, \\ \rho_2^{\text{tot}} = Q_2^{\text{tot}}/V &= 32.41\,\text{veh/km}, \\ \rho_2 = \rho_2^{\text{tot}}/2 &= 16.20\,\text{veh/km} \end{aligned}$$

(c) Because density and flow are extensive quantities, i.e., they are lane additive. In contrast, the speed is an intensive quantity which is not lane additive. Instead, the lane-averaged speed is just the flow-weighted average of the speed on each lane.

(d) With $Q_{\text{in}} = 4\,000\,\text{veh/h}$, the inflow alone brings the main road to its maximum capacity which, therefore, cannot accommodate the additional on-ramp vehicles, so, a breakdown will occur in the merging region. Since prior to that, traffic is in the free-flow branch of the FD with a common speed, so the information on the flow surge travels at 108 km/h=30 m/s. Thus, the surge reaches (the end of the) merging region at time

$$t_{\text{bd}} = 16{:}00 + \frac{9\,000\,\text{m}}{30\,\text{m/s}} = 16{:}05.$$

(e) In Region 1a, we have free flow with a total flow equal to the new demand Q_{in}:

$$\begin{aligned}
Q_{1a}^{\text{tot}} &= 4\,000\,\text{veh/h}, \\
Q_{1a} &= Q_{1a}^{\text{tot}}/2 = 2\,000\,\text{veh/h}, \\
\rho_{1a} &= Q_{1a}/V_0 = 18.5\,\text{veh/km}, \\
\rho_{1a}^{\text{tot}} &= 2\rho_{1a} = 37.0\,\text{veh/km} \\
V_{1a} &= Q_{1a}/\rho_{1a} = V_0 = 30\,\text{m/s}.
\end{aligned}$$

In Region 1b, we have a congested state (right-hand side of the FD) with a total flow given by the outflow capacity minus the ramp demand (assuming that all ramp vehicles can squeeze in):

$$\begin{aligned}
Q_{1b}^{\text{tot}} &= 2Q_{\max} - Q_{\text{on}} = 3\,820\,\text{veh/h}, \\
Q_{1b} &= Q_{1b}^{\text{tot}}/2 = 1\,910\,\text{veh/h}, \\
\rho_{1b} &= \rho^{\text{cong}}(Q_{1b}) = 32.2\,\text{veh/km} \\
\rho_{1b}^{\text{tot}} &= 2\rho_{1b} = 64.3\,\text{veh/km} \\
V_{1b} &= Q_{1b}/\rho_{1b} = 16.5\,\text{m/s}.
\end{aligned}$$

In Region 2, we have just the maximum flow state

$$\begin{aligned}
Q_{2}^{\text{tot}} &= 4\,320\,\text{veh/h}, \\
Q_{2} &= Q_{2}^{\text{tot}}/2 = 2\,160\,\text{veh/h}, \\
\rho_{2} &= Q_{2}/V_0 = 20\,\text{veh/km}, \\
\rho_{2}^{\text{tot}} &= 2\rho_{2} = 40\,\text{veh/km} \\
V_{2} &= Q_{2}/\rho_{2} = V_0 = 30\,\text{m/s}.
\end{aligned}$$

Watch out!

- First, determine whether a region is free or congested
- Then, calculate Q^{tot} to be equal to the demand (free) or the bottleneck capacity (congested)
- Then, get Q by dividing by the number of lanes
- Then, determine the (lane-averaged!) density by inverting the FD taking the free (congested) branch for the free (congested) regions
- Calculate the speed by the hydrodynamic relation $V = Q/\rho$ and the total density by multiplying ρ with the number of lanes
- In all calculations, watch out for unit conversions (particularly, use the SI system and the lane-averaged quantities for inverting the FD).

(f) Shock-wave formula:

$$c = \frac{Q_{1b} - Q_{1a}}{\rho_{1b} - \rho_{1a}} = -1.83\,\text{m/s} = -6.6\,\text{km/h}.$$