Lecture 10: Lane-Changing and other Discrete-Choice Situations

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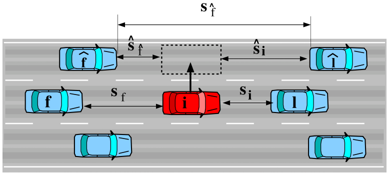
10.1 Overview

- Frome a driver's point of view, there are discrete and continuous actions
- Car-following models imply continuous actions: accelerating and braking
- The third possible continuous action is steering. All other actions such as deciding to change lanes, honking, operating the winkers etc are discrete
- Often, discrete actions/decisions are at a higher (tactical/strategic) layer than the operative actions of accelerating, braking and steering
- Example 1: strategic: determining the route; tactical: deciding on a lane change (e.g., in order to enter/exit); operational: steering if the lane-changing decision is positive (often, the operative modelling of lane changes is just an instantaneous "jump" but it can also done smoothly by trajectory planning)
- Example 2: Tactical: entering a priority road or waiting for a larger gap; operational: accelerating if the "enter" decision is positive

10.2 General Decision Model

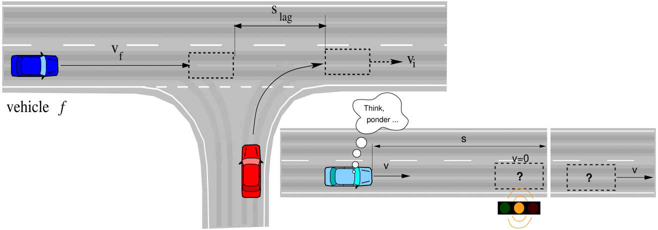
Assume that the driver has a discrete **set $\mathcal{K}$ of alternatives** $k = 1, \ldots, K$

- The alternative set is formulated such that, at any moment, the driver needs to opt for exactly one of them:
  - *Lane changing* (10.3): $k = 1$: stay on your lane; $k = 2$ change to the left; $k = 3$: change to the right



  - *Approaching a traffic light turning red* (10.4): $k = 1$ go on, or $k = 2$ stop
  - *Entering a priority road* (10.5) or *passing a no-signalized intersection* (10.6): $k = 1$ wait for a larger gap; $k = 2$ accept the actual gap and enter the road/cross the intersection
- The alternative set $\mathcal{K}$ is highly dynamic. For example, once we chose an option requiring action ($k > 1$), we have only the new options *pursue* or *revert* this decision
- The alternative set is evaluated either every m time steps or in an event-oriented way

Safety and incentive criteria



- Often, there is a *passive* default action such as "stay on the present lane" or "wait for entering a road or crossing an intersection"
- When deciding on an option requiring action ($k > 1$, if available), two criteria have to be satisfied simultaneously:
  - Safety criterion: Is it safe for me and for all affected traffic participants?
  - Incentive criterion: Is it advantageous with respect to the default ($k = 1$) to take this action? (Under which condition, the incentive is always true?)
- If several options are safe and advantageous, the one with the highest advantage is chosen.
- If all alternatives are unsafe, including the default $k = 1$, the one with the least violation of the safety criterion is chosen

Safety criteria

Let

- i be the decision-making driver
- (j, k) the driver/vehicle j affected by the decision for alternative k
- $a_{\text{mic}}^{(i,k)}$ the microscopic CF-acceleration of the decision maker after opting for alternative k
- $a_{\text{mic}}^{(j,k)}$ the microscopic CF-acceleration of the driver/vehicle j affected by the decision for alternative k

Then, action k is safe if, as a consequence of this action,

$$a_{\text{mic}}^{(j,k)} > -b_{\text{safe}} \ \forall \text{ affected vehicles } j$$

- The only new parameter b_{safe} is the safe deceleration. Everything else is delegated to the CF model
- The set of affected vehicles j includes that of the decision maker i. When he/she crashes into another vehicle (any CF model should return the acceleration $-b_{\text{max}}$ for crashes, i.e., for negative gaps $s < 0$), this clearly is not safe.
- Any carence times, e.g., due to past lane changes of the subject or others, also can be added to the safety restriction (then, we have only the *do-nothing* option $k = 1$)

Incentive criterion

Assume the utility $U_i^k = a_{\text{mic}}^{(j,k)}$ (discuss!) and, for the moment, no random utility (discuss the MNL model!). Then, adopt alternative

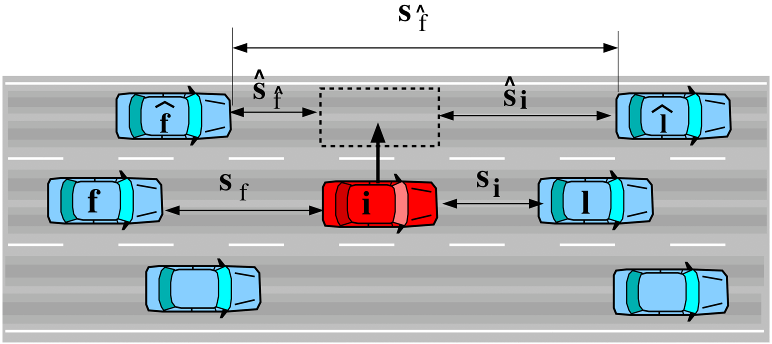
$$k_{\text{selected}} = \arg \max_{k' | \text{safe}} a_{\text{mic}}^{(i,k')} \quad \text{egoistic driver}$$

or, for somewhat altruistic drivers,

$$k_{\text{selected}} = \arg \max_{k' | \text{safe}} a_{\text{mic}}^{(i,k')} + p \sum_{j \neq i} a_{\text{mic}}^{(j,k')} \quad \text{general driver}$$

- The basic "egoistic" criterion is parameter free (makes use of the "bells and whistles" of the CF model) while the general criterion also weights the affected surrounding drivers by a **politeness factor** $p \le 1$
- The choice set includes the "passive" alternative $k = 1$. Suppress frantic changes of mind for marginal utility gain by adding a threshold penalty $a_{\text{thr}} > 0$ to all $k' > 1$
- Strategic/tactical constraints (keep right before leaving), mandatory constraints (lane closing) or traffic regulations ("keep right" directive) are modelled by adding some positive bias a_{bias} to the corresponding alternative

10.3 Lane Changes



Let be

- i: subject vehicle
- l, f: leader and follower of i
- $\hat{l}$, $\hat{f}$: new leader and follower after a change
- $\hat{s}$, $\hat{a}$: new state variables after the lane change

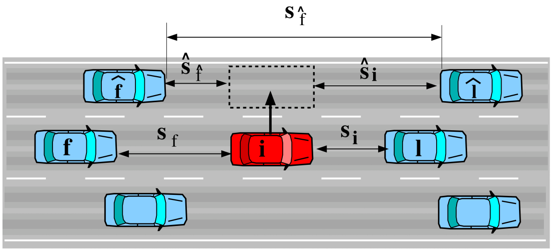
Specialisation of the general safety criteria:

$$\hat{a}_{\hat{f}} = a_{\text{mic}}(\hat{s}_{\hat{f}}, v_{\hat{f}}, v_i) > -b_{\text{safe}} \qquad \text{safety criterion}$$
$$\hat{a}_i = a_{\text{mic}}(\hat{s}_{\hat{l}}, v_{\hat{l}}, v_i) > -b_{\text{safe}} \qquad \text{self-protection}$$

Standard incentive criterion for egoistic drivers:

$$\hat{a}_i - a_i > a_{\text{thr}} + a_{\text{bias}} \qquad \text{egoistic incentive}$$

Polite and not so polite drivers



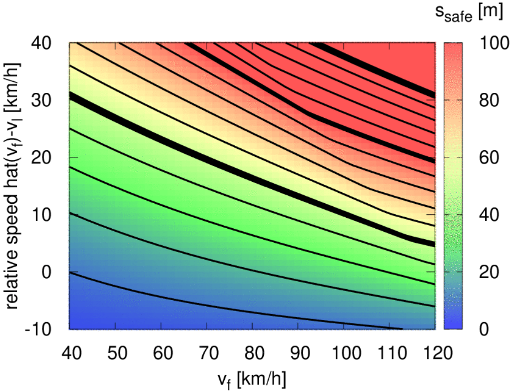
Incentive criterion with consideration of the others:

$$\hat{a}_i - a_i + p\left(\hat{a}_{\hat{f}} - a_{\hat{f}} + \hat{a}_f - a_f\right) > a_{\text{thr}} \pm a_{\text{bias}} \quad \text{MOBIL incentive}$$

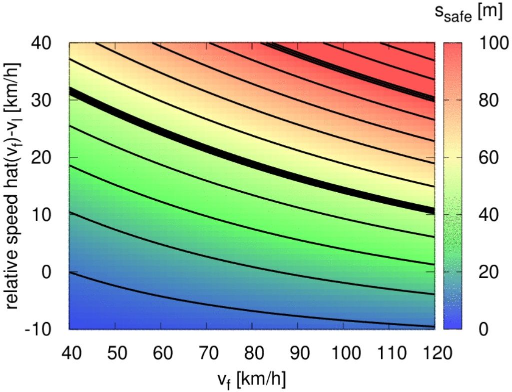
- *Polite* drivers with a **politeness factor $p > 0$** consider accelerations of other drivers not only for safety reasons but also try avoid unnecessary perturbations
- For $p = 1$ (rather altruistic driver), we have the **MOBIL** principle: "**M**inimizing **O**verall **B**raking by **I**ntelligent **L**ane-changing decisions"
- Further parameters: lane-changing threshold a_{thr} and directional bias a_{bias}

Minimum rear and front gaps from the safety criterion

IDM, $v_0 = 120\,\mathrm{km/h}$, $T = 1.2\,\mathrm{s}$, $s_0 = 2\,\mathrm{m}$, $a = 1.5\,\mathrm{m/s^2}$, $b = b_{\text{safe}} = 2\,\mathrm{m/s^2}$

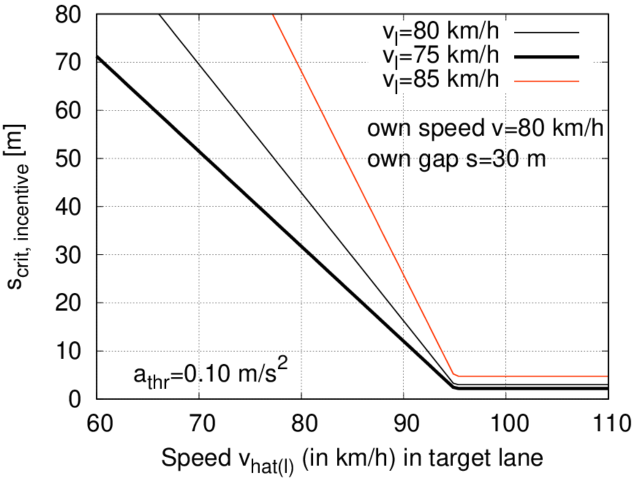


IDMplus, same parameter values



The minimum safe rear gap for lane changing increases with the own speed and, more importantly, with the relative speed of the new follower $\hat{f}$ to the subject vehicle i

IDM incentive criterion



$v_0 = 120\,\mathrm{km/h}$, $T = 1.2\,\mathrm{s}$, $s_0 = 2\,\mathrm{m}$,
$a = 1.5\,\mathrm{m/s^2}$, $b = 2\,\mathrm{m/s^2}$

The critical front gap on the target lane …

- Decreases strongly with the speed on the target lane
- increases with the speed of the "old" leader
- tends to infinity (no desire at all) for a sufficiently large own gap s

Analytic safety criteria for the OVM

- As always, a lane change is safe if both the rear gap (safety criterion proper) and the front gap (self-protection) on the new lane will be large enough
  - For the safety criterion proper, the follower is the new follower $\hat{f}$ on the target lane and the leader the subject vehicle
  - For the self-protection criterion, the follower is the subject and the leader the new leader $\hat{l}$ on the target lane

$$
\begin{aligned}
f_{\text{OVM}}(s_{\text{safe}}, v_f, v_l) &= \frac{1}{\tau}\left(v_{\text{opt}}(s_{\text{safe}}) - v_f\right) \stackrel{!}{\geq} -b_{\text{safe}} \\
v_{\text{opt}}(s_{\text{safe}}) &= v_f - b_{\text{safe}}\tau \qquad |\text{inversion} \\
s_{\text{safe}} &= v_{\text{opt}}^{-1}(v_f - b_{\text{safe}}\tau) := s_{\text{opt}}(v_f - b_{\text{safe}}\tau)
\end{aligned}
$$

Example: triangular FD $s_{\text{opt}}(v) = s_0 + vT$ (only congested branch matters (why?))

$$
s_{\text{safe}}^{\text{OVM,triang}} = s_0 + (v_f - b_{\text{safe}}\tau)T
$$

For stability, the OVM speed adaptation time must obey $v'_{\text{opt}}(s) < 1/(2\tau)$ (calculate this for the triangular FD), typically about $\tau = 0.5\,\text{s}$. With $b_{\text{safe}} = 2\,\text{m/s}^2$: $s_{\text{safe}}^{\text{OVM}} = s_{\text{opt}} - 1\,\text{m}$
the safety criterion essentially reverts to the simple gap-acceptance criterion "Changing is only safe if the target gap is at most marginally lower than the optimal gap"

Analytic safety criteria for the FVDM

Full-velocity-difference model calculated in analogy, just addtl term $\gamma(v_l - v_f)$:

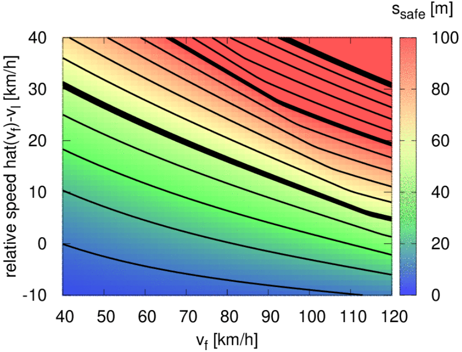
$$
\begin{aligned}
f_{\text{OVM}}(s_{\text{safe}}, v_f, v_l) &= \frac{1}{\tau}(v_{\text{opt}}(s_{\text{safe}}) - v_f) + \gamma(v_l - v_f) \overset{!}{\geq} -b_{\text{safe}} \\
s_{\text{safe}}^{\text{FVDM}} &= s_{\text{opt}}(v_f - (b_{\text{safe}} + \gamma(v_l - v_f))\tau) \\
s_{\text{safe}}^{\text{FVDM,triang}} &= s_0 + (v_f - (b_{\text{safe}} + \gamma(v_l - v_f))\tau)T
\end{aligned}
$$

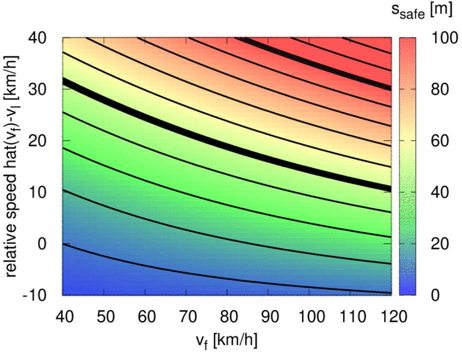
Example

In contrast to the OVM, the FVDM allows for stable traffic for higher adaptation times, e.g., $\tau = 3\,\mathrm{s}$. Furthermore, $\gamma = 0.5\,\mathrm{s}^{-1}$ and $b_{\text{safe}} = 2\,\mathrm{m/s}^2$ are plausible

- If the follower on the target lane with the same speed, the minimum gap is the steady-state gap for a speed $b_{\text{safe}}\tau = 6\,\mathrm{m/s}$ lower than the speed of the subject and new follower
- If the new follower is $4\,\mathrm{m/s}$ faster, the minimum gap is equal to the steady-state gap at the subject's speed $v_l = v$ plus $(v_f - v_) + \gamma\tau(v_f - v) - b_{\text{safe}}\tau = 4\,\mathrm{m/s}$, so the safe gap is even *higher* than the steady-state gap

Analytic safety criteria for the IDM and IDMplus





IDM

$$f_{\text{IDM}}(s_{\text{safe}}, v_f, v_l) = a_{\text{free}}(v_f) - a\left(\frac{s^*(v_f, v_l)}{s_{\text{safe}}}\right)^2 \overset{!}{\geq} -b_{\text{safe}}$$

$$\frac{a_{\text{free}}(v_f) + b_{\text{safe}}}{a} = \left(\frac{s^*(v_f, v_l)}{s_{\text{safe}}}\right)^2$$

$$s_{\text{safe}}^{\text{IDM}} = s^*(v_f, v_l)\sqrt{\frac{a}{a_{\text{free}}(v_f) + b_{\text{safe}}}}$$

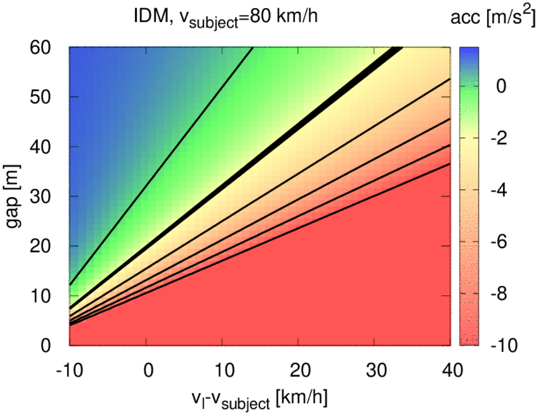
IDMplus

$$s_{\text{safe}}^{\text{IDMplus}} = s^*(v_f, v_l)\sqrt{\frac{a}{a + b_{\text{safe}}}}$$

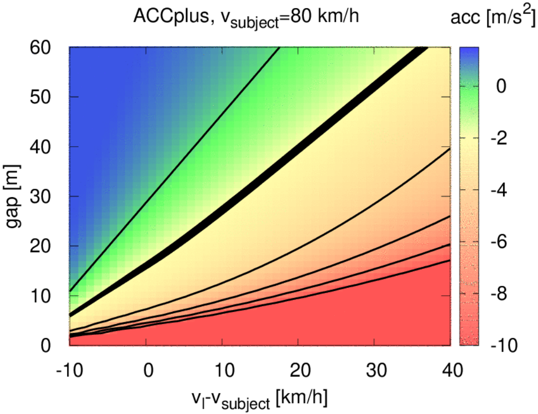
The IDMplus safe gap is just a fixed fraction (< 1) of the desired dynamical gap s^*

Safe criteria for more complicated models

Just contour plot the acceleration function as a function of the gap on the y axis and some other quantity (the thick contour corresponds to $b_{\text{safe}} = 2\,\mathrm{m/s^2}$)



IDM (for comparison)



ACC model (based on IDM+)

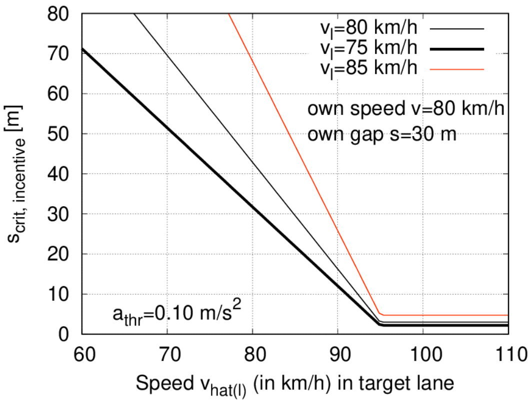
IDM and IDMplus incentive criterion

Since accelerations are compared and the free acceleration is the same before and after the change, the criterion is the same for the IDM and IDM+: Compare the interaction terms $-a(s^*/s)^2$ for the old and new situations ($\Delta = a_{\text{thr}} + a_{\text{bias}}$):

$$\begin{aligned} \text{new acceleration} &\geq \text{old acceleration} + \Delta \\ -a\left(\frac{\hat{s}^*}{\hat{s}}\right)^2 &\geq -a\left(\frac{s^*}{s}\right)^2 + \Delta \\ \left(\frac{\hat{s}^*}{\hat{s}}\right)^2 &\leq \left(\frac{s^*}{s}\right)^2 - \frac{\Delta}{a} \end{aligned}$$

Solve for the minimum target gap $\hat{s}$ for a desire to change lanes:

$$\hat{s} \geq \frac{\hat{s}^*}{\sqrt{\left(\frac{s^*}{s}\right)^2 - \frac{a_{\text{thr}}+a_{\text{bias}}}{a}}}$$



10.3.1. Continuous Lane Changing: Trajectory Planning

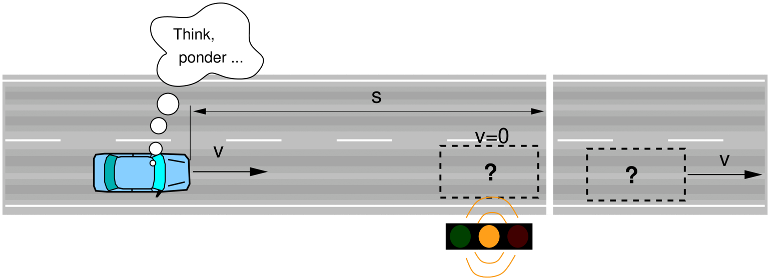
This is an operational implementation of the discrete decision process one level higher for the case that a lane change is safe and desired. Only schematic ($\Rightarrow$ alternative: 2d models):

- Define a reward function taking into account travel time, acceleration, jerk, and, of course, safety criteria (assuming constant speeds or accelerations and no lane changes of the other drivers)
- Define a parameterized trajectory (with, typically, $m = 6$ to allow for a jerk control) and a lane-changing time τ,

$$x(t) = \sum_{j=0}^{n} \beta_{xj} t^j, \quad y(t) = \sum_{j=0}^{m} \beta_{yj} t^j$$

- Determine some of the parameters by the constraints, e.g., $y(0) = \beta_{y0} = y_0$, $\dot{y}(0) = \ddot{y}(0) = \dot{y}(\tau) = \ddot{y}(\tau) = 0$ and the rest, including the lane-changing time, by maximizing the reward

10.4 Approaching a Traffic Light



The criterion to stop at a traffic light turning red is identical to the safety criterion

- with a leading speed $v_l = 0$ (the stopping line does not move)
- The follower is the subject vehicle (as for the self-protection criterion)
- The safe deceleration is a little larger than the comfortable one or the safe deceleration for the lane-changing safety criterion, e.g. $b_{\text{safe}}^{\text{TL}} = 3\,\mathrm{m/s}^2$ (cf. problem)
- Stopping is safe if $s > s_{\text{safe}}$, i.e., the decision "stop" corresponds to "a lane change is safe"
- Otherwise, one is too close for a safe braking and moves on

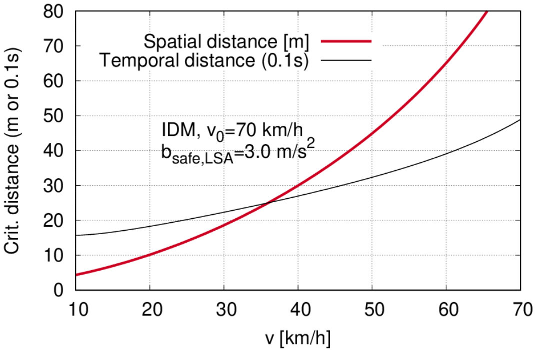
Example: IDM and IDMplus

The dynamical gap for $v_l = 0$ (or $\Delta v = v$) and the IDM safety criterion reformulated for stopping is

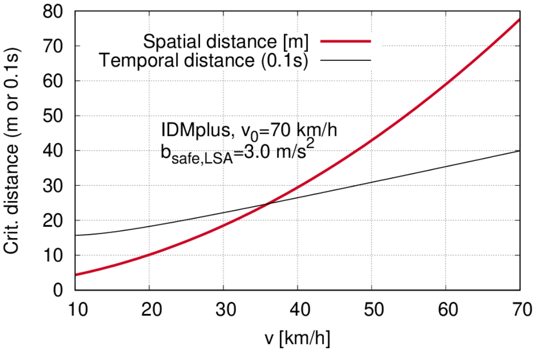
$$\begin{aligned} s^*_{\text{TL}} &= s_0 + vT + v^2/(2\sqrt{ab}), \\ s > s^{\text{IDM}}_{\text{TL}} &= s^* \sqrt{\frac{a}{a_{\text{free}} + b_{\text{safe}}}} \end{aligned}$$

- If $a = b$ and T also denotes the reaction time, the dynamical gap $s^* = s_0 + vT + v^2/(2b)$ is just the minimum gap plus the **stopping distance** at deceleration b (**reaction distance** vT plus the **braking distance** $v^2/(2b)$)
- If, in addition, $v = v_0$ and $b_{\text{safe}} = b$, the IDM vehicle stops if the distance minus s_0 is larger than that stopping distance and gors on, otherwise
- All IDM variants and the ACC model behave similarly. For example, for the IDMplus, the critical distance is $s_{\text{safe}} = cs^*$ with a fixed factor $c = \sqrt{a/(a + b_{\text{safe}})} < 1$

10.4 Approaching a Traffic Light 2



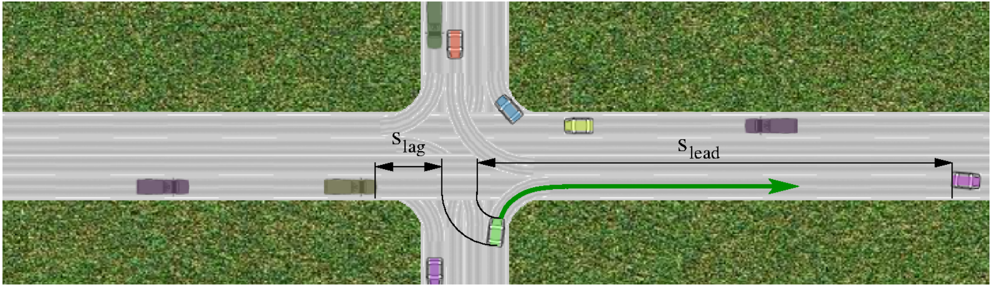
IDM with $v_0 = 70\,\mathrm{km/h}$, otherwise the freeway parameters $T = 1.2\,\mathrm{s}$, $s_0 = 2\,\mathrm{m}$, $a = 1.5\,\mathrm{m/s^2}$, $b = 2\,\mathrm{m/s^2}$



IDMplus with the same parameter values

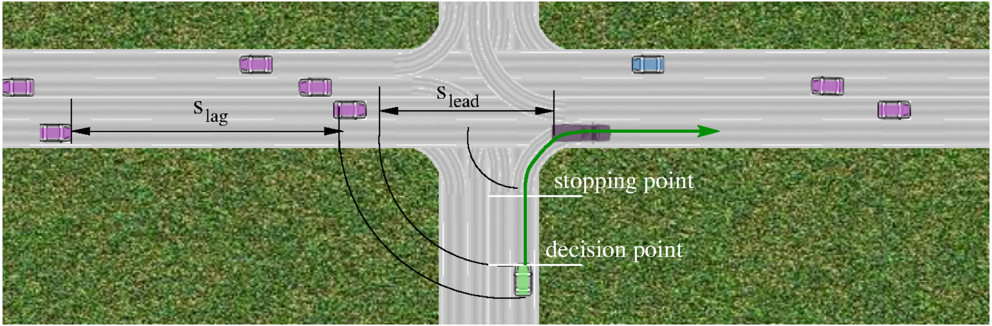
At a speed limit of 50 km/h, the minimum duration for the yellow (amber) traffic light phase is $T_y = 3\,\mathrm{s}$. Assuming a reaction time $T = 1.2\,\mathrm{s}$, what is the implied minimum value for b_{safe}? The critical distance $s_c = v_0 T_y$. Identify this with the stopping distance $v_0 T + v_0^2/(2b_{\text{safe}})$ and solve for the safe deceleration: $b_{\text{safe}} = v_0/(2(T_y - T))$. For $v_0 = 50\,\mathrm{km/h} = 50/3.6\,\mathrm{m/s}$, we obtain $b_{\text{safe}} = 50/3.6^2\,\mathrm{m/s^2} \approx 3.8\,\mathrm{m/s^2}$. Quite a high value! (it gets better for $T_y = 5\,\mathrm{s}$ at 70 km/h)

10.5 Entering a Priority Road



- The incentive criterion is always positive (why?)
- The safety criteria are that for lane changes where the new lead and lag gaps (once on the priority road) are calculated using the arc-length on the approach road to the nominal merging point
- **Stop intersections** are easy: start if you do not collide with the leader (leading gap @ entry is positive) and the lag gap (at starting, not at merging time) is equal to the critical gap at traffic lights
- **Running-start intersections** are more tricky

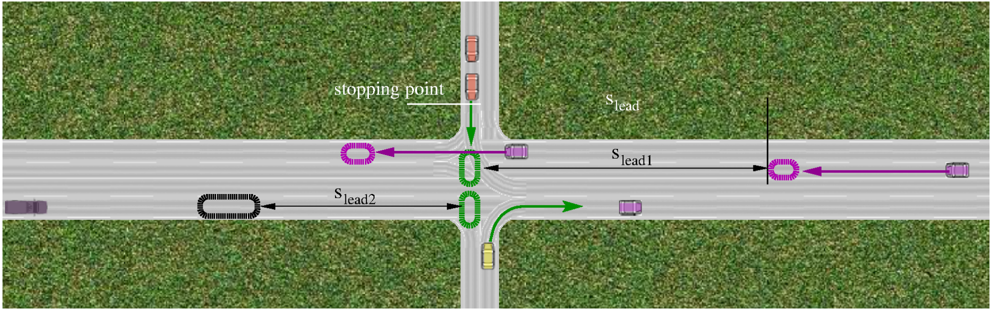
10.5 Entering a Priority Road @ running start



Depending on the visibility conditions and turning curvature, define a **decision point** from which it is possible to estimate the gaps and, if safety is passed, accelerate to enter

- Before the decision point, decelerate to stop as for a stop intersection
- At the decision point, evaluate the safety criteria for the lead and lag gaps *as projected on the main lane* as in a lane-changing situation
- If both criteria are passed, accelerate, turn and enter
- Otherwise, proceed to stop and re-evaluate the criteria regularly (in case of an OK, reevaluate only before the "point of no revision")

10.6 Crossing a Priority Road



Depending on the visibility conditions, define a **decision point** from which it is possible to estimate the gaps and, if safety is passed, accelerate to cross (for a stop intersection, the decision point is at the stopping line)

- Before the decision point, decelerate to stop at the stopping line (as for turning)
- At the decision point, evaluate all **time-to collision** based safety criteria
- If all criteria are passed, accelerate to pass
- Otherwise, proceed to stop or wait and re-evaluate the criteria regularly (in case of an OK, reevaluate only before the "point of no revision")

Time-to collision

Conventional definition:

The **time-to collision** (TTC or T_{TTC}) is the future time interval after which a collision will happen if the conflicting vehicles do not change their speed.

Car-following situation:

$$T_{\text{TTC}} = \frac{s}{v_f - v_l}$$

Conflicting trajectories: time to conflict for a vehicle i to a conflict point at a distance gap s:

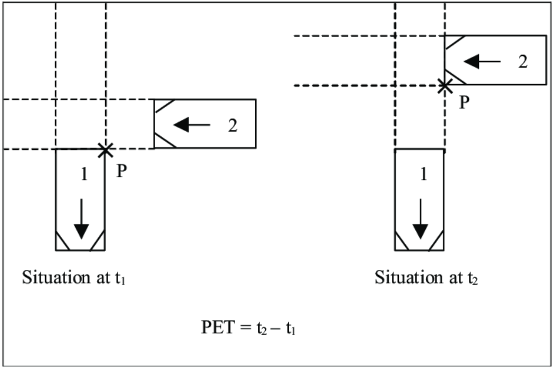
$$T_{\text{TTC}} = \frac{s}{v_i}$$

For a starting vehicle, these definitions are useless **(why?)**. Given an initially stopped vehicle and a starting acceleration a, we have

$$T_{\text{TTC}} = \sqrt{\frac{2s}{a}}$$

Post encroachment time

The criteria are *qualitatively* different to lane changing/merging since only a *simultaneous* occupancy of two or more vehicles on conflict areas of trajectories lead to collisions



The **post encroachment time (PET or T_{PE})** is the time difference $t_2 - t_1$ between the first vehicle leaving the conflict area at t_1 and the second one entering it at t_2

- *Conflict* or **encroachment** area (encroachment=intrusion): generalisation of the trajectory intersection point for extended vehicles (fixed for known trajectories)

- You do not need a "pre encroachment time" **(why?)**
- If $T_{PE} < 0$, we have a collision, if it is smaller than, say, 2 s, we have a **near miss**
- Post-encroachment gap $s_{PE} = v_2 T_{PE}$

Safety criteria for crossing an intersection (most general case)

- Determine the paths (spatial aspect of trajectories) for all OD combinations that can go simultaneously over an intersection (if signal controlled, all that have "green")
- Check safety criteria only for ODs that cross/conflict with paths of priority OD; identify the encroachment areas (EA)
- For a deciding driver i, determine the entering and leaving times t_i and t'_i to/from each of the identified EAs according to plausible assumptions (e.g., constant acceleration to v_0)
- If there are nearby vehicles on the considered conflicting OD, calculate the PETs $t_i - t'_j$ if $t_j < t_i$ (the other driver enters the EA earlier) and $t_j - t'_i$ if $t'_j > t'_i$ (the other driver leaves the EA later) making plausible assumptions for vehicle j (e.g., constant speed, no lane changes)
- If the PET or lowest PET (for slow vehicles j, both entering and leaving conditions may be satisfied simultaneously) is less than T^c_{PE} (e.g., 2 s), safety is violated
- Calculate the safety criterion for all EAs of all conflicting ODs and give a go if all criteria are passed