

Lecture 09: Car-Following Models Based on Driving Strategies

- ▶ 9.1 Motivation
- ▶ 9.2 Gipps' Model
- ▶ 9.3 Intelligent Driver Model
- ▶ 9.4 Derivatives of the Intelligent Driver Model
- ▶ 9.5 Models for Adaptive Cruise Control
- ▶ 9.6 Human-Driver Car-Following Models

9.1 Motivation

The *plausibility criteria* of the last lesson and model completeness are necessary but not sufficient for a realistic simulation. Additional requirements for **car-following models (CF models)** include

- ▶ No accidents \Rightarrow not satisfied by the OVM
- ▶ The accelerations \dot{v} and braking decelerations have to be physically possible, e.g. $-9 \text{ m/s}^2 \leq \dot{v} \leq 4 \text{ m/s}^2 \Rightarrow$ not satisfied by the OVM, Newell's micromodel, or the CA models
- ▶ Furthermore, CF models should reflect a "normal" comfortable driving style in normal situations, e.g., $|\dot{v}| < 2 \text{ m/s}^2$ depending on the driving style \Rightarrow distinguish between emergency and normal driving
- ▶ For highly dynamic situations such as approaching a red traffic lights/standing vehicles, anticipation according to elementary kinematics (e.g., the minimum stopping deceleration $b_{\text{kin}} = v^2/(2s)$) is necessary \Rightarrow incorporate some driving strategy
- ▶ The model parameters should reflect distinct aspects of the driving style

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9.2 Gipps' model

The **Gipps model** explicitly satisfies the kinematics in highly dynamic situations

$$v(t + T) = \min [v_{\text{free}}, v_{\text{safe}}]$$

- ▶ The free-acceleration part obeys, e.g., $v_{\text{free}} = \min(v(t) + aT, v_0)$ with acceleration a or some more complicated acceleration profile.
- ▶ The safe speed is based on following heuristic *worst-case scenario* where a minimum gap s_0 should be kept at all times:
 - ▶ The leader suddenly brakes at deceleration b_l to a full stop,
 - ▶ the follower brakes at deceleration b after a reaction time T . For extra safety, another "brake hitting time" ϑ is assumed (somewhat inconsequential),
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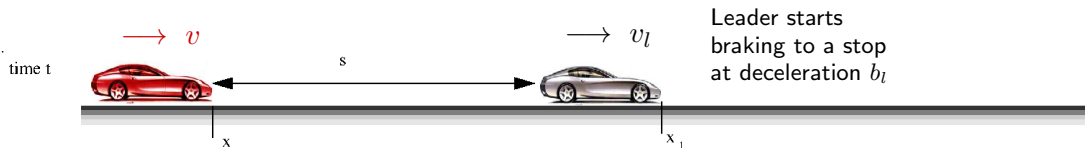
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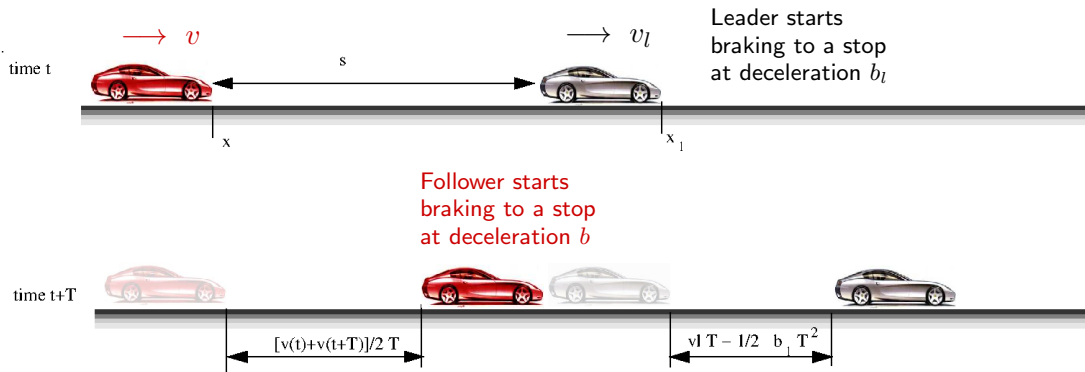
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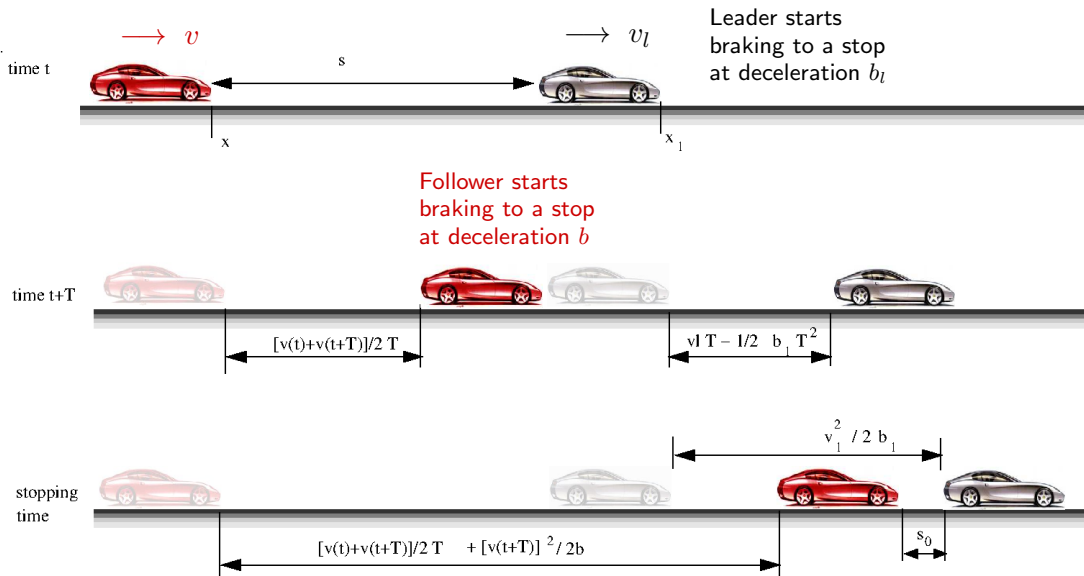
Derivation of the Gipps model: Overview



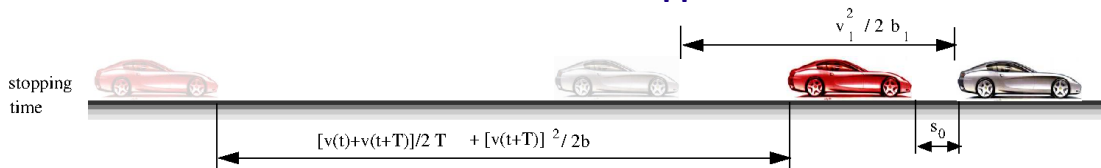
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Derivation of the Gipps model



Find the safe speed $v(t + T) = v_{\text{safe}}$:

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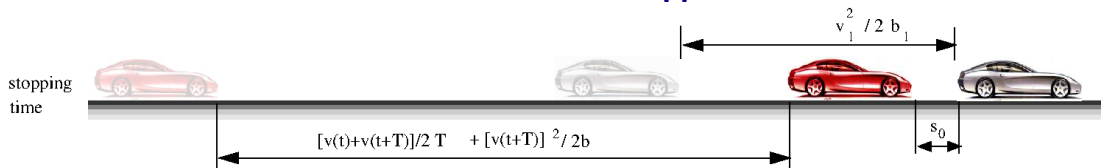
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Assume a "brake hitting time" $\vartheta = T/2 \Rightarrow$ quadratic equation

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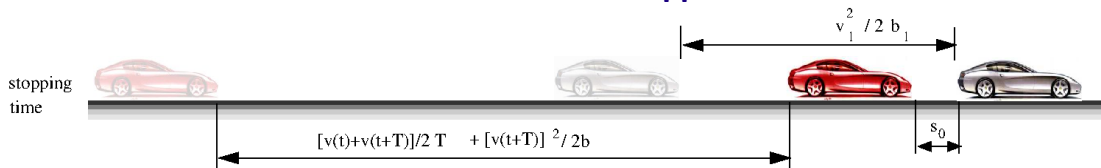
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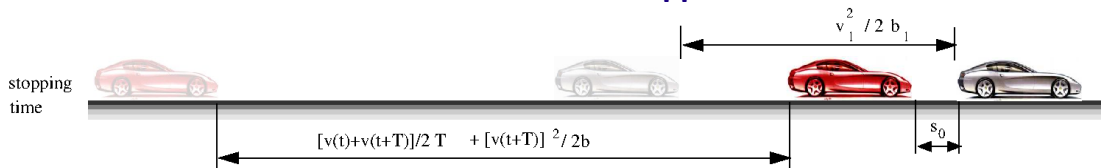
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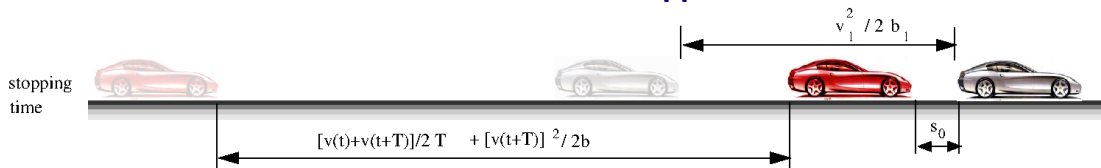
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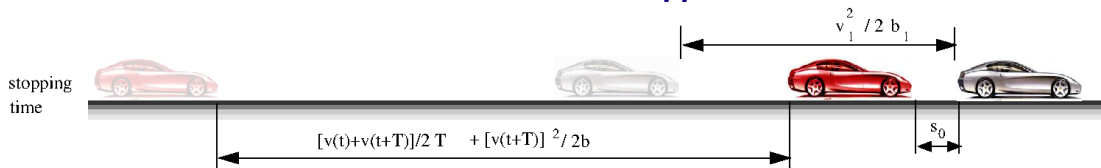
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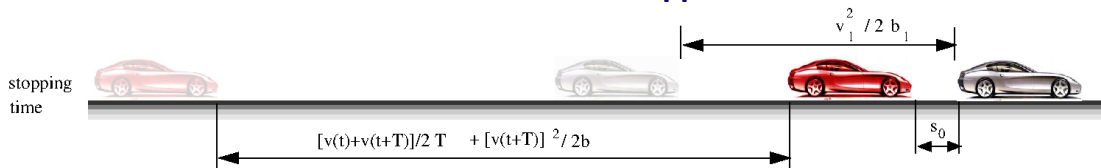
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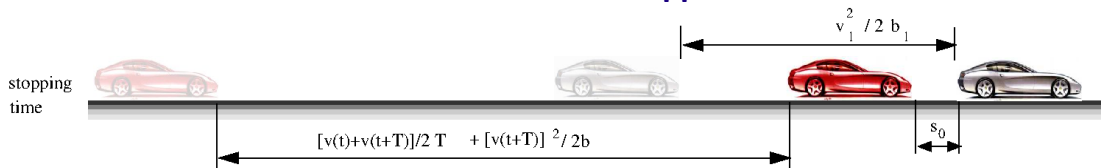
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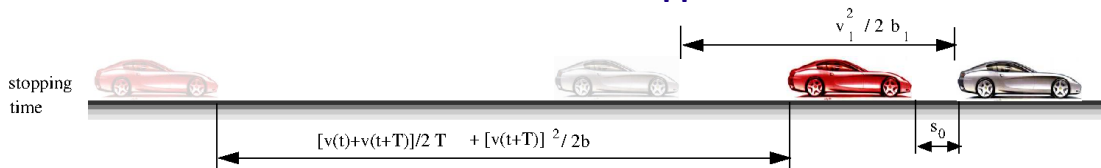
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The simplified Gipps model

The simplified version makes following assumptions:

- ▶ Constant acceleration a in the free-flow regime until reaching the desired speed v_0
- ▶ No acceleration is assumed during the reaction time T and the brake hitting time ϑ is zero. So, these assumptions just calculate the speed which *would* prevent a crash in the worst case if it were adopted instantaneously and held constant during T . Hence, the *reaction distance* of the follower is simply given by $\Delta x_{\text{react}} = v(t)T = v_{\text{safe}}T$
- ▶ The leader and the follower have the same braking capabilities $b_l = b$

This leads to following quadratic equation:

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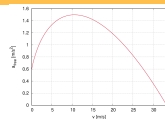
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The final Gipps models

$$v(t + T) = \min [v + a_{\text{free}}(v)T, v_{\text{safe}}(s, v, v_l)] \quad \text{Full Gipps Model}$$

$$a_{\text{free}}(v) = 2.5a \left(1 - \frac{v}{v_0}\right) \sqrt{0.025 + \frac{v}{v_0}}$$



$$v_{\text{safe}}(s, v, v_l) = -b(T/2 + \vartheta) + \sqrt{b^2(T/2 + \vartheta)^2 + 2b(s - s_0) + v_l^2 \frac{b}{b_l} - vbT}$$

The model is for general brake hitting times ϑ . For the standard value $\vartheta = T/2$, simplify $b(T/2 + \vartheta) \rightarrow bT$

$$v(t + T) = \min [v + aT, v_0, v_{\text{safe}}(s, v_l)] \quad \text{Simplified Gipps Model}$$

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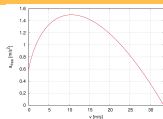
Freeway parameters: $v_0 = 35 \text{ m/s}$, $a = b = b_l = 1.5 \text{ m/s}^2$, $T = 1.1 \text{ s}$, $\vartheta = T/2$, $s_0 = 2 \text{ m}$

City parameters: just reduce the desired speed v_0

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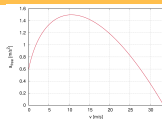
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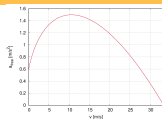
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$$a_{\text{free}}(v) = 2.5a \left(1 - \frac{v}{v_0}\right) \sqrt{0.025 + \frac{v}{v_0}},$$



$$v_{\text{safe}}(s, v, v_l) = -b(T/2 + \vartheta) + \sqrt{b^2(T/2 + \vartheta)^2 + 2b(s - s_0) + v_l^2 \frac{b}{b_l} - vbT}$$

The model is for general brake hitting times ϑ . For the standard value $\vartheta = T/2$, simplify $b(T/2 + \vartheta) \rightarrow bT$

$$v(t + T) = \min [v + aT, v_0, v_{\text{safe}}(s, v_l)] \quad \text{Simplified Gipps Model}$$

$$v_{\text{safe}}(s, v_l) = -bT + \sqrt{b^2T^2 + 2b(s - s_0) + v_l^2}$$

Freeway parameters: $v_0 = 35 \text{ m/s}$, $a = b = b_l = 1.5 \text{ m/s}^2$, $T = 1.1 \text{ s}$, $\vartheta = T/2$, $s_0 = 2 \text{ m}$

City parameters: just reduce the desired speed v_0

Homogeneous steady state and fundamental diagram of the Gipps models I: Free-flow regime

Unlike the past CF-models, the Gipps model(s) do not have an explicit fundamental diagram (FD) given by the OV function \Rightarrow must be calculated by assuming a **stationary steady state**:

- ▶ **Stationarity**: $\frac{d}{dt} = 0$, so $v(t + T) = v(t)$
- ▶ **Homogeneity**: $\frac{d}{dx} = 0$, so $v_l(t) = v(t)$

Free-flow regime:

$$v(t + T) = v(t) \Rightarrow a_{\text{free}}(v) = 0 \Rightarrow v = v_0$$

Does the free-flow Gipps model include any interactions in the free-flow regime?

No, not any! strict separation of regimes by the min-function!

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Homogeneous steady state and fundamental diagram of the Gipps models II: Interaction regime

Here, the second part of the min-function applies:

$$v(t+T) = v = v_{\text{safe}} = v_l$$

$$v = -b(T/2 + \vartheta) + \sqrt{b^2(T/2 + \vartheta)^2 + 2b(s - s_0) + v^2 \frac{b}{b_l} - vbT}$$

$$(v + b(T/2 + \vartheta))^2 = b^2(T/2 + \vartheta)^2 + 2b(s - s_0) + v^2 \frac{b}{b_l} - vbT$$

Quadratic equation for $v_e(s)$ or linear equation for $s_e(v)$:

$$s_e^{\text{Gipps}}(v) = s_0 + vT + v\vartheta + \frac{v^2}{2b} \left(1 - \frac{b}{b_l}\right)$$

Shape of the FD for the special case of the simplified Gipps model?

$s_e = s_0 + vT \Rightarrow$ triangular FD

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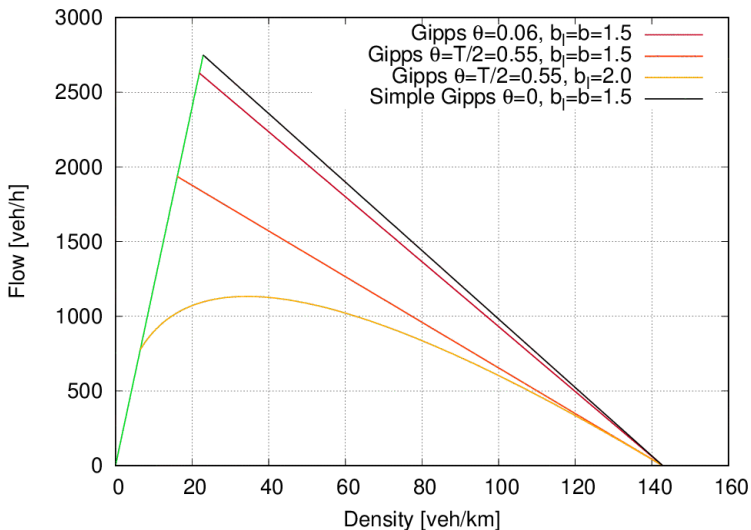
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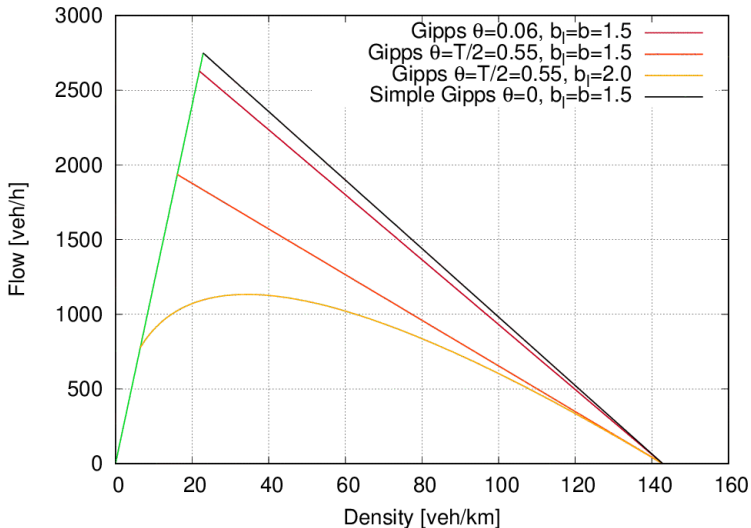
Fundamental diagram of the Gipps model variants



The Drivers of the Gipps model become more defensive

- ▶ with increasing reaction time T and brake hitting times ϑ
- ▶ with increasing implied leader deceleration b_l
- ? Why traffic becomes unstable for $b_l < b$?
- ! Since the follower thinks he/she can brake harder than the leader. Along the whole string of vehicles ...

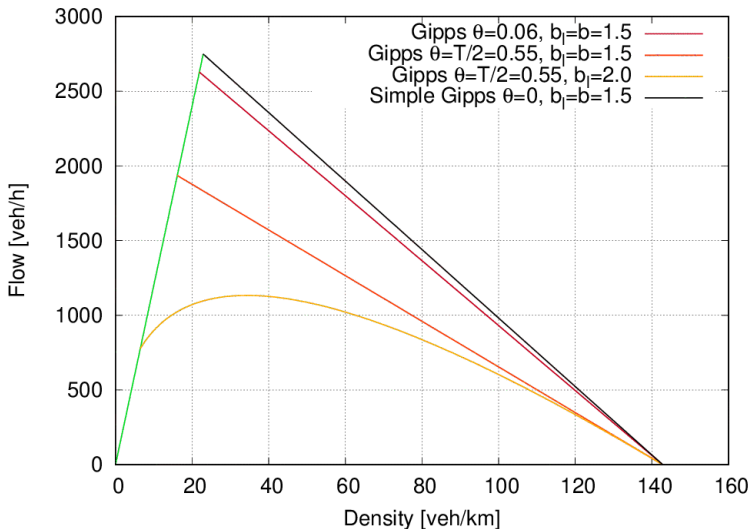
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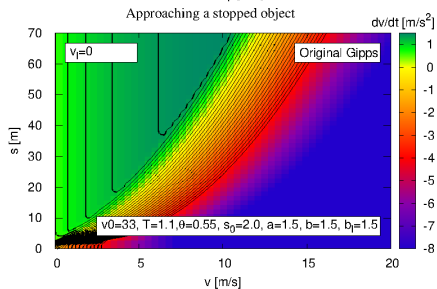
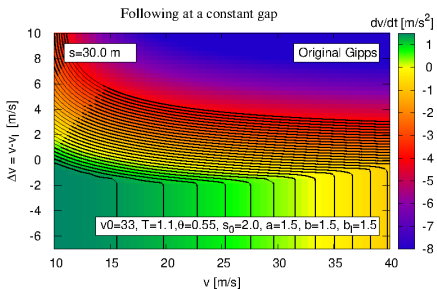
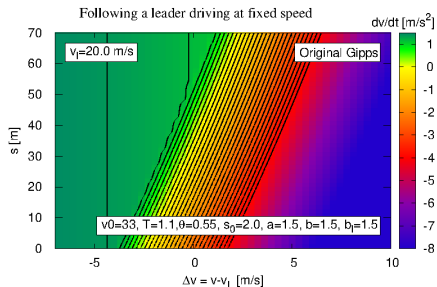
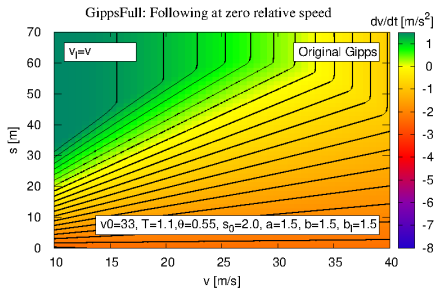
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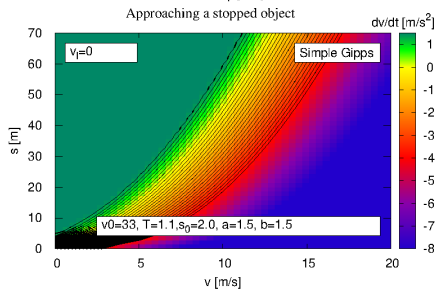
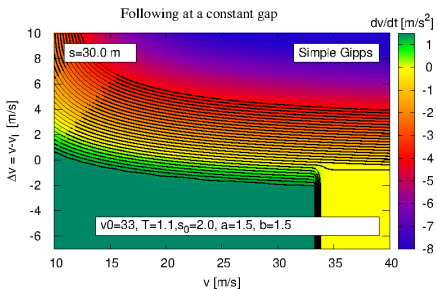
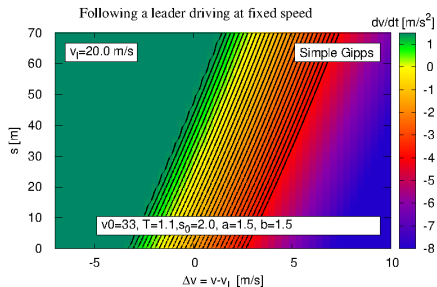
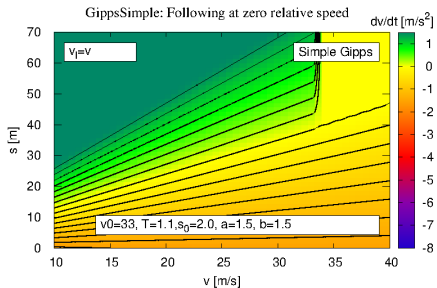
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Gipps model acceleration function

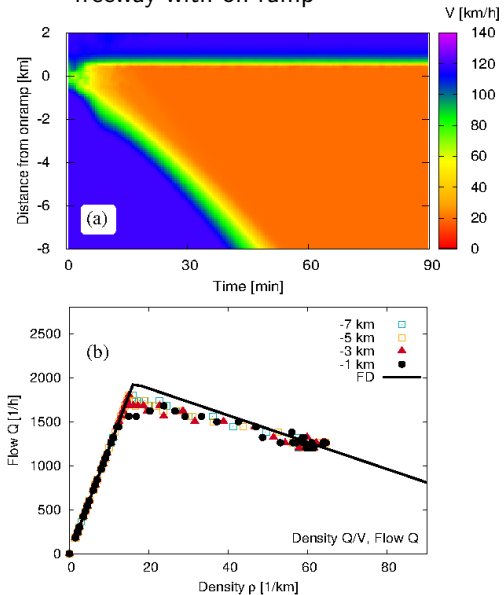


Simplified Gipps model acceleration function

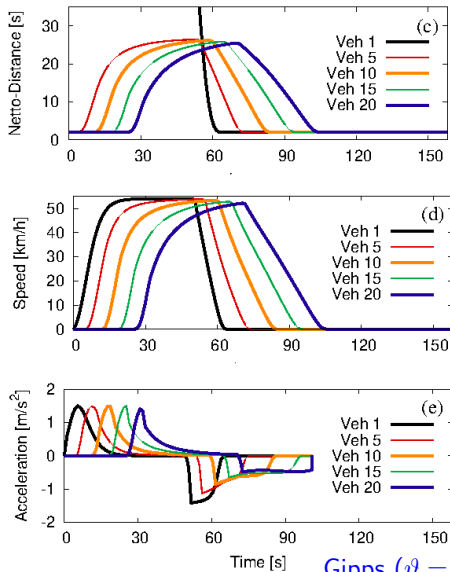


Factsheet of the original Gipps model ($\vartheta = 0.5$)

freeway with on-ramp



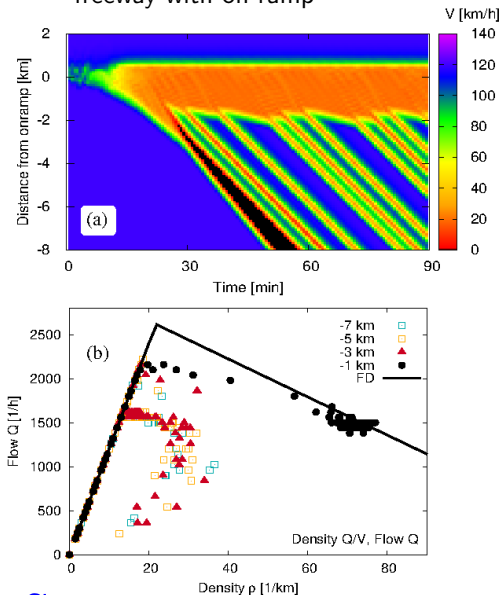
city with traffic lights



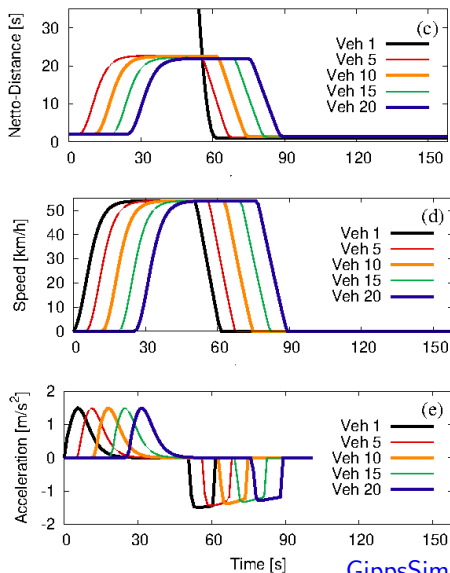
Gipps ($\vartheta = 0.06$) \Rightarrow

Factsheet of the original Gipps model with $\vartheta = 0.06$

freeway with on-ramp

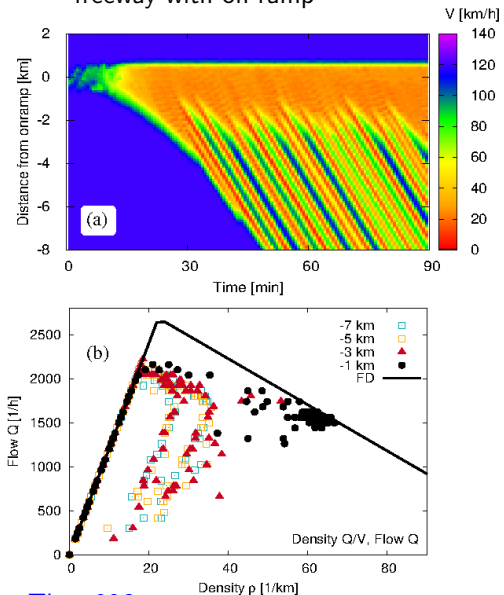


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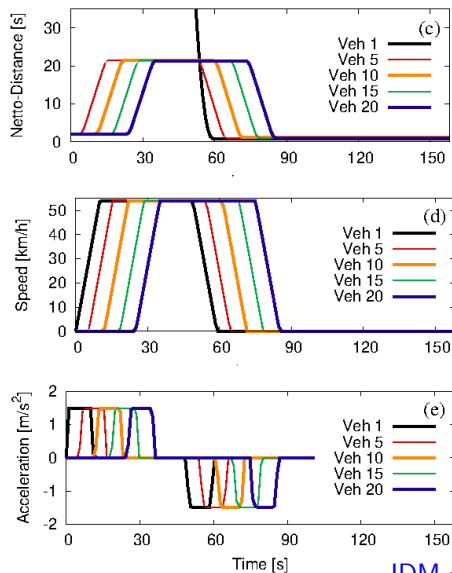


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freeway with on-ramp



city with traffic lights



9.3 Intelligent Driver Model (IDM)

Probably the most parsimonious car-following model satisfying following conditions:

- ▶ All *plausibility conditions* satisfied
- ▶ *smooth driving regime transitions* (i.e., a smooth or even differentiable acceleration function), unlike the Gipps model
- ▶ *collision free* if physically possible
- ▶ *unique feature*: Continuous and stable transition from an emergency to a regular braking maneuver by an *intelligent* driving strategy
- ▶ all model parameters are *intuitive* describing distinct aspects of the driving behavior: aggressive/timid, anticipative/short-sighted, responsive/sleepy, and of course slow/fast

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IDM equations

$$\frac{dv}{dt} = a \left[1 - \left(\frac{v}{v_0} \right)^4 - \left(\frac{s^*(v, v_l)}{s} \right)^2 \right] \quad \text{IDM acceleration}$$

free acceleration: $a[1 - (v/v_0)^4]$, repulsive force: $-a(s^*/s)^2$

$$s^*(v, v_l) = s_0 + \max \left(0, vT + \frac{v(v - v_l)}{2\sqrt{ab}} \right) \quad \text{desired gap}$$

Parameter	Cars High-way	Cars City	Trucks Hwy	Bicycles
Desired speed v_0	120 km/h	50 km/h	80 km/h	20 km/h
Time gap T	1.0 s	1.0 s	1.8 s	0.6 s
Minimum gap s_0	2 m	2 m	3 m	0.4 m
Acceleration a	1.5 m/s ²	2.0 m/s ²	0.5 m/s ²	1.0 m/s ²
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Minimum gap s_0	2 m	2 m	3 m	0.4 m
Acceleration a	1.5 m/s ²	2.0 m/s ²	0.5 m/s ²	1.0 m/s ²
Comf. deceleration b	1.5 m/s ²	2.0 m/s ²	1.0 m/s ²	1.5 m/s ²

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IDM equations

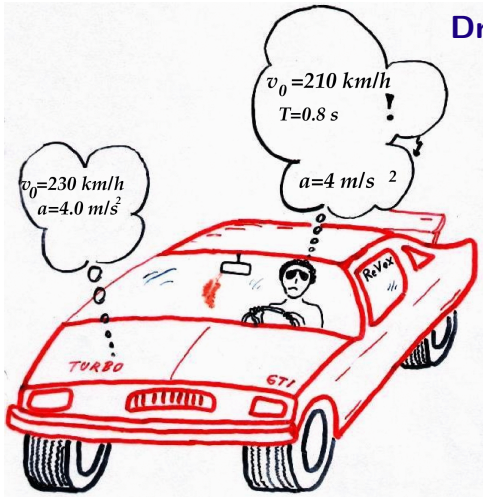
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Driving styles



Aggressive driver:

v_0 , a and b high, T and s_0 low

Experienced responsive driver:

a high, b low, rest normal

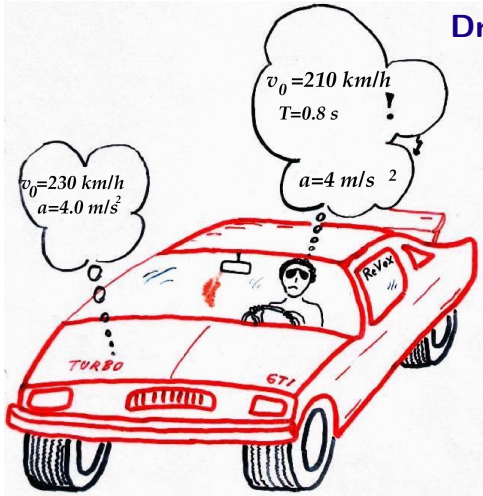
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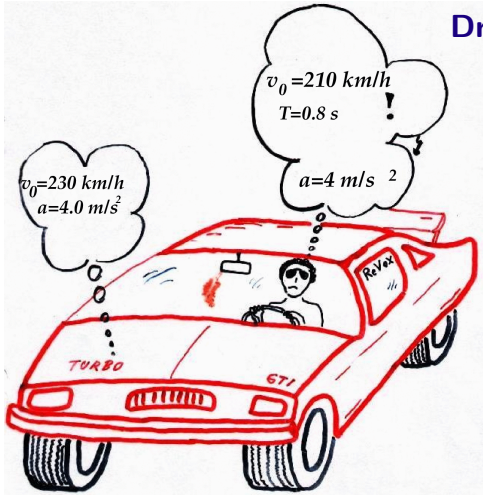
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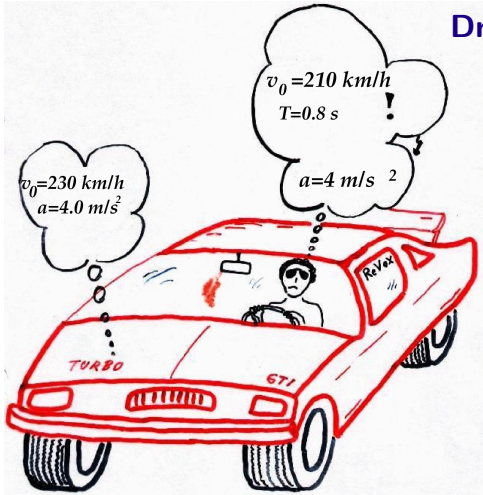
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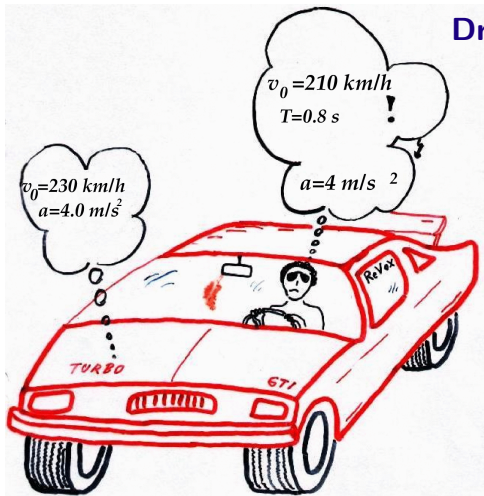
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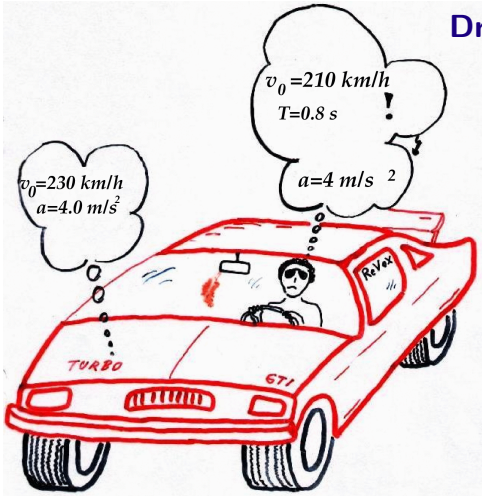
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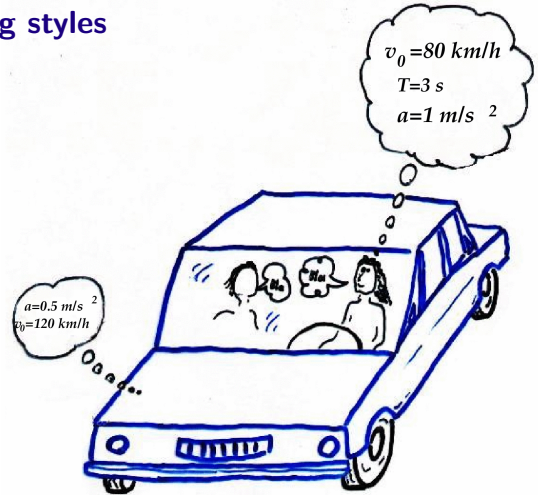


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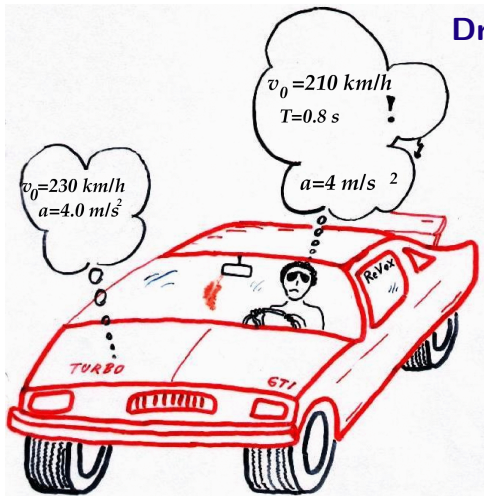
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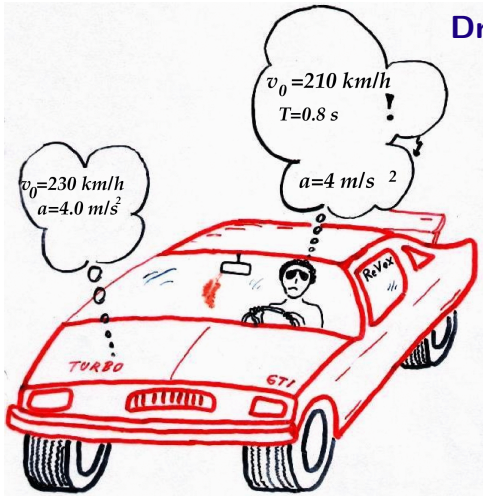
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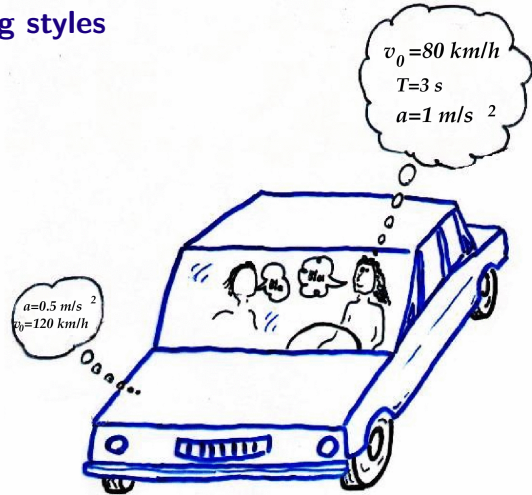


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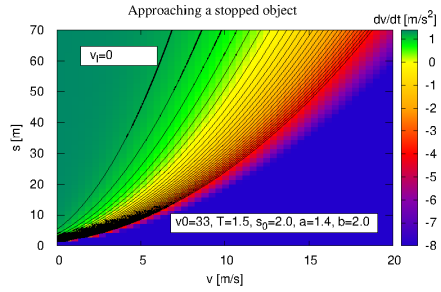
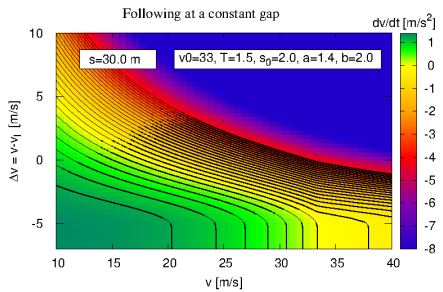
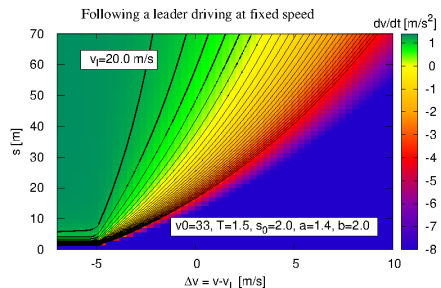
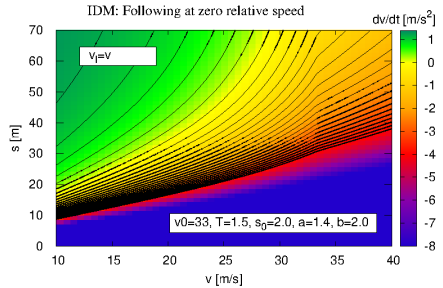
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IDM acceleration function



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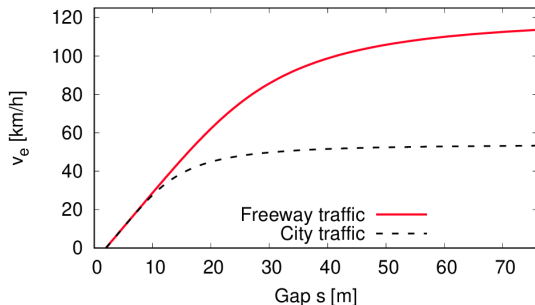
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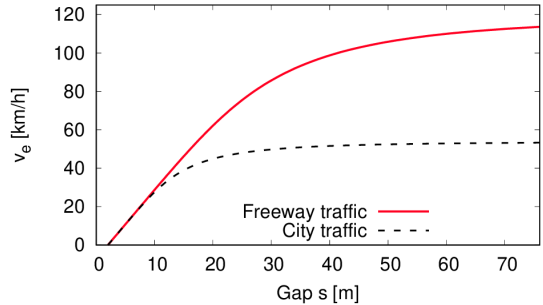
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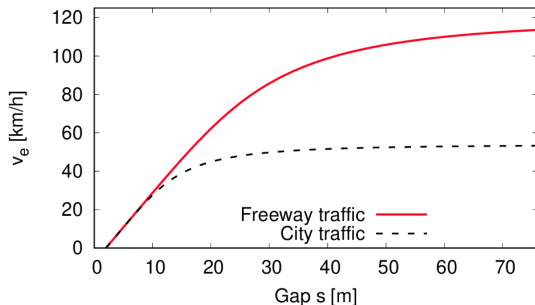
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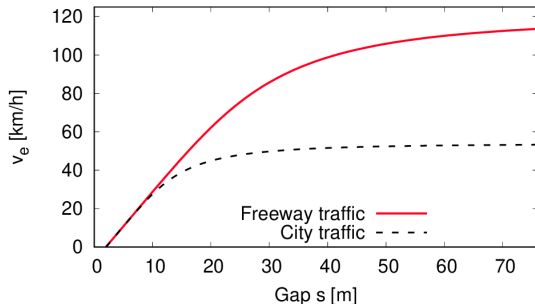
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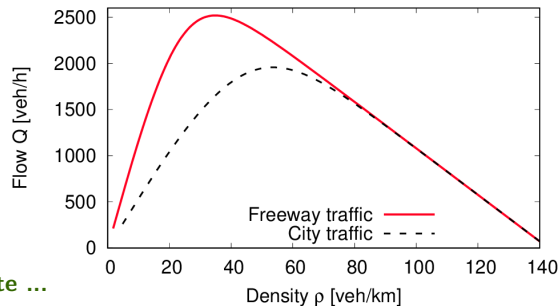
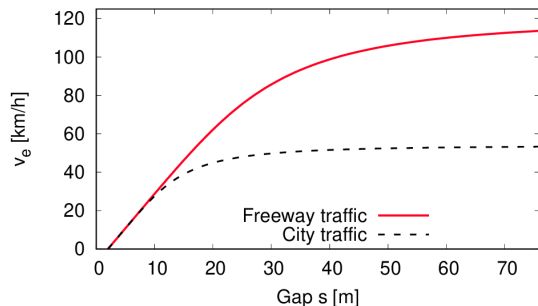
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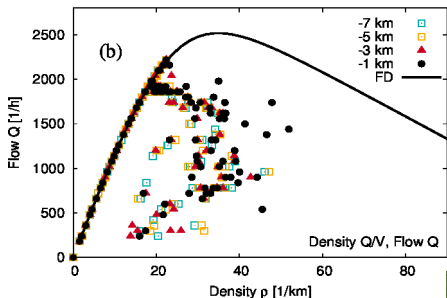
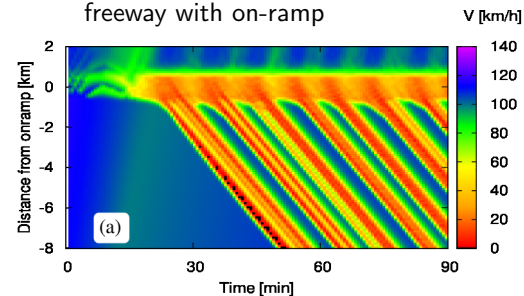
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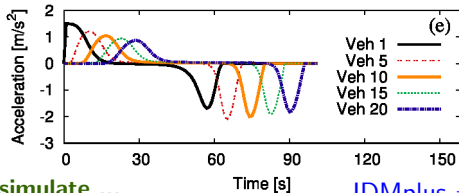
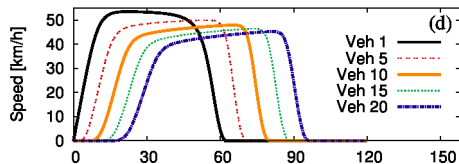
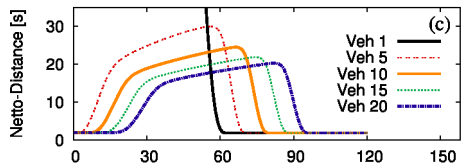


Factsheet of the Intelligent-Driver Model (IDM)

freeway with on-ramp



city with traffic lights



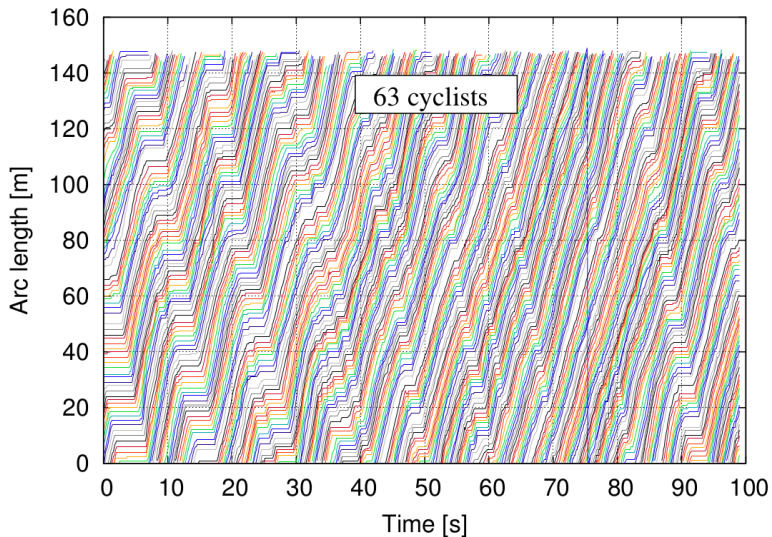
← Gipps



simulate ...

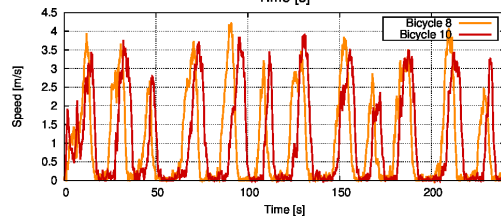
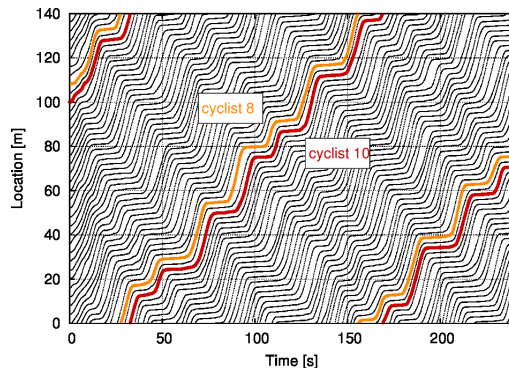
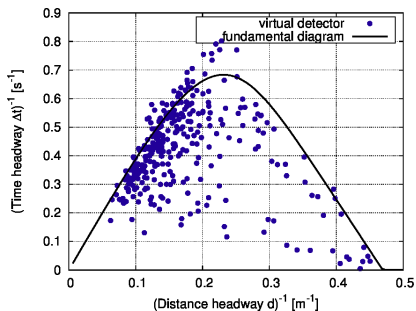
IDMplus →

IDM for bicycle traffic? Single-file bicycle traffic experiment



simulate ...

Simulating single-file bicycle traffic with the IDM



$$l_{\text{bike}} = 1.7 \text{ m}, v_0 = 4 \text{ m/s}, T = 0.6 \text{ s},$$

$$s_0 = 0.4 \text{ m}, a = 0.8 \text{ m/s}^2, b = 1.5 \text{ m/s}^2$$

9.4 Derivatives of the Intelligent Driver Model

For a realistic driving feeling or for use as the core of an ACC controller, the IDM still has several deficiencies:

- ▶ When reaching the desired speed, the steady-state time gap

$$T = \frac{s_e(v) - s_0}{v} = \frac{T}{\sqrt{1 - (v/v_0)^4}}$$

becomes significantly larger than T leading to a somewhat unrealistic platoon behaviour in the *city with traffic lights* situation.

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IDM with triangular fundamental diagram: IDM+

The time-gap deficiency can be solved by following modification:

$$\frac{dv}{dt} = \min [a (1 - (v/v_0)^4), a (1 - (s^*/s)^2)] \quad \text{IDM+}$$

- ▶ The acceleration function is no longer smooth but still continuous
- ▶ Instead of the continuous transition of the IDM, the IDM+ has two distinct regimes: free acceleration (the first expression of the min function is relevant), and interacting (the second expression matters)
- ▶ Steady-state time gap: $\frac{dv}{dt} = 0 \Rightarrow$ if $v < v_0$, the second expression in the min-condition matters
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- ▶ Steady-state time gap: $\frac{dv}{dt} = 0 \Rightarrow$ if $v < v_0$, the second expression in the min-condition matters
 $\Rightarrow s = s^*(v, v) = s_0 + vT \Rightarrow$ constant time gap and triangular FD
- ▶ The *intelligent* braking strategy is not affected (see the following plots)

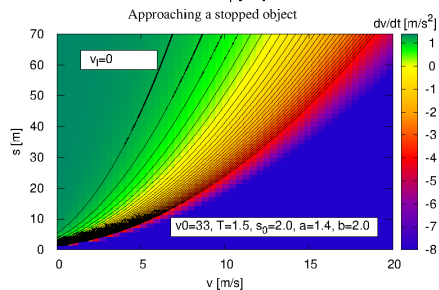
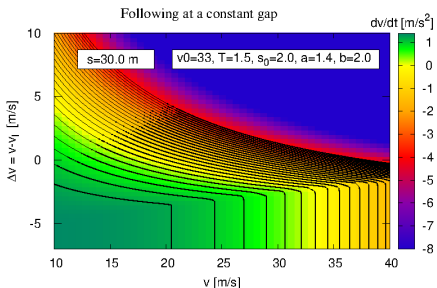
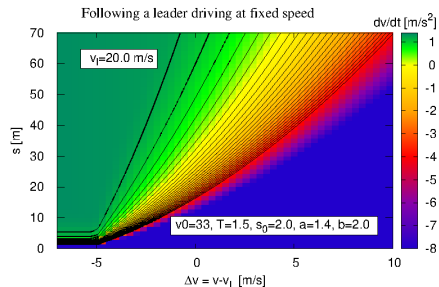
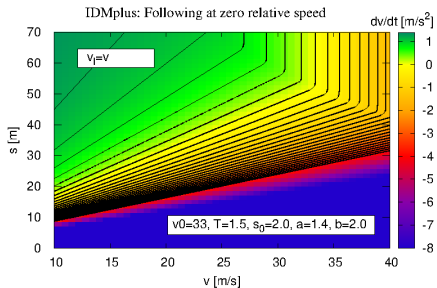
IDM with triangular fundamental diagram: IDM+

The time-gap deficiency can be solved by following modification:

$$\frac{dv}{dt} = \min [a (1 - (v/v_0)^4), a (1 - (s^*/s)^2)] \quad \text{IDM+}$$

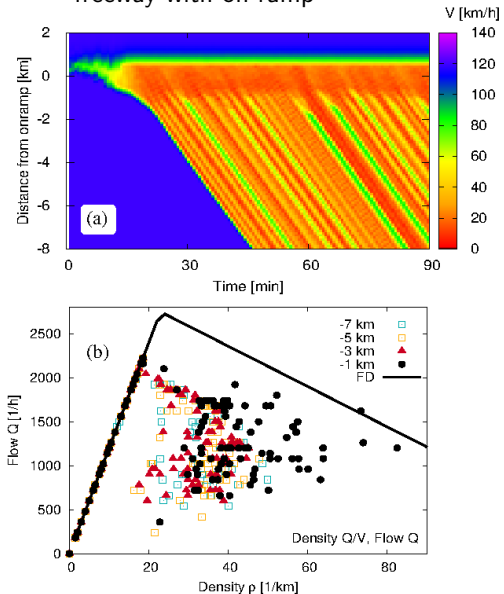
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IDM+ acceleration function

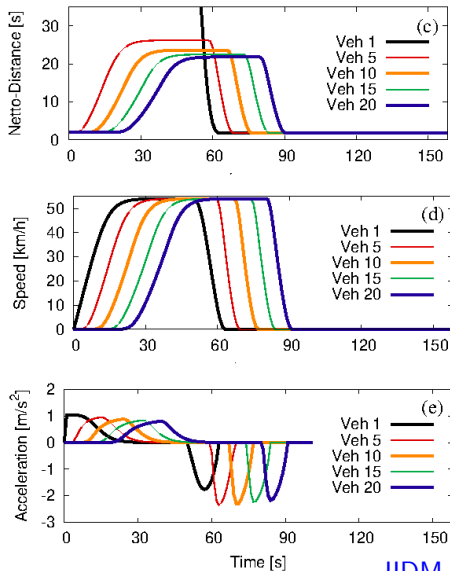


Factsheet of the Improved IDM (IDM+)

freeway with on-ramp



city with traffic lights

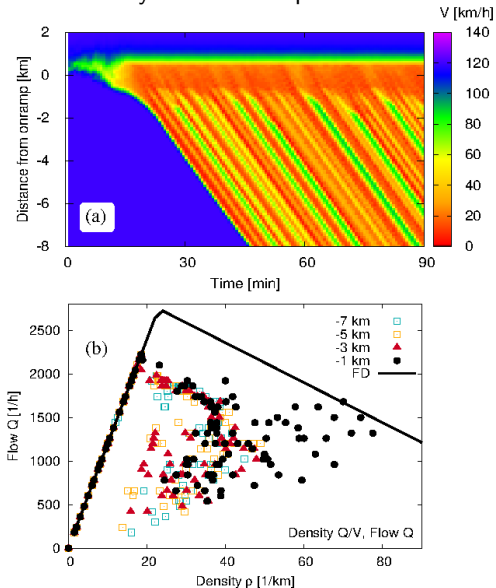


Another IDM with triangular fundamental diagram: IIDM

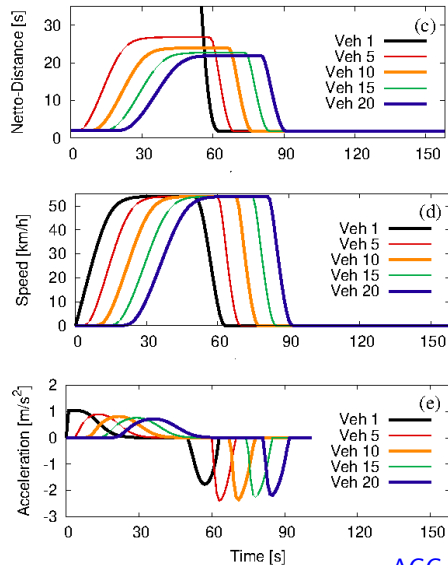
- ▶ Another possibility to obtain an IDM-like model with a triangular FD and the intelligent braking strategy unaffected
- ▶ In contrast to the IIDM, the acceleration function is smooth
- ▶ However, this implies a more complicated formulation (not shown)

Factsheet of the Improved IDM (IIDM)

freeway with on-ramp



city with traffic lights



9.5 Models for Adaptive Cruise Control

- ▶ Besides a triangular FD (i.e., constant time gaps in the following regime), an ACC model needs to be robust against changing leading objects caused, e.g., by active or passive lane changes
- ▶ This is realized by replacing the worst-case heuristics of the IDM by a more realistic “constant acceleration heuristics”: Human drivers also do not expect a full braking maneuver to the stop *out of the blue* (and would not be able to handle it)
- ▶ In contrast, because the ACC model does only have insignificant reaction delays (all IDM variants presented in this lecture have zero reaction time!), the ACC controller could even handle this
- ▶ The actual model is not shown, just the results

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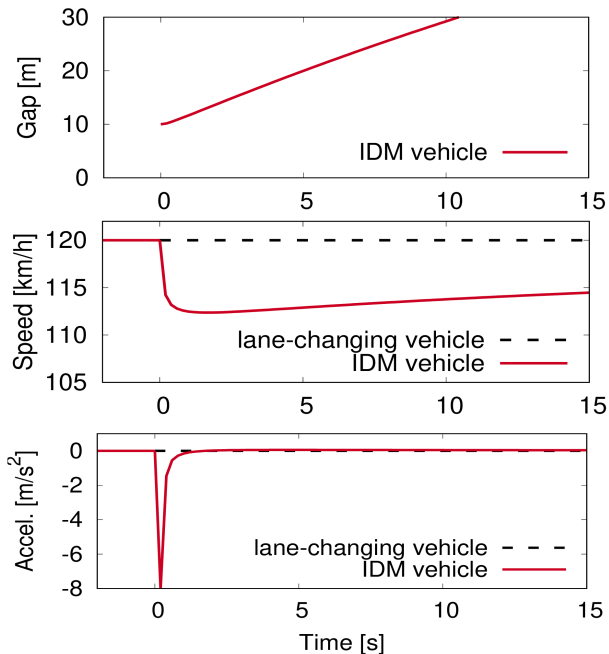
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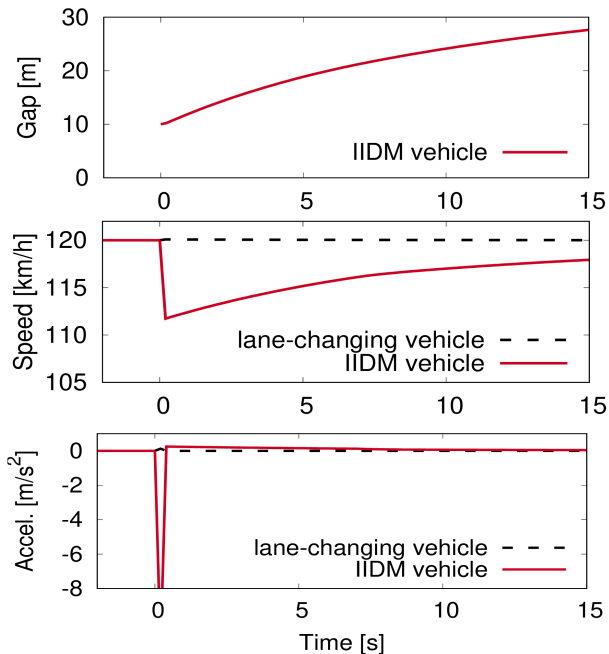
Response to a close cut-in maneuver at same speed: IDM



Lane-changing vehicle:
 same speed 120 km/h as
 the follower, cuts in leaving
 a gap of 10 m

IDM responds too panically

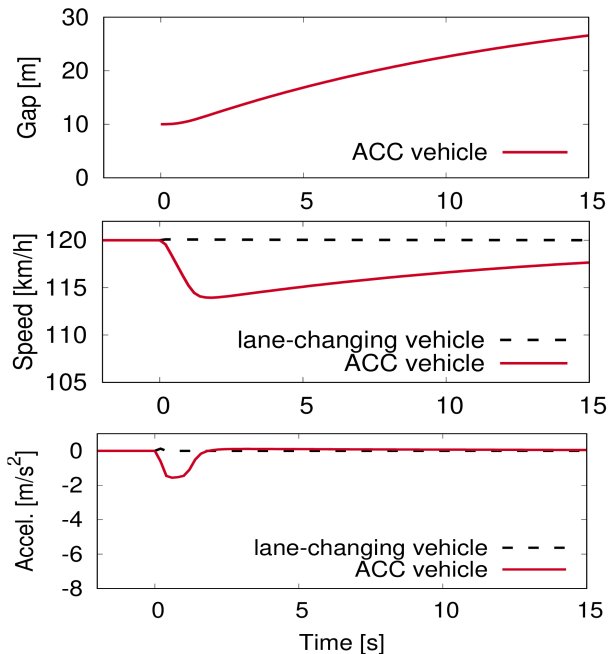
Response to a close cut-in maneuver at same speed: IIDM



Lane-changing vehicle:
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IIDM response similarly
 panically but has a better
 following behaviour
 afterwards

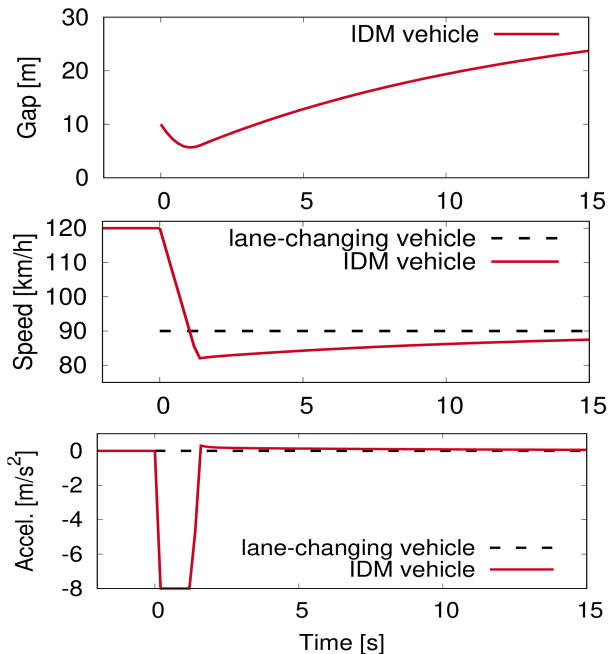
Response to a close cut-in maneuver at same speed: ACC



Lane-changing vehicle:
same speed 120 km/h as
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The (IDM+-based) ACC
model has a *cool* immedi-
ate response and a plausi-
ble following behaviour af-
terwards

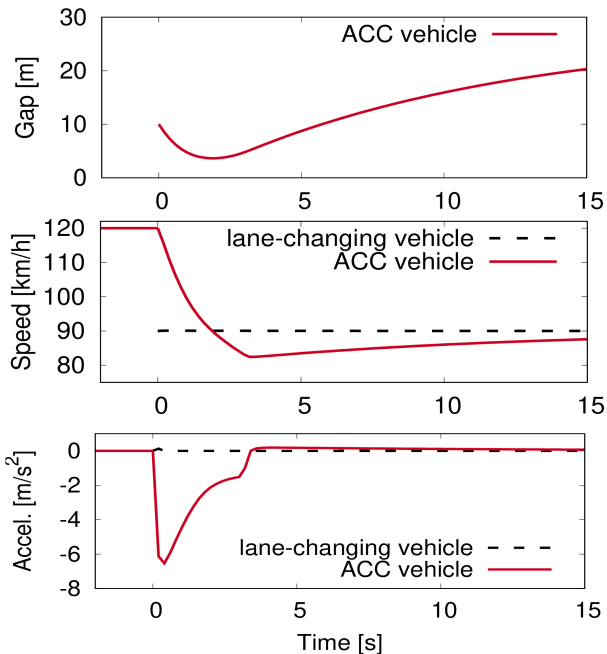
Response to a critical cut-in maneuver: IDM



Lane-changing vehicle:
30 km/h slower than the
follower, cuts in leaving a
gap of just 10 m

IDM switches to emergency
mode which is right in this
situation

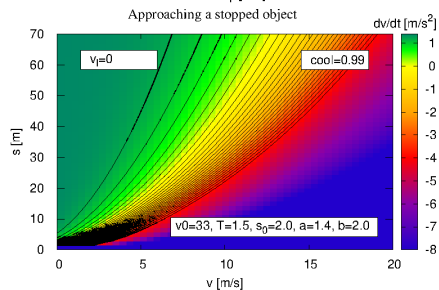
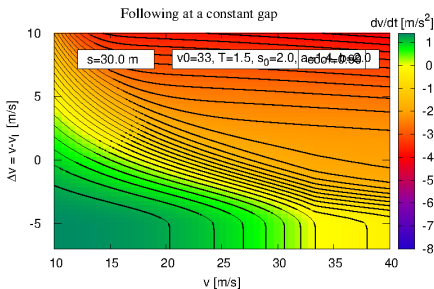
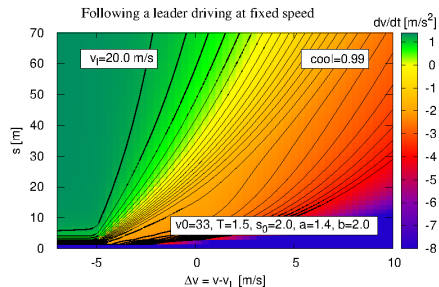
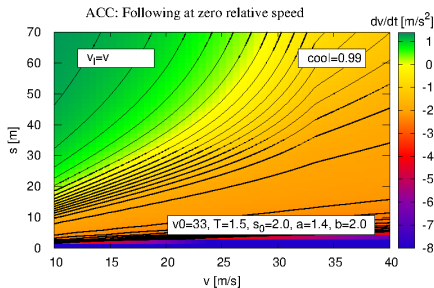
Response to a critical cut-in maneuver: ACC model



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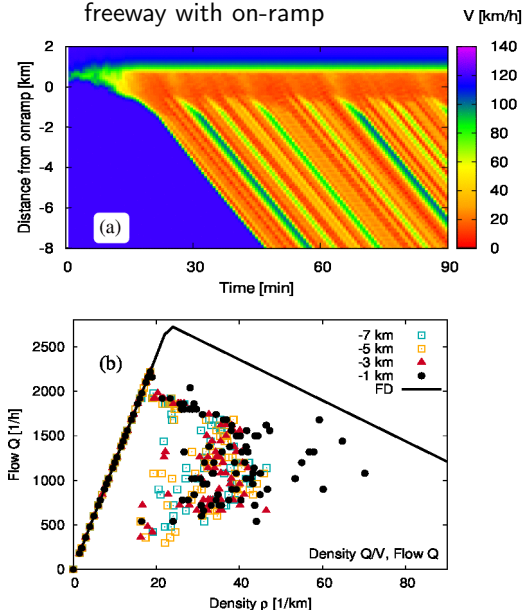
Also the ACC model loses
its *coolness* which is com-
pletely justified in this situ-
ation

ACC model acceleration function

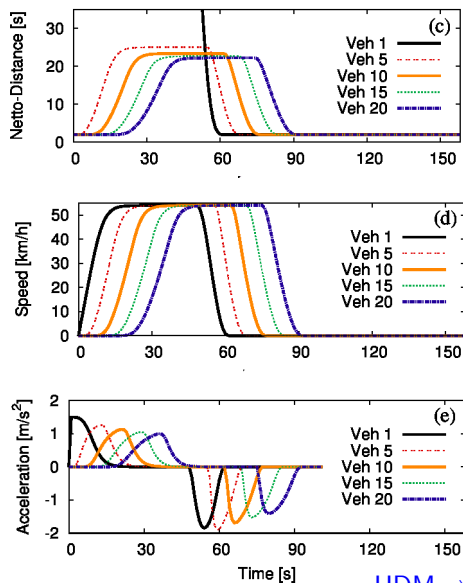


Factsheet of the ACC model

freeway with on-ramp



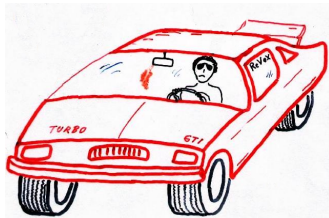
city with traffic lights



← IDM+

HDM →

9.6 Human-Driver Car-Following Models

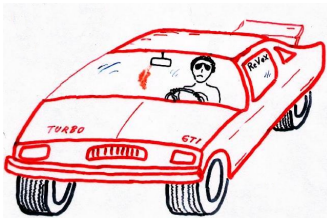


In contrast to ACC controllers, humans have ...



- ▶ Significant **reaction times**
⇒ state $s(t - T_r)$, $v_l(t - T_r)$
- ▶ Response **thresholds** (⇒ Wiedemann)
- ▶ Risk **attitude** (⇒ **Prospect Theory**)
- ▶ Correlated **estimation errors** in s , v , and v_l and general acceleration noise
- ▶ Temporal **anticipation**:
 $s(t + T_a) = s(t) + T_a (v_l(t) - v(t))$
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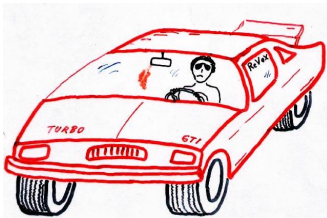


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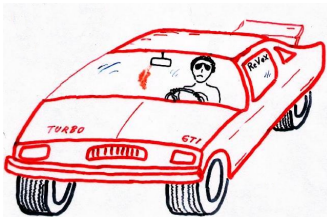


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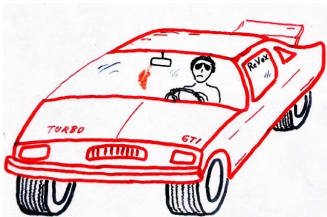


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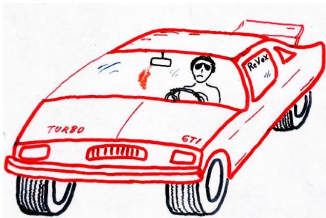


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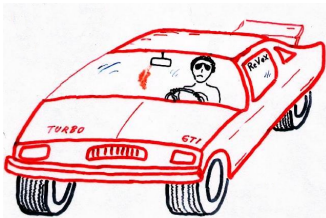


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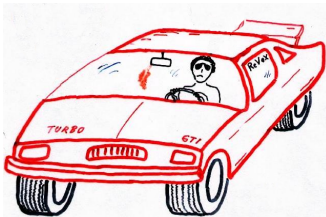


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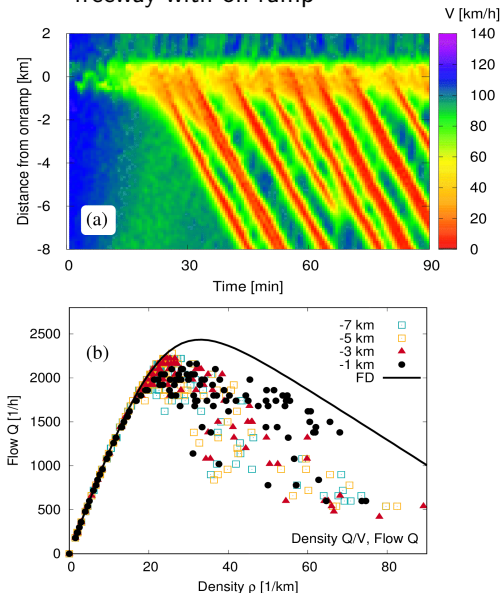
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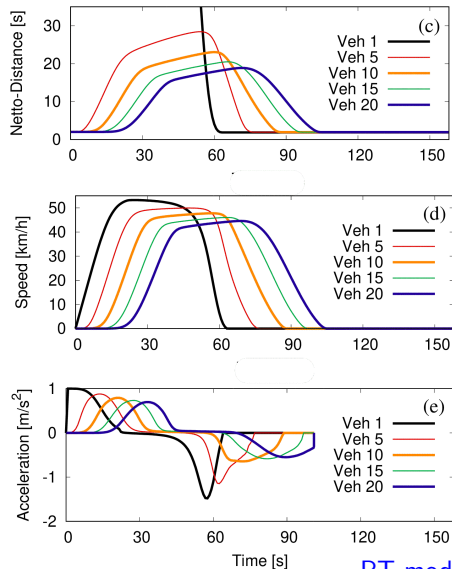
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Factsheet of the IDM-based Human Driver Model

freeway with on-ramp



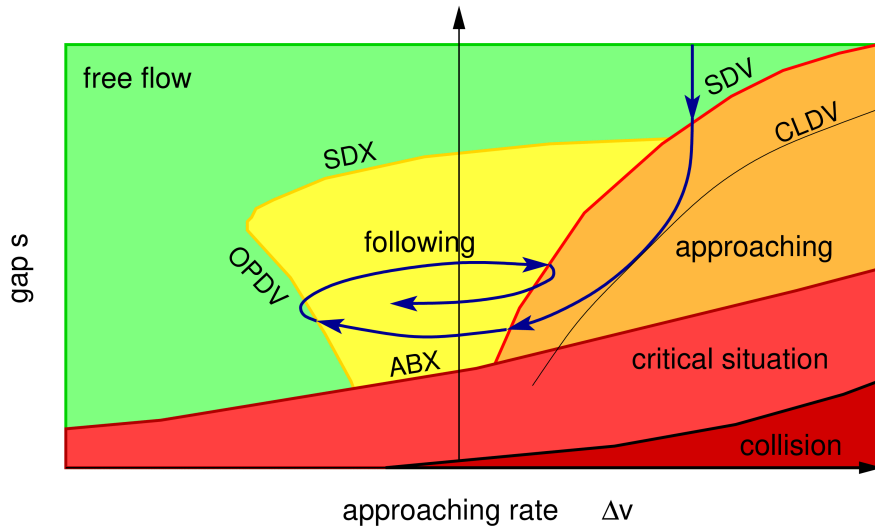
city with traffic lights



⇐ ACC model

PT model ⇒

Response thresholds: Wiedemann trajectories in space-relative speed space



Base model in VISSIM

CF models based on risk perception: Prospect Theory of Kahneman and Tversky

Prospect theory is a variant of **Expected Utility Theory (EUT)**:

- ▶ Given is a decision situation where, depending on the action a , a discrete set of outcomes $k \in \mathcal{K}(a)$ with utilities $U_k(a)$ can happen with probabilities $P_k(a)$
- ▶ The *Homo Oeconomicus*' action a tries to maximize the **expected utility**

$$E(U) = \sum_{k \in \mathcal{K}(a)} P_k(a) U_k(a) \stackrel{!}{=} \max_a$$

- ▶ The actions a can be discrete such as accepting an offer or not, or continuous such as deciding on an acceleration
- ▶ In **Prospect Theory**, both the probabilities and the utilities get a subjective bias and the outcome weighted in this way is called a *prospect*:
 - ▶ Small probabilities are overestimated (for probabilities > 0.5 , the complement probability is considered)
 - ▶ At a certain framing reference, the sensitivity to utility changes is at its maximum
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Examples

1. Taking part in a lottery: a lot costs 1 €, the probability of winning 95 € (outcome 1) is 1 %:
 - ▶ Action "Y": $P_1 = 0.01, U_1 = 95 - 1 = 94, P_2 = 0.99, U_2 = -1$,
Action "N": Only outcome $k = 2$ with certainty ($P_2 = 1, U_2 = 0$)
 - ▶ EUT: $E(\text{"Y"}) = 0.01 \cdot 94 + 0.99 \cdot (-1) = -0.05, E(\text{"N"}) = 0 \Rightarrow$ decision "N"
 - ▶ PT: The loss aversion and the reference effect shift the decision towards "N", the positively biased probability P_1 towards it \Rightarrow depends on the person
2. Signing an insurance contract. The insurance costs 1 € and protects from a damage of 95 € (outcome 1) occurring at a probability of 1 %
 - ▶ Action "Y": $P_1 = 0.01, U_1 = -1, P_2 = 0.99, U_2 = -1$,
Action "N": $P_1 = 0.01, U_1 = -95, P_2 = 0.99, U_2 = 0$
 - ▶ EUT: $E(\text{"Y"}) = -1, E(\text{"N"}) = -0.95 \Rightarrow$ decision "N"
 - ▶ PT: Here, the loss aversion and the subjective increase of P_1 probably prevails over the reference effect and the insurance is taken ("Y")
3. Sitting in a vehicle and deciding on the acceleration (continuous-valued action) a . Outcomes $k = 1$: "crash" and $k = 2$: "no crash" where $P_1(a) = 1 - P_2(a)$ increases with a

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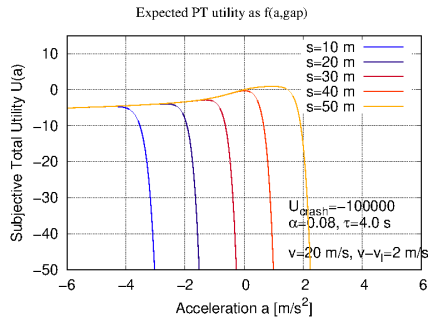
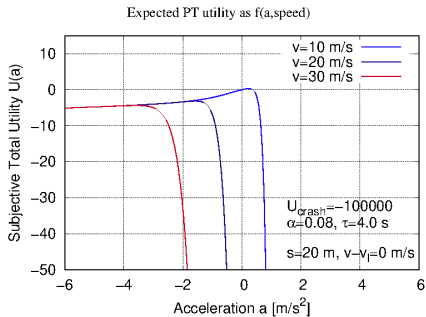
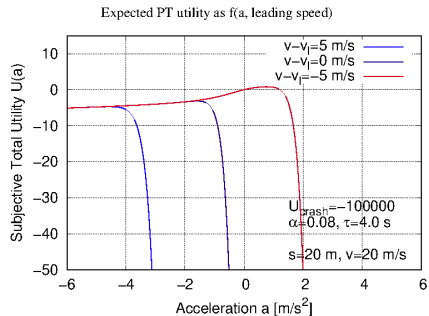
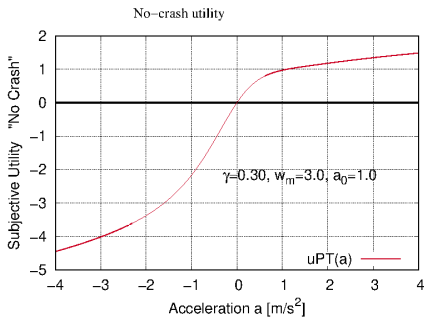
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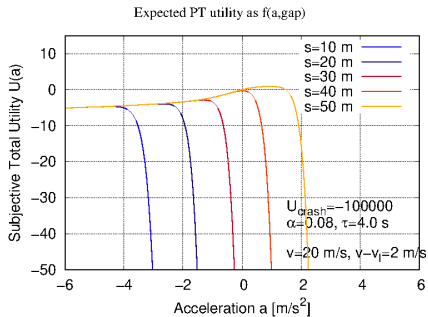
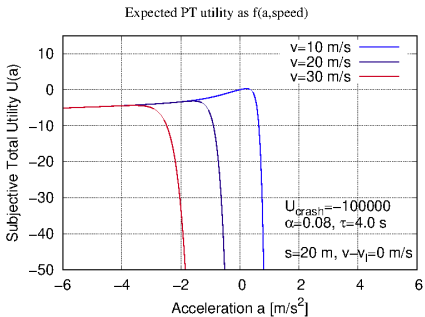
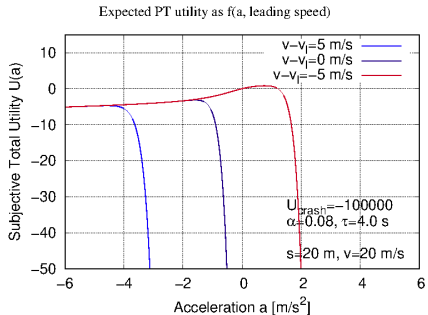
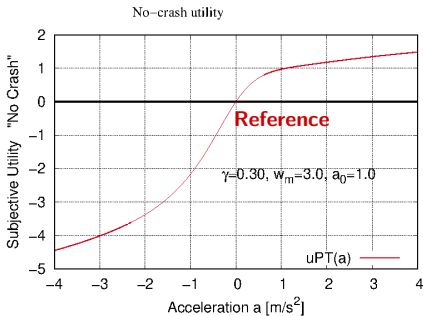
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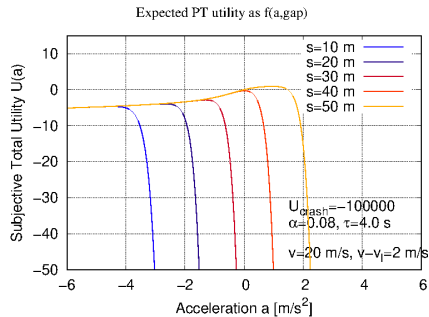
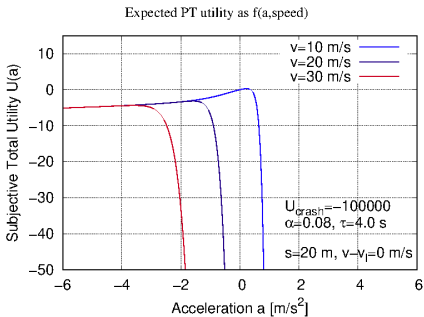
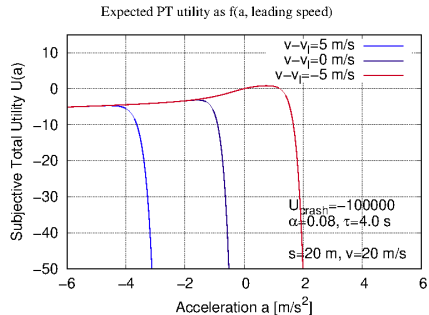
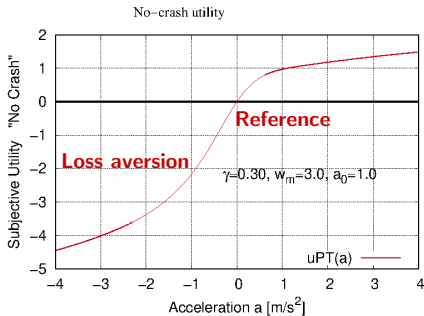
Prospect-theoretic utilities



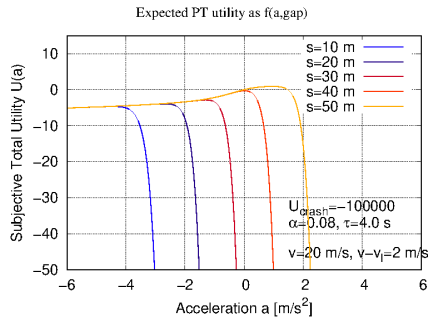
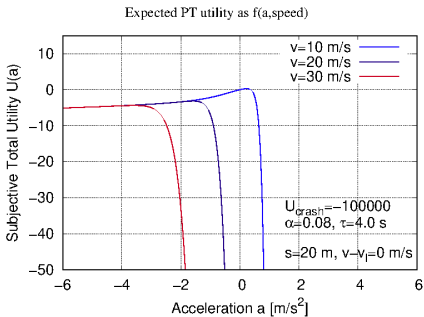
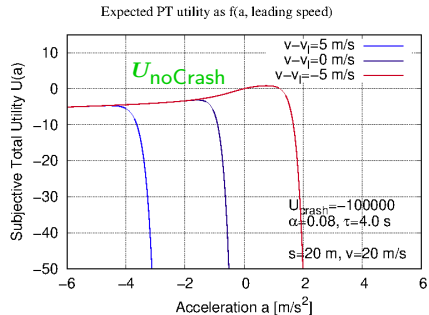
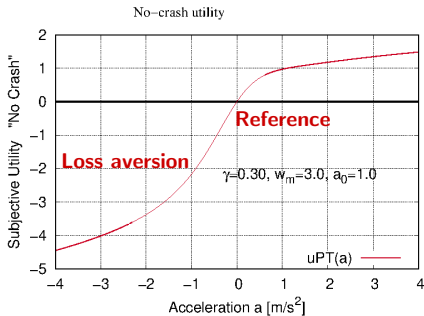
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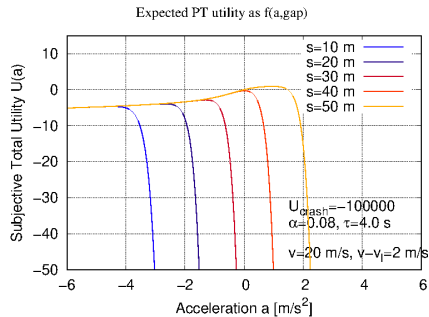
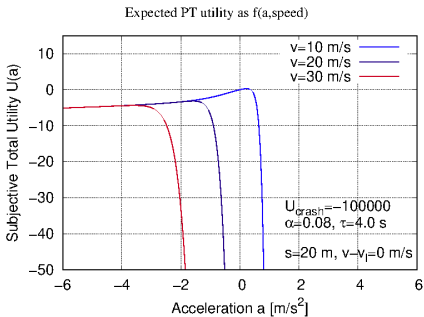
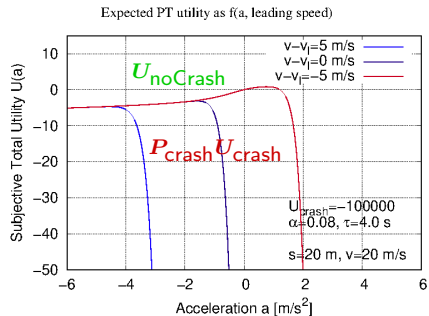
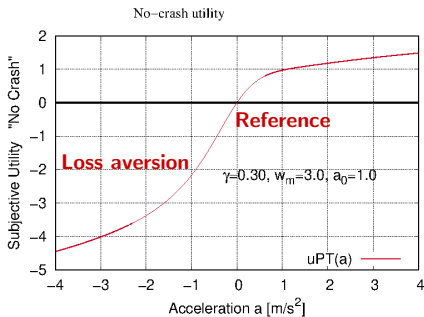
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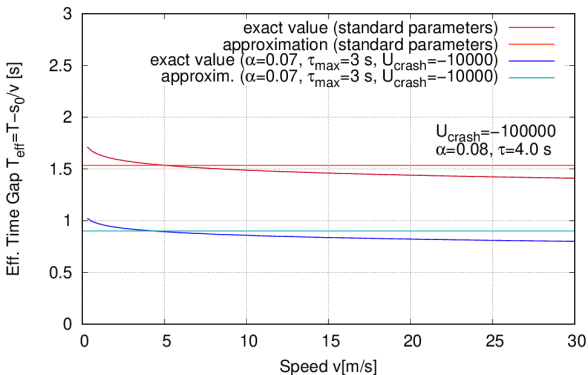
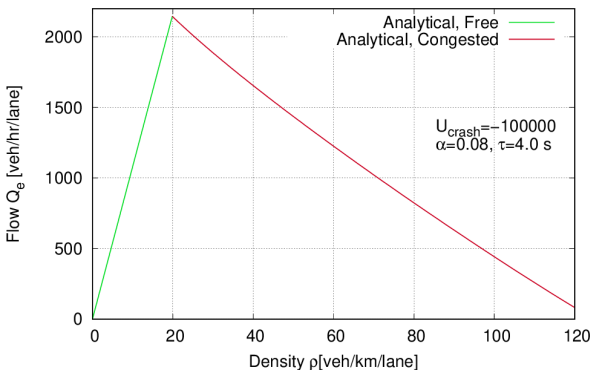
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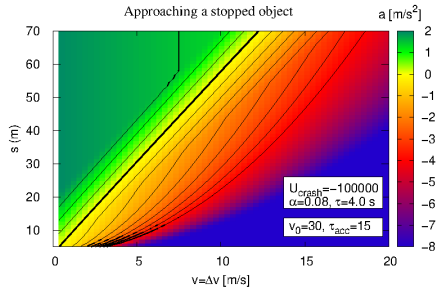
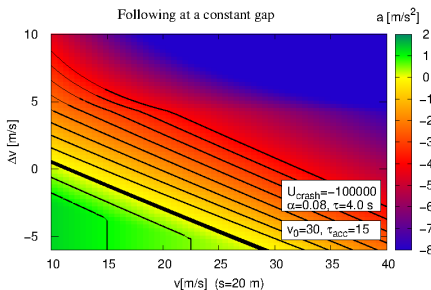
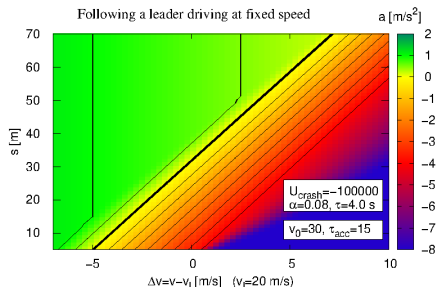
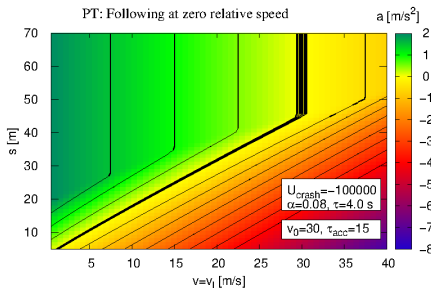


Fundamental diagram and steady-state gap



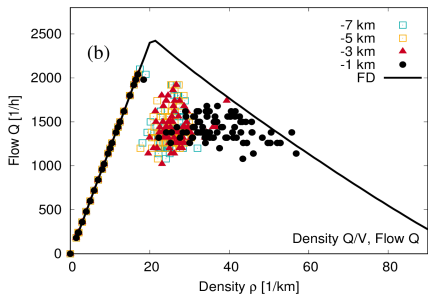
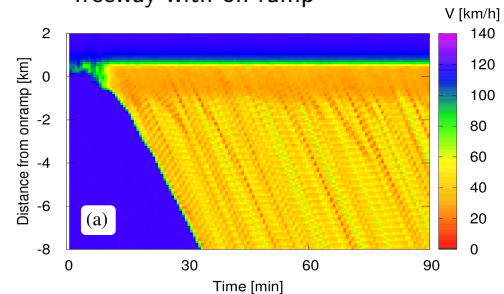
$$T_{\text{eff}} \approx \alpha \tau_a \sqrt{2 \ln(-P_{\text{crash}})}$$

PT model acceleration function



Factsheet of the PT model based on Kahneman and Twersky

freeway with on-ramp



city with traffic lights

