Lecture 11: Models for Pedestrian Flow

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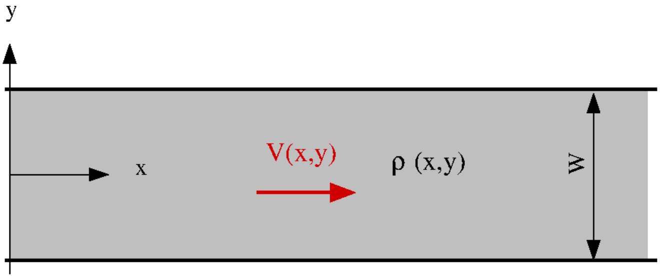
11.1 Macroscopic Models for Unidirectional Flow

- Basic difference to (lane-based) vehicular traffic: full *two-dimensionality*
- Can also be applied to disordered traffic of other *self-driven agents* such as non-lanebased traffic flow, cycling, Marathon runs, inline-skating, and crosscountry-ski events
- In contrast to microscopic pedestrian models such as the **Social-Force Model**, macroscopic pedestrian models are only suited for essentially unidirectional pedestrian flows
- Because of the two dimensions, we need to redefine density and introduce a new macroscopic quantity: **flow density**

11.1.1 Elementary Macroscopic Variables of Twodimensional Flow



- **2d density:** $\rho(x,y,t) = \rho(\boldsymbol{x},t)$ $[\text{pedestrians}/\text{m}^2$ or $\text{peds}/\text{m}^2]$,
- **Local velocity:** $\boldsymbol{V}(\boldsymbol{x},t) = (V, V_y)$ $[\text{m}/\text{s}]$.
- **Flow density:** $\boldsymbol{J}(\boldsymbol{x},t) = (J, J_y) = \rho \boldsymbol{V}$ $[\text{peds}/(\text{ms})]$
- **Effective 1d density** [peds/m]:

$$\rho_{1\text{d}}(x,t) = \int_{y=-W/2}^{W/2} \rho(x,y)\ \text{d}y \approx \rho W$$

- **Flow** [peds/s]

$$Q = \int_{y=-W/2}^{W/2} J(x,y)\ \text{d}y \approx \rho V W \approx \rho_{1\text{d}} V$$

Hydrodynamic relation and continuity equation

- hydrodynamic relations (vectorial and effective):

$$\boldsymbol{J} = \rho \boldsymbol{V}, \quad J = \rho V, \quad Q = \rho_{\mathrm{1d}} V = \rho W V$$

- Vectorial continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = 0$$

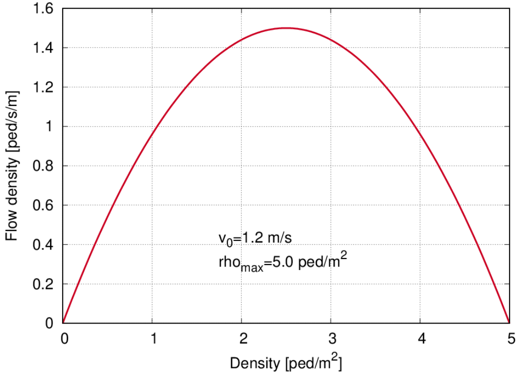
- Effective continuity equation:

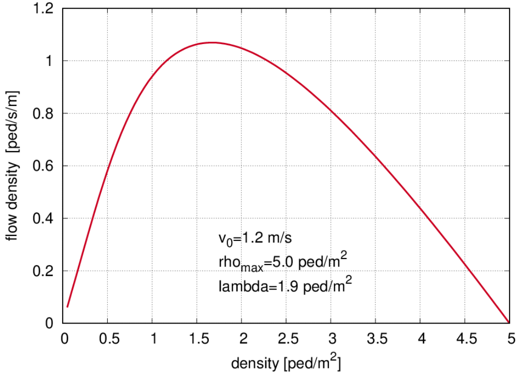
$$\frac{\partial \rho_{\mathrm{1d}}}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

important:

- In the effective continuity equation, lateral motion is introduced implictly: For example, in a stationary situation, $\frac{\partial}{\partial t} = 0$, we have $Q = \rho W V =$ const.. If $W(x)$ narrows funnel-like, the 2d-density ρ increases by concentric lateral motion

11.1.2 Fundamental Diagramm





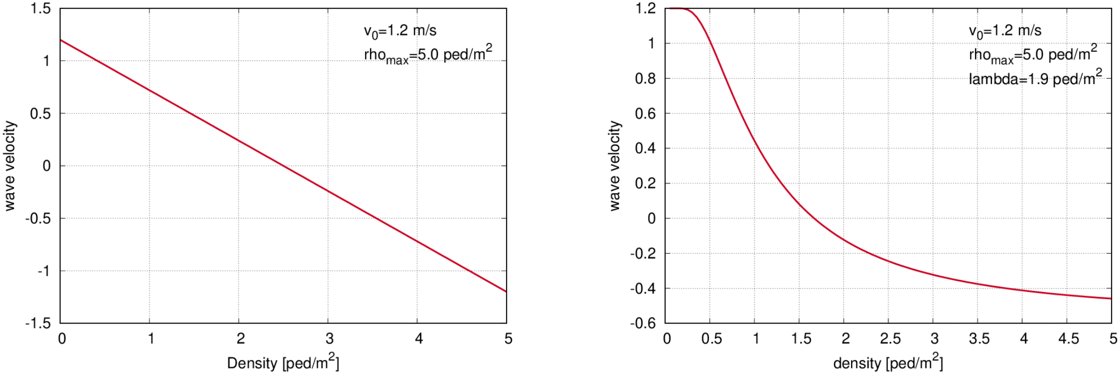
Parabolic FD a la Greenshields:

$$
\begin{aligned}
V_e(\rho) &= V_0\left(1-\frac{\rho}{\rho_{\text{max}}}\right), \\
J_e(\rho) &= V_0\rho\left(1-\frac{\rho}{\rho_{\text{max}}}\right)
\end{aligned}
$$

Weidmann FD:

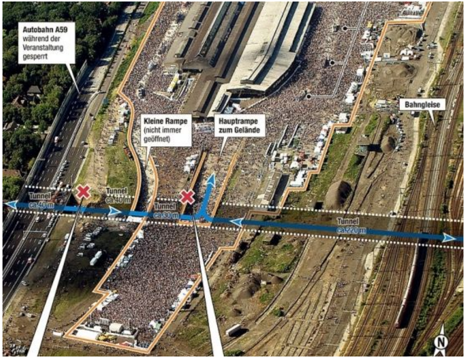
$$
\begin{aligned}
v_e(\rho) &= v_0\left\{1-\exp\left[-\lambda\left(\frac{1}{\rho}-\frac{1}{\rho_{\text{max}}}\right)\right]\right\} \\
J_e(\rho) &= \rho V_e(\rho)
\end{aligned}
$$

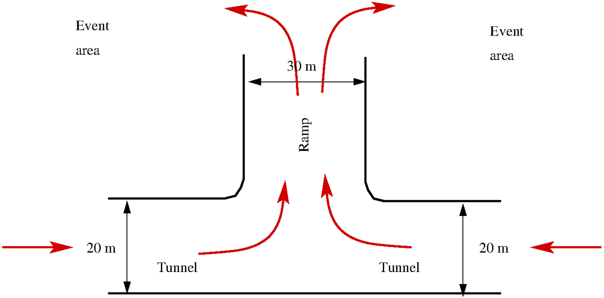
Wave velocities



- Wave velocity $w(\rho) = J'(\rho)$
- Notice: derivative with respect to 2d density. Double density means $1/\sqrt{2}$ times the average distance but more interacting people

11.1.3 Application I: Loveparade in Duisburg





Example for macroscopic event and evacuation planning:

- Assume unidirectional flow and a capacity density of $J_{\text{max}} = 1\,\text{ped/s/m}$ (a little bit lower than that of the Weidmann FD)
- Identify the bottleneck: The ramp to the event site at a width $W = 30\,\text{m}$ (the two tunnels have a summed cross section of $W = 40\,\text{m}$)
- Calculate the bottleneck strength assuming no further obstacles:

$$C = W J_{\text{max}} = 30\,\text{ped/m} = 108\,000\,\text{ped/h}$$

- Best case for three hours approach time (continuous unidirectional flow, no obstacles):

$$n = 324\,000\,\text{persons}$$

Application II: Mixed traffic

Application III: planning Marathon sports events

Planning of a starting scheme

- Define several starting groups i with a maximum number n_i of athletes, each, ordered to expected performance (best are first)
- Define a time delay τ between the starting shots of every group ("wave start"; the individual athlete's time starts when passing an electronic RFID gate)
- Identify the bottlenecks (often near the start) and estimate the flow profile from the expected speeds, their dispersion, and the start time delays
- Check if the maximum flow is below the bottleneck capacity; otherwise, change n_i and τ

Application IV: Large-scale pedestrian streams in Mekka

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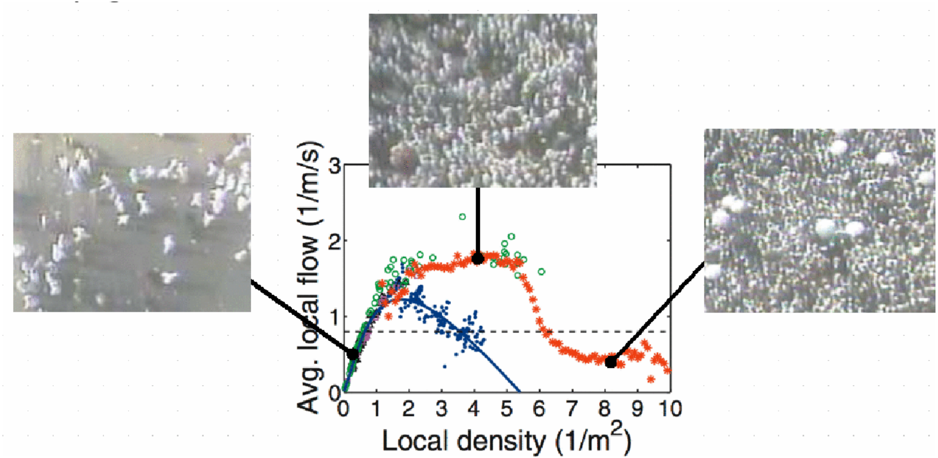


11.2 Microscopic Model I: Social-Force Model

- In contrast to macroscopic models, microscopic pedestrian models are suited to model any pedestrian motion, whether directed or not:
- Any pedestrian can have its individual destination
- The models of this class are fully twodimensional; all pedestrians can move anywhere inside allowed regions
- The first and most prominent representative is the **Social Force Model (SFM)**
- More generally, the SFM is a model for **self-driven particles** or **active particles**, sometimes also called **agents** (no stirring or shaking involved)
- In analogy to Newtonian forces, the SFM pedestrian is driven by **social forces**
- In extended models, additional physical forces are modelled in case of a direct contact. However, this makes the equations hard to solve because it entails **stiff differential equations** (involving several time scales)

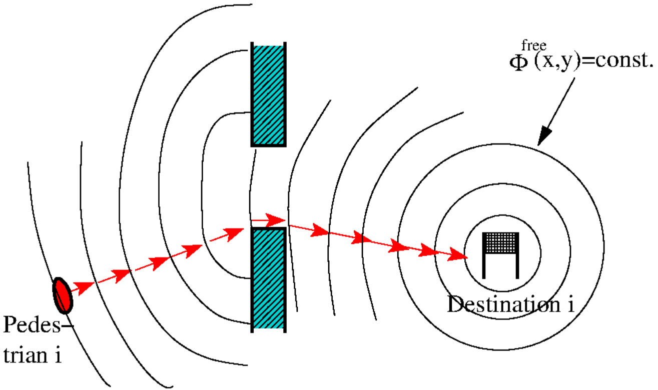
Overall specification of the Social-Force Model

Pedestrians are active particles of mass 1, i.e., force equals acceleration:

$$\dot{\boldsymbol{v}}_i \equiv \frac{\mathrm{d}\boldsymbol{v}_i}{\mathrm{d}t} = \boldsymbol{f}_i^{\text{free}} + \sum_{j \neq i} \boldsymbol{f}_{ij}^{\text{int}} + \sum_k \boldsymbol{f}_{ik}^{\text{walls}}.$$

- The **free force** $\boldsymbol{f}_i^{\text{free}}$ directs the pedestrians to their respective destinations. It can be modelled by a gradient of a **free potential**, e.g., indicating the shortest distance to the destination
- The pedestrian-pedestrian **interaction forces** $\boldsymbol{f}_{ij}^{\text{int}}$ acting from pedestrian j onto i are generally repulsive to avoid collisions and depends on the distance, directions, and the velocity vectors of both pedestrians
- The **wall forces** keep the pedestrians on the walkable area, i.e., away from the boundaries or from fixed compact obstacles. Both can be modelled as an **obstacle floor field**
- *The floor fields and free potentials are static scalar fields $\Phi(x, y)$ defined on the walkable area. They can be pre-calculated for all boundaries, obstacles, and possible destinations*

11.2.1 Free potential and force

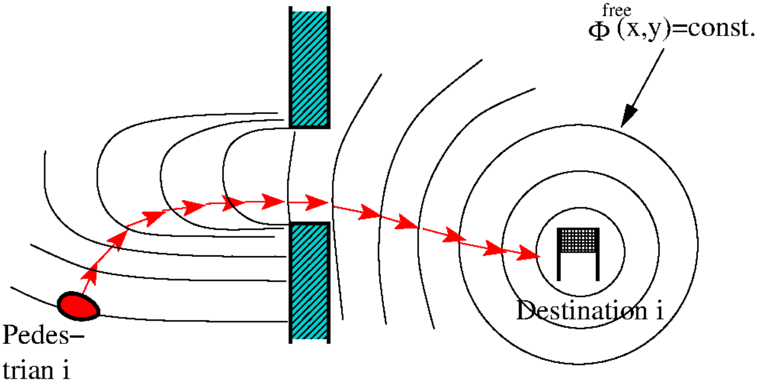


- Free force corresponds to a vectorial free-traffic OVM

$$\boldsymbol{f}_i^{\text{free}} = \frac{\boldsymbol{v}_{0i} - \boldsymbol{v}_i}{\tau}$$

- The desired velocity $\boldsymbol{v}_{0i} = v_0 \boldsymbol{\nabla} \Phi^{\text{free}}$ is given by the gradient $\boldsymbol{\nabla} = (\partial x, \partial y)$ of the free potential
- The free potential $\Phi^{\text{free}}(x, y)$ gives the distance to the destination on the shortest possible path
- Model parameters: **speed adaptation time τ** and **desired speed** (magnitude of $\boldsymbol{v}_{0i}$) Give plausible values for τ and v_0

Changing the pedestrian's preferences



- If the preference is to avoid "pancake" like crowding to leave doors etc, change the floor field to no longer reflect directly the shortest distance
- Here pedestrians prefer a queue rather than a "pancake"
- Rather than the gradient, just take v_0 times the unit vector of the gradient (Why was the gradient always a unit vector, in the last slide?)

11.2.2 Pedestrian-pedestrian interactions

Assumptions:

- The forces should be *repulsive*. ("Mexican-hat" potentials modelling attractive forces for couples, families, or friends will not be considered but can be added easily.)
- Generally, the forces should depend on the velocity vectors (speeds and directions) of both pedestrians. This also includes simple anticipation heuristics over anticipation time τ_a
- Unlike physical forces, there is no momentum conservation (*actio=reactio*). Instead, forces from objects in viewing direction are stronger than that the nearly vanishing ones on the back. This anisotropy is the basis for fundamental diagrams
  - ? Compare with car-following models
  - ? What would a fundamental diagram look like for interaction forces satisfying *actio=reactio*?

$\Rightarrow$ general structure for a force from pedestrian/vehicle/compact obstacle j onto pedestrian i:

$$\boldsymbol{f}_{ij} = \boldsymbol{f}_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j, \boldsymbol{v}_i, \boldsymbol{v}_j; \tau_a)$$

Further simplifying assumptions for the interactions

The general structure implies two functions $f_x(.)$ and $f_y(.)$ with eight dynamic arguments (the components of the two position and velocity vectors), each.

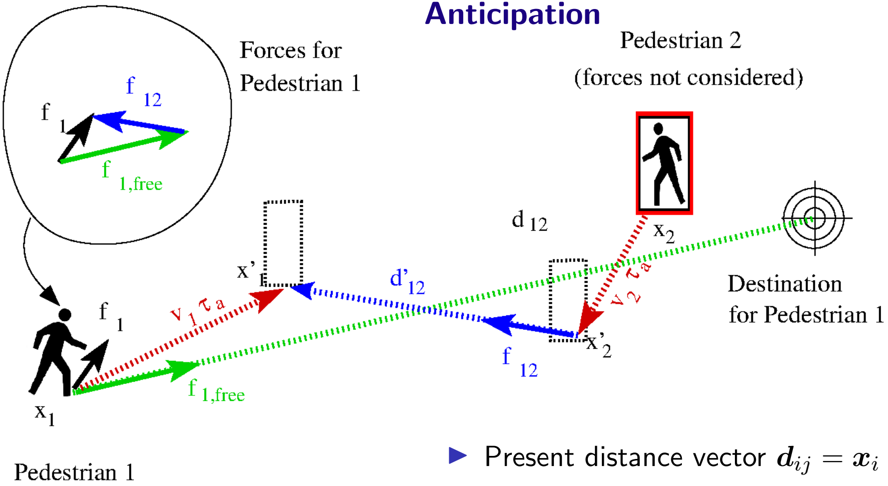
Rather complicated $\Rightarrow$ Simplify further

- Translational invariance: Dependence on the **distance vector $\boldsymbol{d}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$**, only, and not separately on $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ in difference vectors, the subject always comes first!
- Without the viewing angle dependency, the forces can be written as a gradient of an **interaction potential**, $\boldsymbol{f}_{\text{pot}} = \boldsymbol{\nabla}_{\boldsymbol{d}}\Phi^{\text{int}}(\boldsymbol{d}, \boldsymbol{v}_i - \boldsymbol{v}_j)$
- The potential/potential force is either Galilei invariant, i.e., depends on the **relative velocity vector $\boldsymbol{v}_i - \boldsymbol{v}_j$**, only (**Elliptical specification II**), or the velocity of the interacting pedestrian/object j is ignored (**Elliptical specification I**), or there is neither velocity dependence nor anticipation (**Circular specification**).
  (*in any case, the free part and the obstacles break this invariance on the system level*)
- The viewing angle dependency is just a multiplicative prefactor $w(\cos\phi)$ of the cosine of the viewing angle, $\cos\phi = -\boldsymbol{d}.\boldsymbol{v}_i/(|\boldsymbol{d}|\ |\boldsymbol{v}_i|)$

$\Rightarrow$ Force expression reduced to a scalar potential of only four dynamical variables:

$$\boldsymbol{f}_{ij} = w(\cos\phi)\boldsymbol{f}_{ij}^{\text{pot}}(\boldsymbol{d}, \boldsymbol{v}_i - \boldsymbol{v}_j) = -w(\cos\phi)\boldsymbol{\nabla}_{\boldsymbol{d}}\Phi^{\text{int}}(\boldsymbol{d}, \boldsymbol{v}_i - \boldsymbol{v}_j)$$

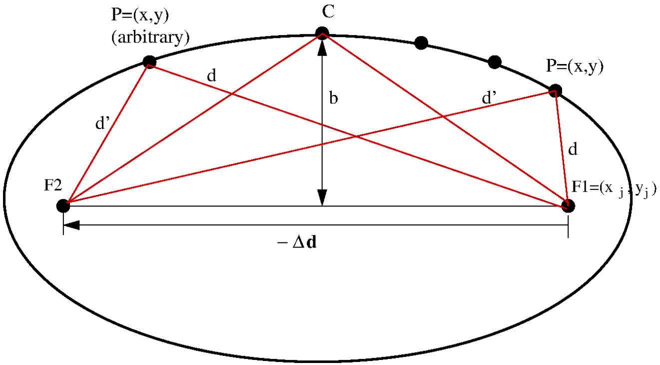
Anticipation



- Present distance vector $\boldsymbol{d}_{ij} = \boldsymbol{x}_i - \boldsymbol{x}_j$
- Anticipated distance change:
  - Elliptical specification II: $\Delta \boldsymbol{d}_{ij} = \tau_a(\boldsymbol{v}_i - \boldsymbol{v}_j)$
  - Elliptical specification I: $\Delta \boldsymbol{d}_{ij} = \tau_a \boldsymbol{v}_i$
  - Circular specification (no anticipation): $\Delta \boldsymbol{d}_{ij} = \boldsymbol{0}$
- Anticipated distance vector: $\boldsymbol{d}'_{ij} = \boldsymbol{d}_{ij} + \Delta \boldsymbol{d}_{ij}$

? Check, in which specification pedestrian i will pass pedestrian j to the left or right to avoid a collision

Constructing the interaction potential for the elliptical specifications



- Determine the present collision point $F_1 = \boldsymbol{x}_j$ for i and the *present* location $F_2 = \boldsymbol{x}_j - \Delta \boldsymbol{d}$ for an *anticipated* collision after τ_a
- Define ellipses by the focal points F1 and F2 and the present distance vector $\boldsymbol{d}$
- Equipotential lines have a constant **semi-minor axis** b
- Hammer two nails at F1 and F2 and attach the ends of a string of length $L > \overline{F_1 F_2}$ to the nails. Tighten the string with a pencil and draw. You will draw an ellipse which therefore satisfies

$$\overline{F_1 P} + \overline{P F_2} = L$$

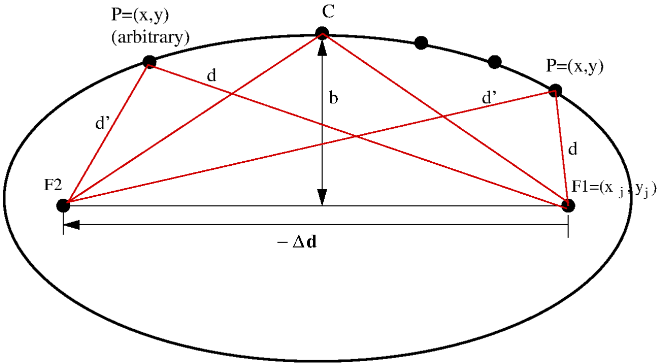
$$|\boldsymbol{d}| + |\boldsymbol{d} + \Delta \boldsymbol{d}| = L$$

This also applies to point C defining the minor semi-axis b of the ellipse:

$$|\boldsymbol{d}| + |\boldsymbol{d} + \Delta \boldsymbol{d}| = L = 2\sqrt{\left(\frac{|\Delta \boldsymbol{d}|}{2}\right)^2 + b^2}$$

$$b(\boldsymbol{d}) = \frac{1}{2}\sqrt{(|\boldsymbol{d}| + |\boldsymbol{d} + \Delta \boldsymbol{d}|)^2 - |\Delta \boldsymbol{d}|^2}$$

Constructing the interaction potential for the elliptical specifications



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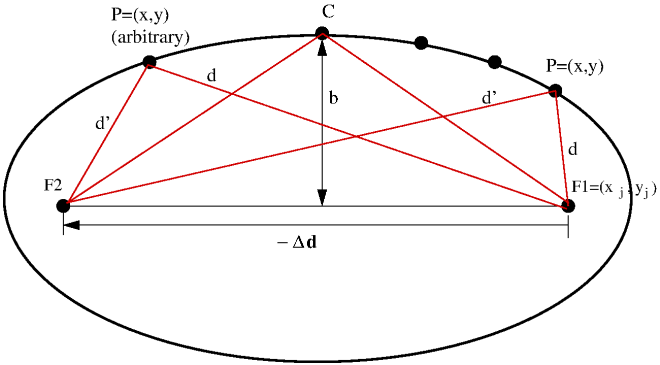
Constructing the interaction potential for the elliptical specifications



This also applies to point C defining the minor semi-axis b of the ellipse:

$$|\boldsymbol{d}| + |\boldsymbol{d} + \Delta\boldsymbol{d}| = L = 2\sqrt{\left(\frac{|\Delta\boldsymbol{d}|}{2}\right)^2 + b^2}$$

$$b(\boldsymbol{d}) = \frac{1}{2}\sqrt{(|\boldsymbol{d}| + |\boldsymbol{d} + \Delta\boldsymbol{d}|)^2 - |\Delta\boldsymbol{d}|^2}$$

- Determine the present collision point $F_1 = \boldsymbol{x}_j$ for i and the *present* location $F_2 = \boldsymbol{x}_j - \Delta\boldsymbol{d}$ for an *anticipated* collision after τ_a
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$$\overline{F_1 P} + \overline{P F_2} = L$$

$$|\boldsymbol{d}| + |\boldsymbol{d} + \Delta\boldsymbol{d}| = L$$

The SFM interaction potential

- The focal line $\overline{F_1 F_2}$ defining generated ellipses with $b=0$ means collision with pedestrian/object j within the anticipation horizon $[t, t+\tau_a]$
- $\Rightarrow$ potential should be highest for $b(x,y)=0$ and decrease with b:

$$\Phi^{\text{int}}(\boldsymbol{d}) = AB\ \exp\left(\frac{-b(\boldsymbol{d})}{B}\right), \quad b(\boldsymbol{d}) = \frac{1}{2}\sqrt{(|\boldsymbol{d}| + |\boldsymbol{d}+\Delta\boldsymbol{d}|)^2 - |\Delta\boldsymbol{d}|^2}$$

- Potential depends on the present distance vector $\boldsymbol{d} = \boldsymbol{x_i} - \boldsymbol{x}_j$ and on the anticipated distance vector change
  - Elliptical specification II: $\Delta\boldsymbol{d} = \tau_a(\boldsymbol{v_i} - \boldsymbol{v}_j)$
  - Elliptical specification I: $\Delta\boldsymbol{d} = \tau_a \boldsymbol{v_i}$
  - Circular specification: $\Delta\boldsymbol{d} = \boldsymbol{0}$
- parameters:
  - **interaction strength A**, typically values around $A = 2\,\mathrm{m/s^2}$
  - **range B**, typically values around $B = 1\,\mathrm{m}$
- For the circular definition without anticipation, we have $\Phi^{\text{int}}(\boldsymbol{d}) = ABe^{-|d|/B}$

Questions

? Potentials if both pdestrians have the same velocity

Proposal of an improved and simpler potential: Anticipated circular specification

The *elliptical specification II* shows the most plausible behaviour and performs best in calibration/validation. Still it has some imperfections:

- Gradient, i.e., the derived social forces, diverge towards the focal points of the ellipse
- Ad-hoc nature. Why ellipses and the complicated construction?
- Answer: Use the circular potential but *centered at a position where the two pedestrians come closest within the anticipation horizon*

Anticipated circular specification

1. Calculate the time interval τ' of closest encounter within the anticipation time horizon:
   - anticipated distance vector $\boldsymbol{d}_a(\tau') = \boldsymbol{d} + (\boldsymbol{v}_i - \boldsymbol{v}_j)\tau'$
   - Minimum distance reached ($\boldsymbol{d}_a$ perpendicular to $\boldsymbol{v}_i - \boldsymbol{v}_j$) at time

$$\tau_{\text{min}} = -\frac{\boldsymbol{d} \cdot (\boldsymbol{v} - \boldsymbol{v}_j)}{(\boldsymbol{v} - \boldsymbol{v}_j)^2}$$

   - anticipated time

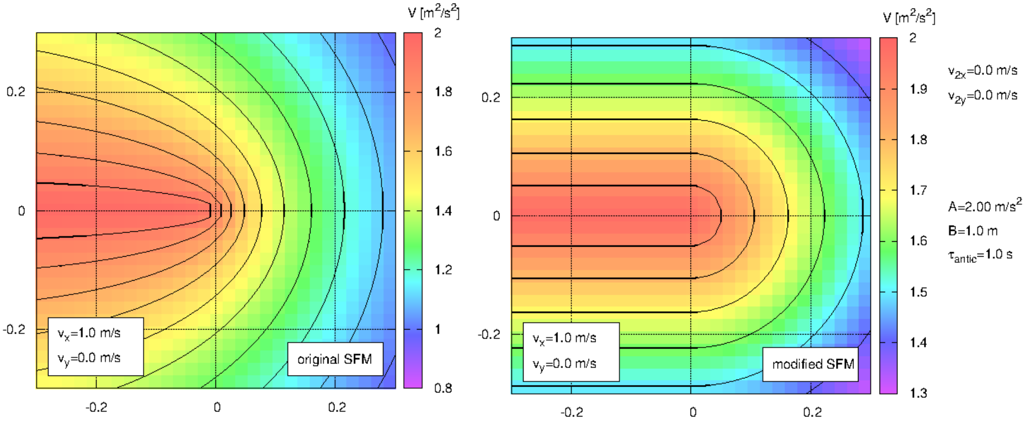
$$\tau' = \max\left(0, \min\left(\tau_a, \tau_{\text{min}}\right)\right)$$

2. Construct circular potential around the distance vector at t':

$$\Phi^{\text{int,mod}}(\boldsymbol{d}) = AB \exp\left(-\frac{|\boldsymbol{d}'|}{B}\right), \quad \boldsymbol{d}' = \boldsymbol{d} + (\boldsymbol{v}_i - \boldsymbol{v}_j)\tau'$$

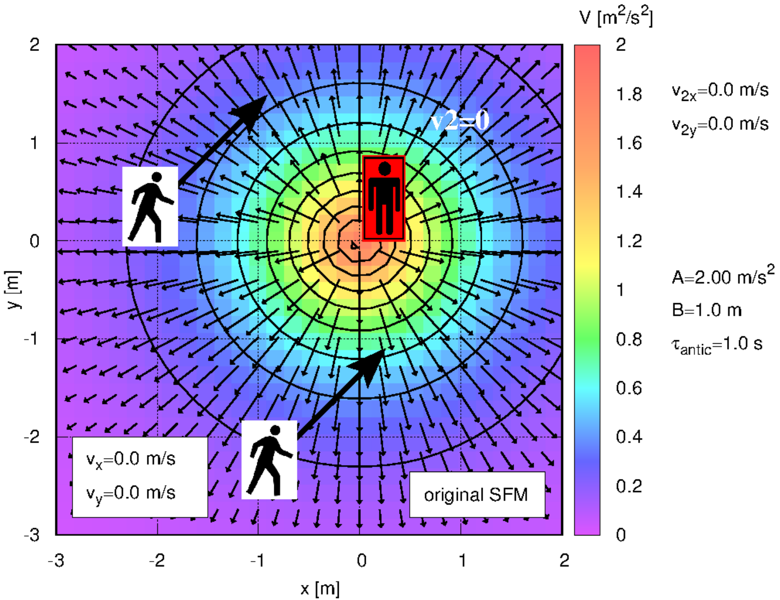
- If the time of shortest encounter lies in the past, we have just the normal circular potential
- Since the potential center *generally is different for each point* (x, y), we, in effect, have an elliptical-like potential but without divergent gradients

Comparison of potentials



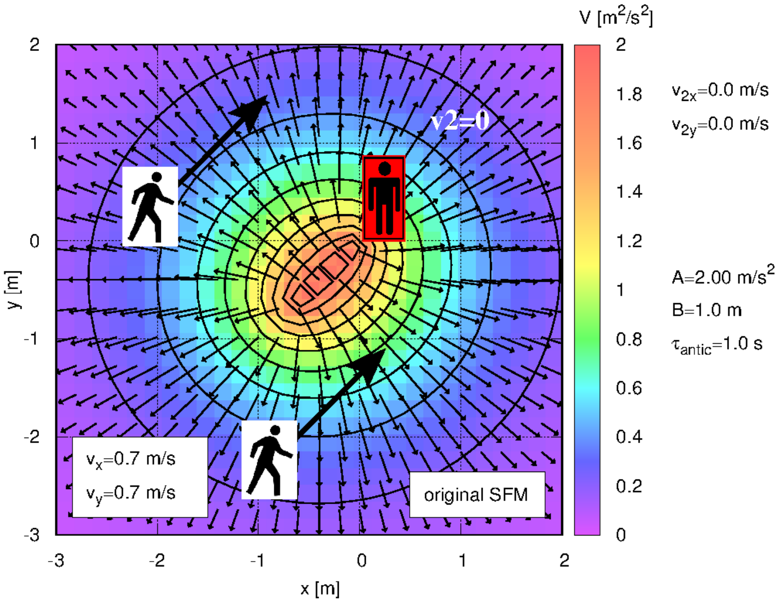
Detail of original (left) and modified (right) interaction potentials for $\boldsymbol{v}_1 = (1,0)$ and $\boldsymbol{v}_2 = \boldsymbol{0}$ near the focal point $F_1 = (0,0)$ of the original SFM.

SFM potential and forces on a pedestrian walking in NE direction ...



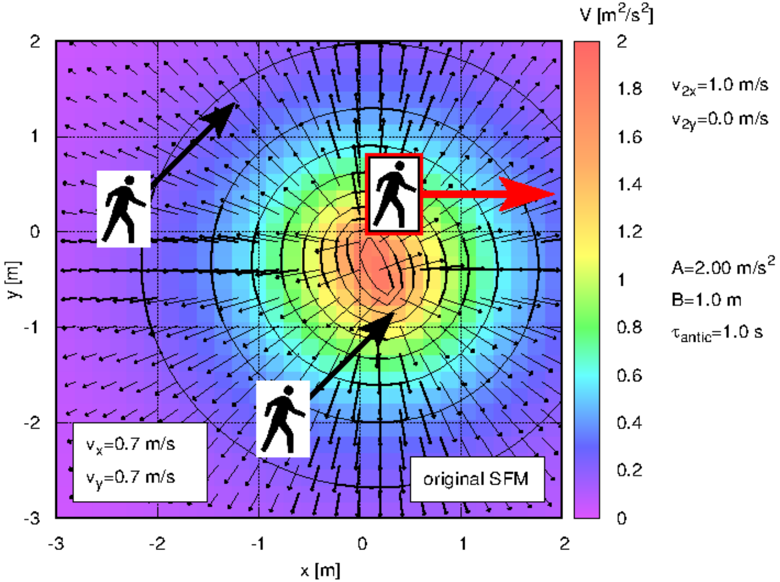
- caused by a standing or moving pedestrian/compact object (circular specification)
- In the circular specification, the own velocity only influences the directional dependence (not shown here). The velocity of the interacting pedestrian is ignored

SFM potential and forces on a pedestrian walking in NE direction ...



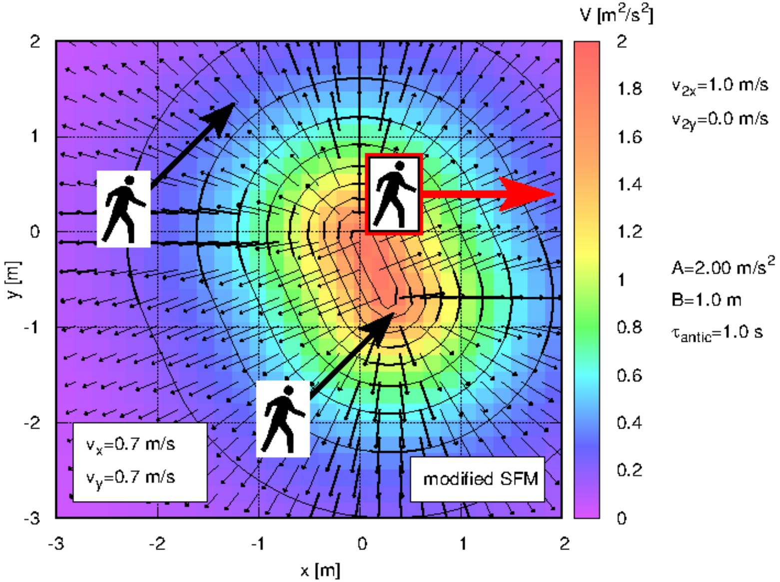
- caused by a standing pedestrian/compact object (specification II)
- or by standing/moving objects (specification I)
- Note the different potentials for moving targets in spec I and II

SFM potential and forces on a pedestrian walking in NE direction ...



- caused by a moving pedestrian (specification II)

SFM potential and forces on a pedestrian walking in NE direction ...



- caused by a moving pedestrian (improved/modified specification II)
- Although circular for a given point (x, y), it becomes elongated as a function of (x, y) because the center (closest anticipated distance) changes with (x, y)

Deriving the force field from the potential

- The potential Φ^{int} is de-facto a "hill" with a ridge along the positions where the subject pedestrian i expects a collision within the anticipation horizon.
- The potential force $-\boldsymbol{\nabla}\Phi^{\text{int}}$ is directed "downhill", i.e., away from potential collision points
- Use the fact, that, for arbitrary constant vectors $\boldsymbol{a}$, we have

$$\boldsymbol{\nabla}_d|\boldsymbol{d}+\boldsymbol{a}| = \frac{\boldsymbol{d}+\boldsymbol{a}}{|\boldsymbol{d}+\boldsymbol{a}|} = \boldsymbol{e}_{\boldsymbol{d}+\boldsymbol{a}}$$

Circular potential

$$\begin{aligned} \boldsymbol{f}^{\text{pot}} = -\boldsymbol{\nabla}_d\Phi^{\text{int}}(\boldsymbol{d}) &= -\boldsymbol{\nabla}_d\left(ABe^{-\frac{|\boldsymbol{d}|}{B}}\right) \\ &= Ae^{-\frac{\boldsymbol{d}}{B}}\boldsymbol{\nabla}_d|\boldsymbol{d}| \end{aligned}$$

$$\boldsymbol{f}^{\text{pot}} = Ae^{-\frac{\boldsymbol{d}}{B}}\boldsymbol{e}_d \quad \text{circular potential}$$

Deriving the force field for the elliptical specifications I and II

The two speciifcations differ only in the constant $\Delta \boldsymbol{d} = \boldsymbol{v}_i \tau_a$ (I) and $\Delta \boldsymbol{d} = (\boldsymbol{v}_i - \boldsymbol{v}_j)\tau_a$, respectively

$$\Phi^{\text{int}}(\boldsymbol{d}) = ABe^{-\frac{b(\boldsymbol{d})}{B}}, \quad b(\boldsymbol{d}) = \frac{1}{2}\sqrt{(|\boldsymbol{d}| + |\boldsymbol{d} + \Delta \boldsymbol{d}|)^2 - |\Delta \boldsymbol{d}|^2}$$

$$\begin{aligned}
\boldsymbol{f}^{\text{pot}} = -\boldsymbol{\nabla}_d \Phi^{\text{int}}(\boldsymbol{d}) &= -\boldsymbol{\nabla}_d \left(ABe^{-\frac{b(\boldsymbol{d})}{B}} \right) \\
&= Ae^{-\frac{\boldsymbol{d}}{B}} \, \boldsymbol{\nabla}_d (b(\boldsymbol{d})) \\
&= Ae^{-\frac{\boldsymbol{d}}{B}} \frac{|\boldsymbol{d}| + |\boldsymbol{d} + \Delta \boldsymbol{d}|}{4b} (\boldsymbol{e}_{\boldsymbol{d}} + \boldsymbol{e}_{\boldsymbol{d}+\Delta \boldsymbol{d}}) \\
&\overset{\text{thread and nails}}{=} Ae^{-\frac{\boldsymbol{d}}{B}} \frac{1}{4}\sqrt{4 + \left(\frac{\Delta \boldsymbol{d}}{b}\right)^2} (\boldsymbol{e}_{\boldsymbol{d}} + \boldsymbol{e}_{\boldsymbol{d}+\Delta \boldsymbol{d}}),
\end{aligned}$$

$$\boldsymbol{f}^{\text{pot}} = Ae^{-\frac{b(\boldsymbol{d})}{B}} \sqrt{1 + \left(\frac{\Delta \boldsymbol{d}}{2b(\boldsymbol{d})}\right)^2} \left(\frac{\boldsymbol{e}_{\boldsymbol{d}} + \boldsymbol{e}_{\boldsymbol{d}+\Delta \boldsymbol{d}}}{2}\right)$$

Elliptical specifications I and II

Deriving the force field for the anticipated circular specification

This is like the potential for the circular specification, but at the anticipated position $\boldsymbol{d} + \Delta \boldsymbol{d}'$ (shifted from $\boldsymbol{d}$ by a constant vector):

$$\boldsymbol{f}^{\text{pot}} = A \exp\left(-\frac{|\boldsymbol{d} + \Delta \boldsymbol{d}'|}{B} \right) \boldsymbol{e}_{d+\Delta \boldsymbol{d}'} \quad \text{Anticipated circular specification}$$

with

$$\begin{aligned} \Delta \boldsymbol{d}' &= (\boldsymbol{v}_i - \boldsymbol{v}_j)\tau', \\ \tau' &= \max\left(0, \min\left(\tau_a, \tau_{\text{min}}\right)\right), \\ \tau_{\text{min}} &= -\frac{\boldsymbol{d} \cdot (\boldsymbol{v}_i - \boldsymbol{v}_j)}{(\boldsymbol{v}_i - \boldsymbol{v}_j)^2} \end{aligned}$$

- If $\boldsymbol{v}_i = \boldsymbol{v}_j$, both the anticipated circular potential and the elliptical specification II revert to the circular potential but not the elliptical specification I. (why?)

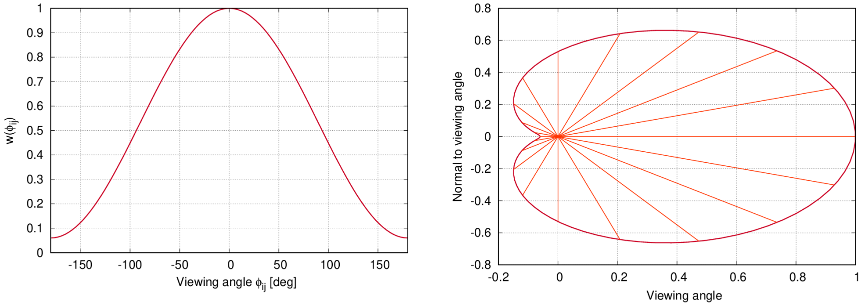
Directionality

The potential force ignores that forces from pdestrians/objects ahead are stronger than that in the back and that also relative speed should matter. Multiplicative approach:

$$\boldsymbol{f}^{\text{int}} = w\boldsymbol{f}^{\text{pot}}$$

(1) Classical SFM dependency on the viewing angle:

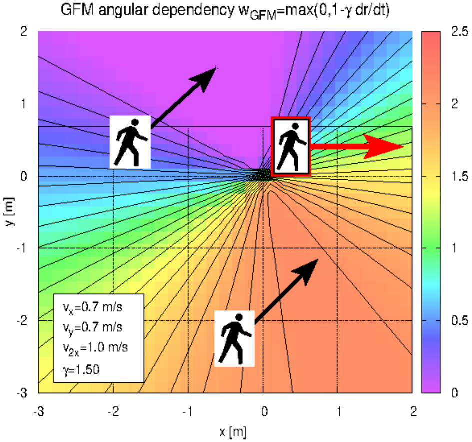
$$w(\cos\phi) = \lambda + (1-\lambda)\left(\frac{1+\cos\phi}{2}\right), \quad \cos\phi = -\boldsymbol{e}_{v_i}\cdot\boldsymbol{e}_d$$



Directionality (FVDM approach)

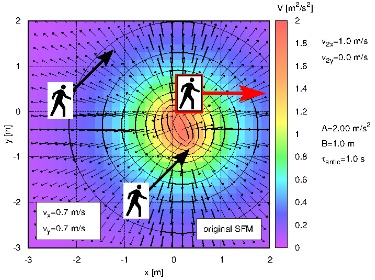
(2) Dependence on the approaching rate (generalisation of the FVDM relative speed sensitivity):

$$w(\dot{d}) = \max(0, 1 - \gamma \dot{d}), \quad \dot{d} = \boldsymbol{e}_d \cdot (\boldsymbol{v}_i - \boldsymbol{v}_j) \tag{1}$$



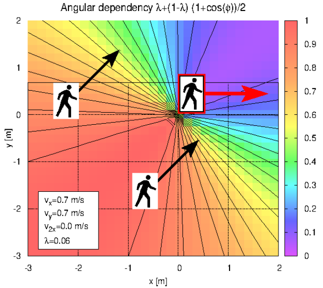
Depen– dency approaching rate

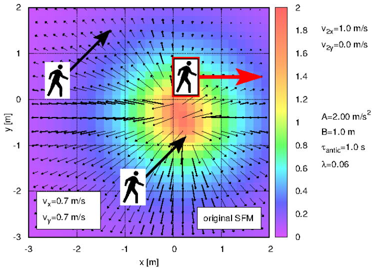
From a symmetric to a directed asymmetric interaction: Pedestrian at (x, y) walking to a target pedestrian moving to the right



\+

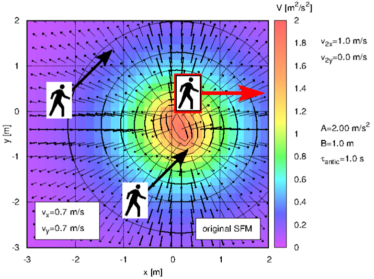
Angular depen– dency position



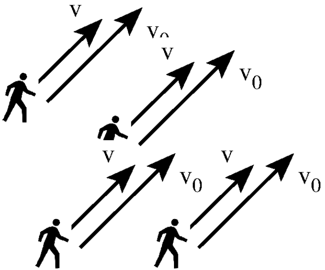


From the asymmetric interaction to the full force: adding the free force

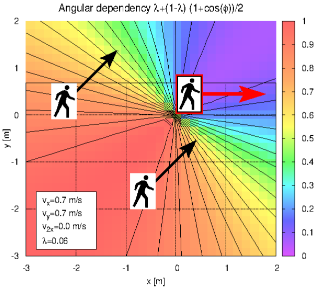
+ Potential force



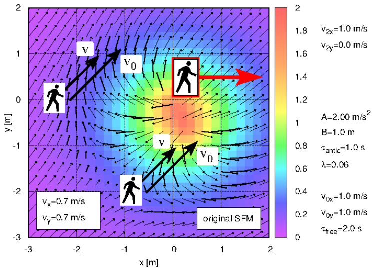
+ Free force



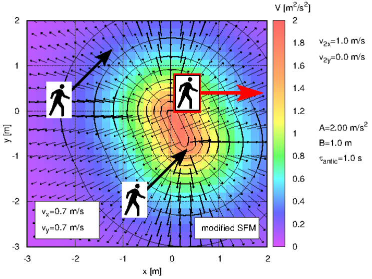
+ Angular depen–dency

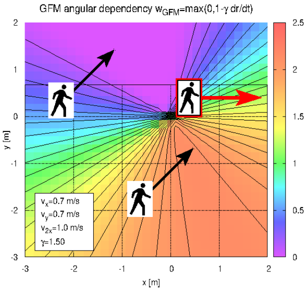


+ Full force



Viewing angle vs. approaching rate directional weighting (example anticipated circular potential)

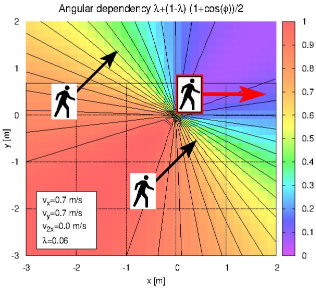


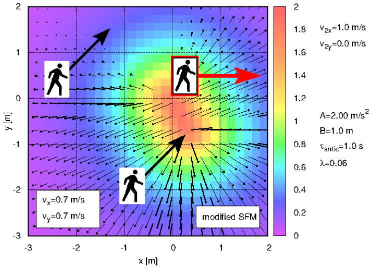


Depen–
dency
approaching
rate

+

Angular
depen–
dency
position





11.2.3 Potential and force by obstacles

- Basically, obstacles and boundaries of walkable areas are like standing pedestrians: One should not collid with/transgress them
- Small compact obstacles (poles, signposts, trees, pillars) can, in fact, be handled like standing pedestrians
- For extended obstacles/boundaries this would be inefficient (a cordon of many standing virtual pedestrians) and also biased (the additive effect of the forces exaggerates the effect)

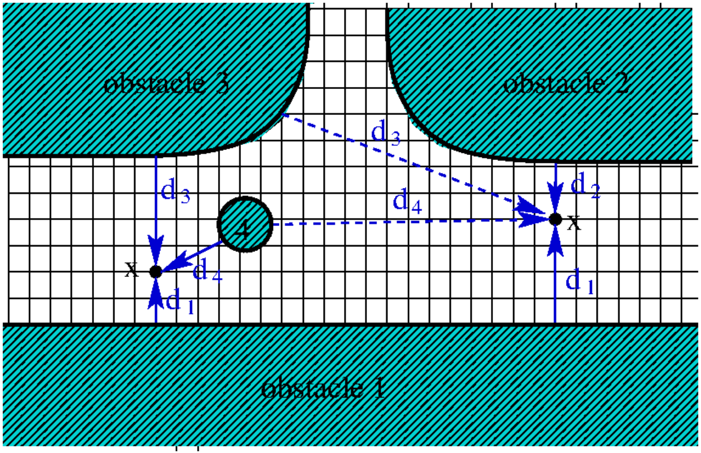
? What to do ?

! Use the fact that obstacles, boundaries etc are really immobile so as to precalculate a global floor potential from all obstacles and boundaries

! Add anticipation in the same way as for defining the anticipated circular potential: Calculate the anticipation point and take the gradient at that point

! Multiply a factor for the viewing angle and/or approaching rate as when interacting with other pedestrians

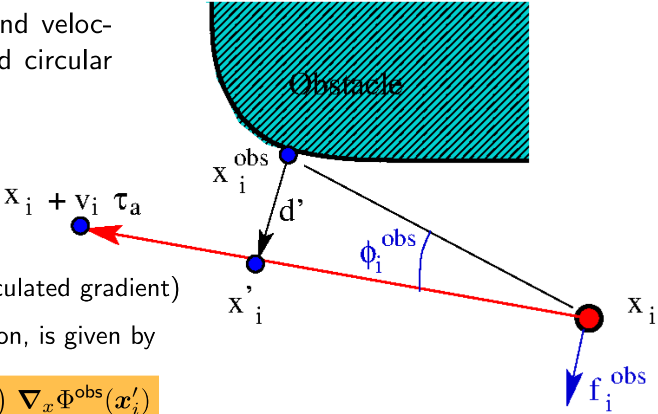
Potential and force by obstacles: precalculate the obstacle floor field



- Identify all obstacles and boundary objects
- Define a grid on the walkable area. For each gridpoint (at $\boldsymbol{x}$) and each obstacle object k, determine the distance vector $\boldsymbol{d}_k(\boldsymbol{x})$ to the neares point of this object
- Ignore obstacles that are shielded or too far away
- The global floor potential is given by

$$\Phi^{\text{obs}}(\boldsymbol{x}) = AB \sum_k e^{-\frac{\boldsymbol{d}_k(\boldsymbol{x})}{B}}$$

Obstacle social forces on pedestrians



The procedure for a pedestrian at $\boldsymbol{x}_i$ and velocity $\boldsymbol{v}_i$ is the same as for the anticipated circular interaction potential:

- Calculate the anticipated position $\boldsymbol{x}_i'$ along the line $\{\boldsymbol{x}_i + t'\boldsymbol{v}_i : t' \in [0.\tau_a]\}$ with the shortest distance $\boldsymbol{d}'$ to *any* obstacle (or with the highest precalculated gradient)
- The obstacle social force, i.e., acceleration, is given by

$$\boldsymbol{f}_i^{\text{obs}} = -w\left(\cos\phi_i^{\text{obs}}\right)\,\boldsymbol{\nabla}_x\Phi^{\text{obs}}(\boldsymbol{x}_i')$$

- A bilinear interpolation on the precalculated grid is enough for calculating the gradient
- If only a single obstacle is relevant, we have the anticipated circular specification:

$$\boldsymbol{f}^{\text{obs}} = w\left(\cos\phi_i^{\text{obs}}\right)\, Ae^{-\frac{\boldsymbol{d}'}{B}}\,\boldsymbol{e}_{d'}$$

11.2.4 Model Parameters and Fundamental Diagram

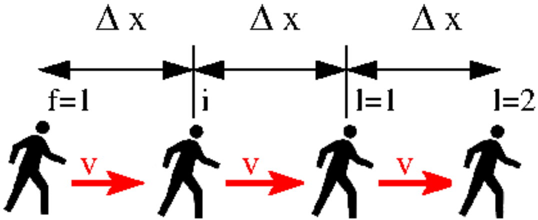
Overview of the model parameters of the SFM

Parameter	normal walking	Marathon runners	comment
Desired speed v_0	1.2 m/s	3 m/s	free traffic
Speed adaptation time τ	1 s	1.5 s	free traffic
Interaction strength A	$2\,\mathrm{m/s^2}$	$3\,\mathrm{m/s^2}$	of the order of the maximum acceleration
Interaction range B	$1\,\mathrm{m}$	$2\,\mathrm{m}$	decays by a factor $1/e$ per distance increment B
Anticipation time τ_a	$1\,\mathrm{s}$	$2\,\mathrm{s}$	anticipation for collisions assuming constant velocities
directionality λ	0.06	0.03	isotropic *actio=reactio*: $\lambda = 1$
relative speed sensitivity γ	1.5	1.0	alternative formulation of the directionality (as in the FVDM)

Fundamental diagram

- Because of the different possible geometric configurations, a fully 2d fundamental diagram (FD) is not unique
- A simpler approach is to define a **single-file fundamental diagram**
- Because the number of interacting persons in single files increases linearly with distance rather than quadratically, a single-file FD as a function of the 1d density $\rho_{1\text{d}}$ also approximates a 2d FD as a function of the 2d density ρ

As usual in FDs, we have identical pedestrians with identical (center-center) distances $\Delta x = 1/\rho_{1\text{d}}$ and identical speeds

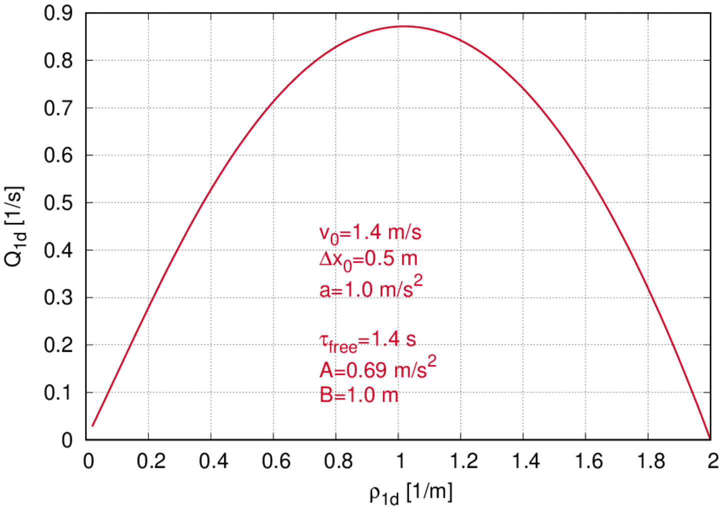


Derivation for full interactions without shielding

$$
\begin{aligned}
\frac{\mathrm{d}v_i}{\mathrm{d}t} &= \frac{v_0 - v_i}{\tau} + \sum_{l=1}^{\infty} f_{il} + \sum_{m=1}^{\infty} f_{im} \\
&= \frac{v_0 - v_i}{\tau} - 1 \sum_{l=1}^{\infty} A e^{-l\Delta x/B} + \lambda \sum_{m=1}^{\infty} A e^{-m\Delta x/B} \\
&= \frac{v_0 - v_i}{\tau} - A(1-\lambda) \sum_{l=1}^{\infty} e^{-l\Delta x/B} \\
&= \frac{v_0 - v_i}{\tau} - A(1-\lambda) \left(\sum_{l=0}^{\infty} e^{-l\Delta x/B} - 1 \right) \\
&\overset{\text{geometric series}}{=} \frac{v_0 - v_i}{\tau} - A(1-\lambda) \left(\frac{1}{1 - e^{-\Delta x/B}} - 1 \right) \overset{!}{=} 0
\end{aligned}
$$

$$
v_i(\Delta x) \to V(\Delta x) = v_0 - \tau A(1-\lambda) \left(\frac{e^{-\Delta x/B}}{1 - e^{-\Delta x/B}} \right)
$$

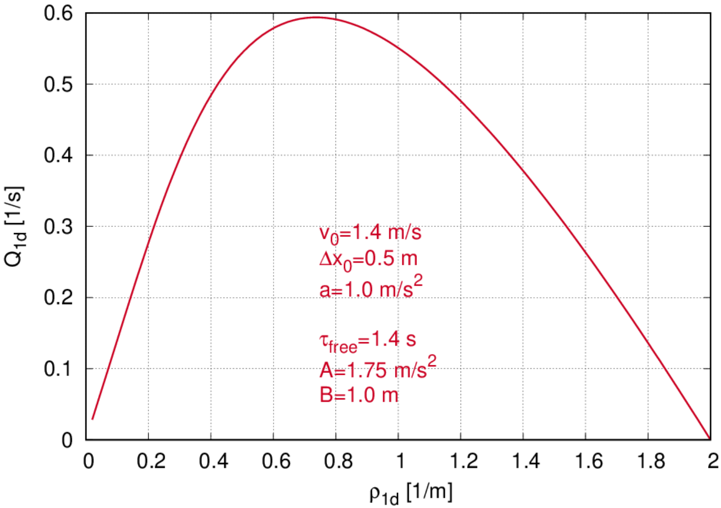
SFM fundamental diagrams for a single file without shielding



The value of A such that $V(0.5\,\mathrm{m}) = 0$

$$Q(\rho_{1d}) = \rho_{1d} V(1/\rho_{1d}), \quad V(\Delta x) = v_0 - \tau A(1-\lambda)\left(\frac{e^{-\Delta x/B}}{1-e^{-\Delta x/B}}\right)$$

SFM FD for a single file with NN interactins (one front and back pedestrian)



The value of A such that $V_{\mathrm{NN}}(0.5\,\mathrm{m}) = 0$

$V_{\mathrm{NN}}(\Delta x)$ corresponds to third line of the derivation with only $l = 1$:

$$Q(\rho_{1\mathrm{d}}) = \rho_{1\mathrm{d}} V_{\mathrm{NN}}(1/\rho_{1\mathrm{d}}), \quad V_{\mathrm{NN}}(\Delta x) = v_0 - \tau A(1-\lambda)e^{-\Delta x/B}$$

11.3: An alternative approach: PLEdestrian