Lecture 09: Car-Following Models Based on Driving Strategies

9.1 Motivation

The *plausibility criteria* of the last lesson and model completeness are necessary but not sufficient for a realistic simulation. Additional requirements for **car-following models (CF models)** include

- No accidents $\Rightarrow$ not satisfied by the OVM
- The accelerations $\dot{v}$ and braking decelerations have to be physically possible, e.g. $-9\,\mathrm{m/s^2} \le \dot{v} \le 4\,\mathrm{m/s^2}$ $\Rightarrow$ not satisfied by the OVM, Newell's micromodel, or the CA models
- Furthermore, CF models should reflect a "normal" comfortable driving style in normal situations, e.g., $|\dot{v}| < 2\,\mathrm{m/s^2}$ depending on the driving style $\Rightarrow$ distinguish between emergency and normal driving
- For highly dynamic situations such as approaching a red traffic lights/standing vehicles, anticipation according to elementary kinematics (e.g., the minimum stopping deceleration $b_{\text{kin}} = v^2/(2s)$) is necessary $\Rightarrow$ incorporate some driving strategy
- The model parameters should reflect distinct aspects of the driving style

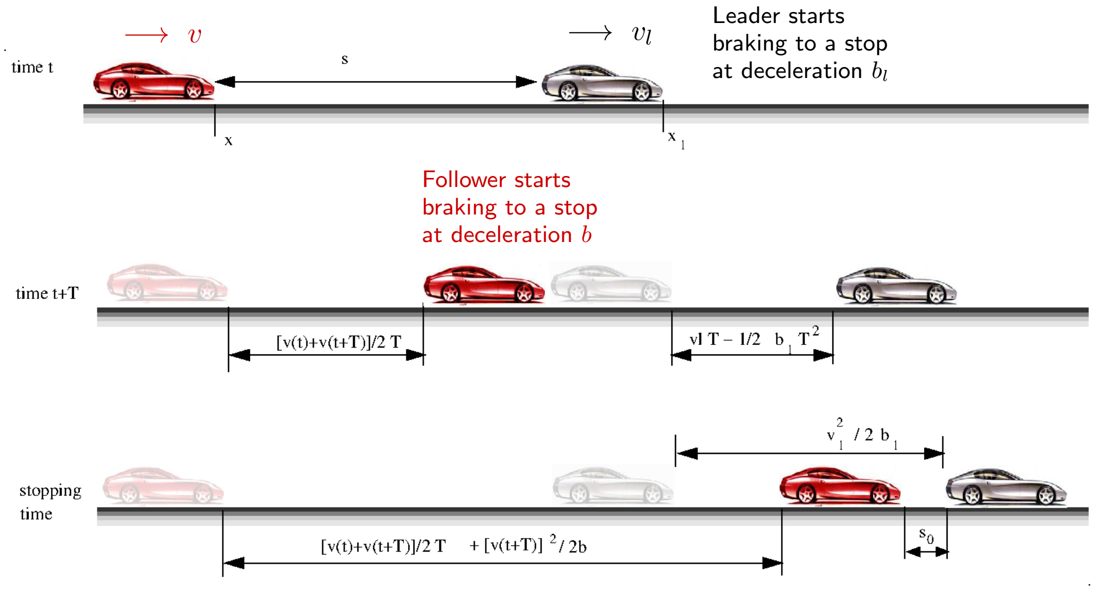
9.2 Gipps' model

The **Gipps model** explicitely satisfies the kinematics in highly dynamic situations

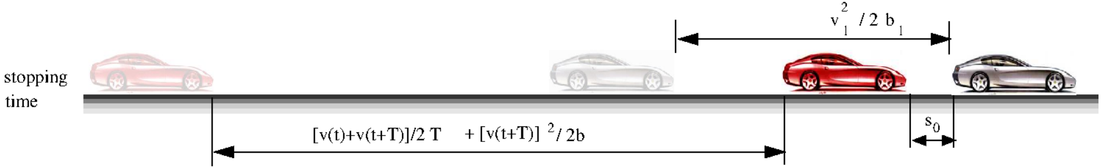
$$v(t+T) = \min\left[v_{\text{free}}, v_{\text{safe}}\right]$$

- The free-acceleration part obeys, e.g., $v_{\text{free}} = \min(v(t) + aT, v_0)$ with acceleration a or some more complicated acceleration profile.
- The safe speed is based on following heuristic *worst-case scenario* where a minimum gap s_0 should be kept at all times:
  - The leader suddenly brakes at deceleration b_l to a full stop,
  - the follower brakes at deceleration b after a reaction time T. For extra safety, another "brake hitting time" ϑ is assumed (somewhat inconsequential),
  - constant acceleration from $v(t)$ to v_{safe} during the reaction time T, constant speed v_{safe} during the brake hitting time ϑ

Derivation of the Gipps model: Overview



Derivation of the Gipps model



Find the safe speed $v(t+T) = v_{\text{safe}}$:

$$x^{\text{stop}} - x = \frac{v(t) + v_{\text{safe}}}{2} T + v_{\text{safe}}\vartheta + \frac{v_{\text{safe}}^2}{2b}$$

$$x_l^{\text{stop}} - x_l = \frac{v_l^2}{2b_l}$$

$$s_0 \stackrel{!}{=} s_{\text{stop}} = s + (x_l^{\text{stop}} - x_l) - (x^{\text{stop}} - x)$$
$$= s + \frac{v_l^2}{2b_l} - \left(\frac{v(t) + v_{\text{safe}}}{2} T + v_{\text{safe}}\vartheta + \frac{v_{\text{safe}}^2}{2b} \right)$$

Assume a "brake hitting time" $\vartheta = T/2 \Rightarrow$ quadratic equation

$$v_{\text{safe}}^2 + 2bTv_{\text{safe}} + bvT - v_l^2 \frac{b}{b_l} - 2b(s - s_0) = 0$$

The simplified Gipps model

The simplified version makes following assumptions:

- Constant acceleration a in the free-flow regime until reaching the desired speed v_0
- No acceleration is assumed during the reaction time T and the brake hitting time ϑ is zero. So, these assumptions just calculate the speed which *would* prevent a crash in the worst case if it were adopted instantaneously and held constant during T. Hence, the *reaction distance* of the follower is simply given by $\Delta x_{\text{react}} = v(t)T = v_{\text{safe}}T$
- The leader and the follower have the same braking capabilities $b_l = b$

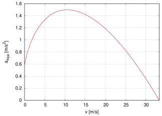
This leads to following quadratic equation:

$$v_{\text{safe}}^2 + 2bTv_{\text{safe}} - v_l^2 - 2b(s - s_0) = 0$$

The final Gipps models

$$v(t+T) = \min\left[v + a_{\text{free}}(v)T, v_{\text{safe}}(s, v, v_l)\right] \quad \text{Full Gipps Model}$$

$$a_{\text{free}}(v) = 2.5a\left(1 - \frac{v}{v_0}\right)\sqrt{0.025 + \frac{v}{v_0}},$$



$$v_{\text{safe}}(s, v, v_l) = -b(T/2 + \vartheta) + \sqrt{b^2(T/2+\vartheta)^2 + 2b(s - s_0) + v_l^2\frac{b}{b_l} - vbT}$$

The model is for general brake hitting times ϑ. For the standard value $\vartheta = T/2$, simplify $b(T/2+\vartheta) \to bT$

$$v(t+T) = \min\left[v + aT, v_0, v_{\text{safe}}(s, v_l)\right] \quad \text{Simplified Gipps Model}$$

$$v_{\text{safe}}(s, v_l) = -bT + \sqrt{b^2T^2 + 2b(s - s_0) + v_l^2}$$

Freeway parameters: $v_0 = 35\,\text{m/s}$, $a = b = b_l = 1.5\,\text{m/s}^2$, $T = 1.1\,\text{s}$, $\vartheta = T/2$, $s_0 = 2\,\text{m}$
City parameters: just reduce the desired speed v_0

Homogeneous steady state and fundamental diagram of the Gipps models I: Free-flow regime

Unlike the past CF-models, the Gipps model(s) do not have an explicit fundamental diagram (FD) given by the OV function ⇒ must be calculated by assuming a **stationary steady state**:

- **Stationarity**: $\frac{\mathrm{d}}{\mathrm{d}t} = 0$, so $v(t+T) = v(t)$
- **Homogeneity**: $\frac{\mathrm{d}}{\mathrm{d}x} = 0$, so $v_l(t) = v(t)$

Free-flow regime:

$$v(t+T) = v(t) \;\Rightarrow\; a_{\text{free}}(v) = 0 \;\Rightarrow\; v = v_0$$

Does the free-flow Gipps model include any interactions in the free-flow regime?

No, not any! strict separation of regimes by the min-function!

Homogeneous steady state and fundamental diagram of the Gipps models II: Interaction regime

Here, the second part of the min-function applies:

$$
\begin{aligned}
v(t+T) &= v = v_{\text{safe}} = v_l \\
v &= -b(T/2+\vartheta) + \sqrt{b^2(T/2+\vartheta)^2 + 2b(s-s_0) + v^2\frac{b}{b_l} - vbT} \\
\left(v + b(T/2+\vartheta)\right)^2 &= b^2(T/2+\vartheta)^2 + 2b(s-s_0) + v^2\frac{b}{b_l} - vbT
\end{aligned}
$$

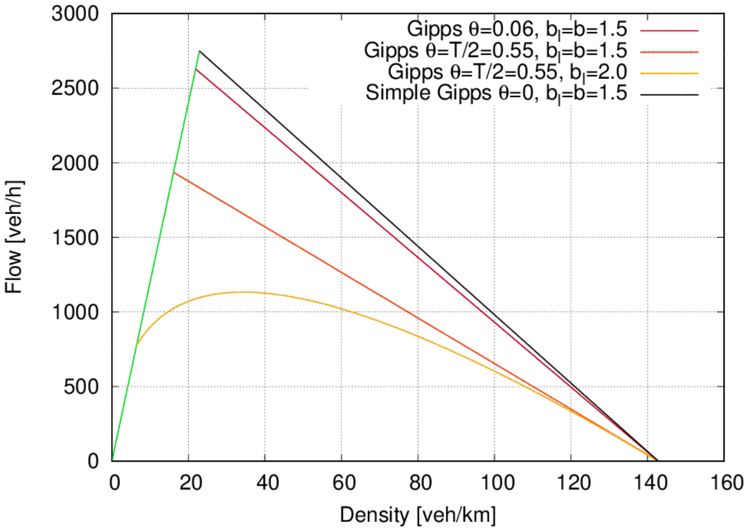
Quadratic equation for $v_e(s)$ or linear equation for $s_e(v)$:

$$
s_e^{\text{Gipps}}(v) = s_0 + vT + v\vartheta + \frac{v^2}{2b}\left(1 - \frac{b}{b_l}\right)
$$

Shape of the FD for the special case of the simplified Gipps model?

$s_e = s_0 + vT \Rightarrow$ triangular FD

Fundamental diagram of the Gipps model variants



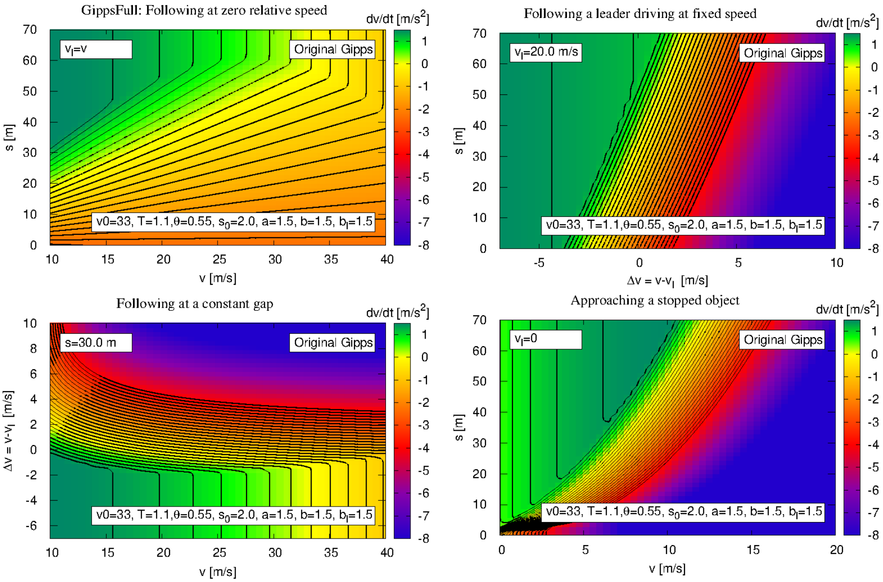
The Drivers of the Gipps model become more defensive

- with increasing reaction time T and brake hitting times ϑ
- with increasing implied leader deceleration b_l

? Why traffic becomes unstable for $b_l < b$?

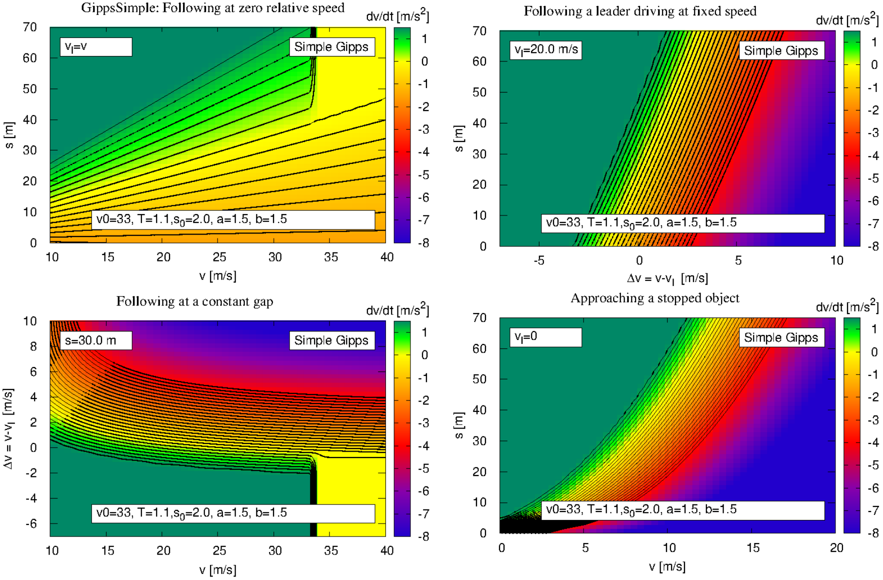
! Since the follower thinks he/she can brake harder than the leader. Along the whole string of vehicles ...

Gipps model acceleration function



⇒ accGippsSimple

Simplified Gipps model acceleration function



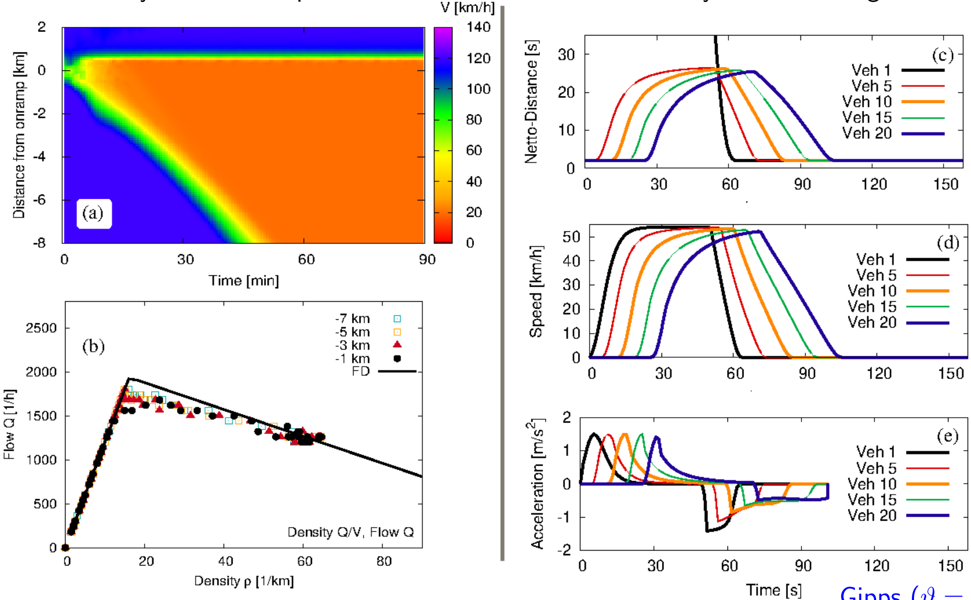
⇐ accGipps

⇒ accIDM

Factsheet of the original Gipps model ($\vartheta = 0.5$)

freeway with on-ramp

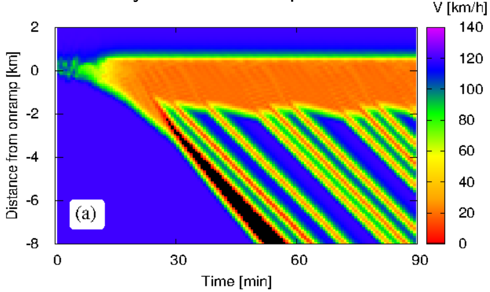
city with traffic lights

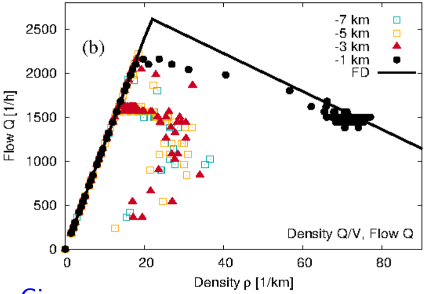


Gipps ($\vartheta = 0.06$) $\Rightarrow$

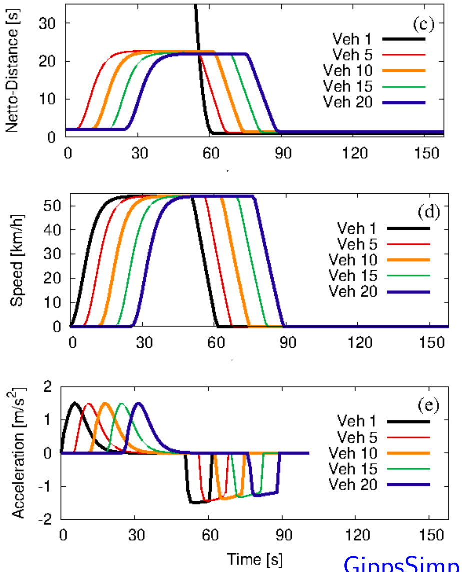
Factsheet of the original Gipps model with $\vartheta = 0.06$

freeway with on-ramp





city with traffic lights

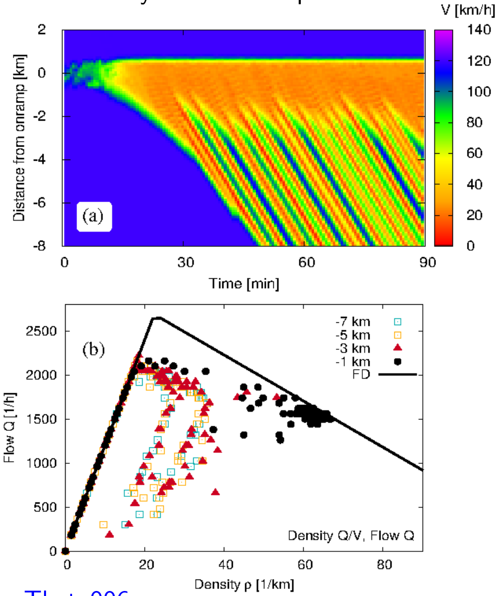


⇐ Gipps

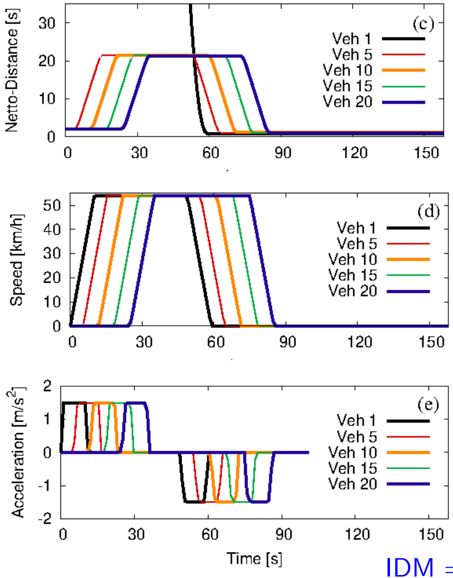
GippsSimple ⇒

Factsheet of the simplified Gipps model

freeway with on-ramp



city with traffic lights



⇐ GippsTheta006

IDM ⇒

9.3 Intelligent Driver Model (IDM)

Probably the most parsimonious car-following model satisfying following conditions:

- All *plausibility conditions* satisfied
- *smooth driving regime transitions* (i.e., a smooth or even differentiable acceleration function), unlike the Gipps model
- *collision free* if physically possible
- *unique feature:* Continuous and stable transition from an emergency to a regular braking maneuver by an *intelligent* driving strategy
- all model parameters are *intuitive* describing distinct aspects of the driving behavior: aggressive/timid, anticipative/short-sighted, responsive/sleepy, and of course slow/fast

IDM equations

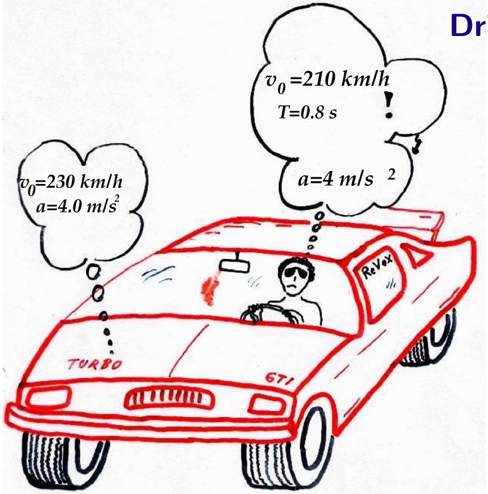
$$\frac{\mathrm{d}v}{\mathrm{d}t} = a\left[1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s^*(v,v_l)}{s}\right)^2\right]$$ IDM acceleration

free acceleration: $a[1-(v/v_0)^4]$, repulsive force: $-a(s^*/s)^2$

$$s^*(v,v_l) = s_0 + \max\left(0, vT + \frac{v(v-v_l)}{2\sqrt{ab}}\right)$$ desired gap

Parameter	Cars Highway	Cars City	Trucks Hwy	Bicycles
Desired speed v_0	$120\,\mathrm{km/h}$	$50\,\mathrm{km/h}$	$80\,\mathrm{km/h}$	$20\,\mathrm{km/h}$
Time gap T	$1.0\,\mathrm{s}$	$1.0\,\mathrm{s}$	$1.8\,\mathrm{s}$	$0.6\,\mathrm{s}$
Minimum gap s_0	$2\,\mathrm{m}$	$2\,\mathrm{m}$	$3\,\mathrm{m}$	$0.4\,\mathrm{m}$
Acceleration a	$1.5\,\mathrm{m/s^2}$	$2.0\,\mathrm{m/s^2}$	$0.5\,\mathrm{m/s^2}$	$1.0\,\mathrm{m/s^2}$
Comf. deceleration b	$1.5\,\mathrm{m/s^2}$	$2.0\,\mathrm{m/s^2}$	$1.0\,\mathrm{m/s^2}$	$1.5\,\mathrm{m/s^2}$

Driving styles



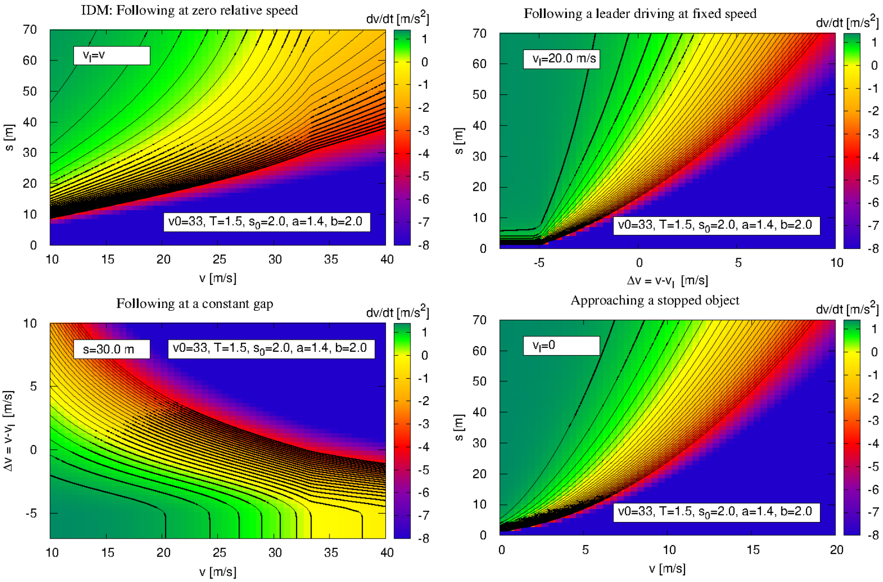
Aggressive driver:
v_0, a and b high, T and s_0 low

Experienced responsive driver:
a high, b low, rest normal

Relaxed driver:
v_0, a low, b normal, T and s_0 high

Experienced defensive driver:
v_0, a normal, b low, T and s_0 high

IDM acceleration function



⇐ accGippsSimple

⇒ accIDM+

IDM properties I: steady state

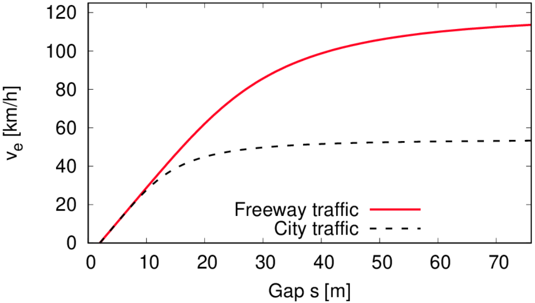
? Calculate the homogeneous steady state

! $\frac{\mathrm{d}v}{\mathrm{d}t} = 0, \quad s^* = s_0 + vT$

$$0 = a\left[1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{s_0+vT}{s}\right)^2\right]$$

can be solved for $s = s_e(v)$:

$$s_e(v) = \frac{s_0+vT}{\sqrt{1-(v/v_0)^4}}$$

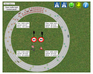


? How to derive a macroscopic fundamental diagram (FD) out of $s_e(\rho)$

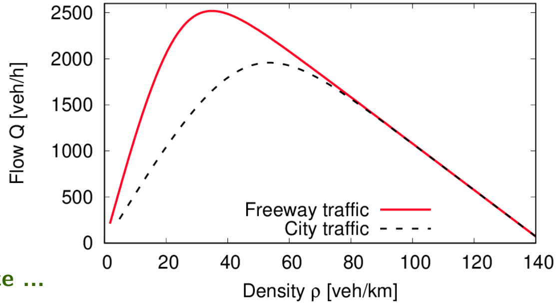
! Only possible as a parametric function of the speed v. With the vehicle length l, we have $s = 1/\rho - l \;\Rightarrow$

$$\rho(v) = \frac{1}{s_e(v)+l},$$

$$Q(v) = v\rho(v)$$



simulate ...



IDM properties II: the "intelligent" braking stratey

- "Extreme" assumptions $s_0 = T = 0$, $v_l = 0$, so $s^* = v^2/(2\sqrt{ab})$
- Consider only the repulsive term:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -a\left(\frac{s^*}{s}\right)^2 = -\frac{av^4}{4abs^2} = -\left(\frac{v^2}{2s}\right)^2\frac{1}{b} \stackrel{!}{=} -\frac{b_{\text{kin}}^2}{b}$$

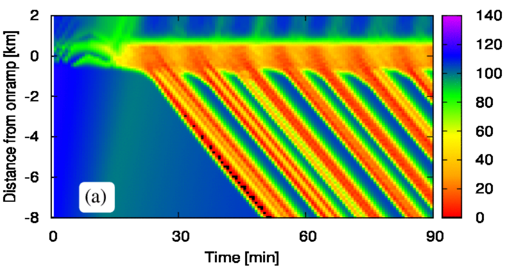
- At a given dynamic state, the *kinematic deceleration* $b_{\text{kin}} = \frac{v^2}{2s}$ is the minimum deceleration avoiding a crash. When, this situation is *dynamically* "safe" or "under control"? If $b_{\text{kin}} \le b$
- How manages the IDM to not brake too early but bring a critical situation under control? Rewriting $\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{b_{\text{kin}}^2}{b}$ reveals the trick:
  - Situation safe or $b_{\text{kin}} < b \Rightarrow |\frac{\mathrm{d}v}{\mathrm{d}t}| < b_{\text{kin}}$ $\Rightarrow$ brake *less* than b_{kin}: too early is bad!
  - Situation critical or $b_{\text{kin}} > b \Rightarrow |\frac{\mathrm{d}v}{\mathrm{d}t}| > b_{\text{kin}}$ $\Rightarrow$ brake *more* than b_{kin}: get situation under control!

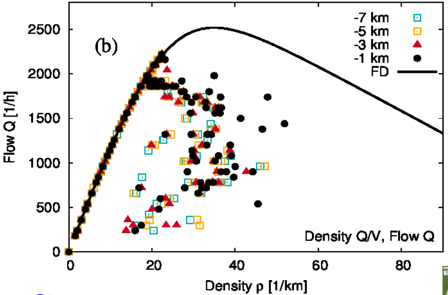
In both cases, the comfortable deceleration b is *dynamically* reached!

simulate ...

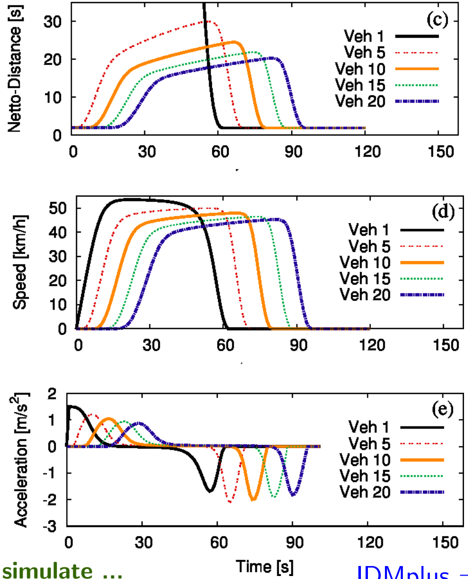
Factsheet of the Intelligent-Driver Model (IDM)

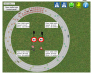
freeway with on-ramp





city with traffic lights



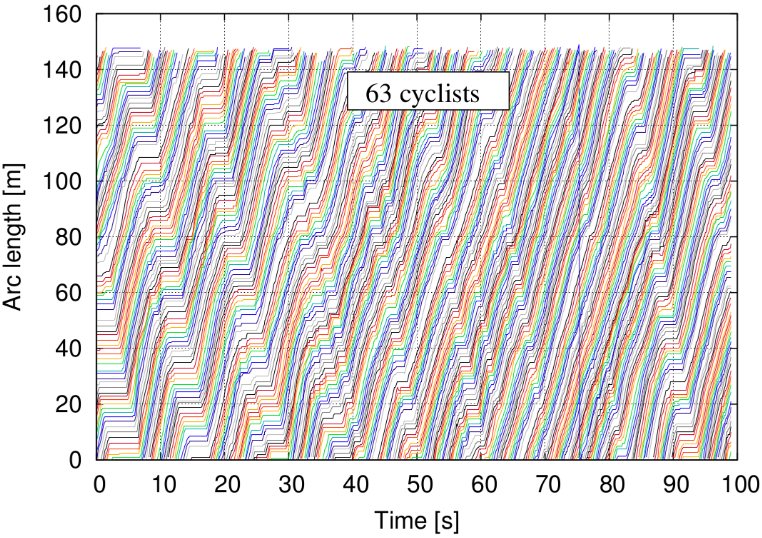


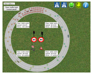
simulate ...

⇐ Gipps

IDMplus ⇒

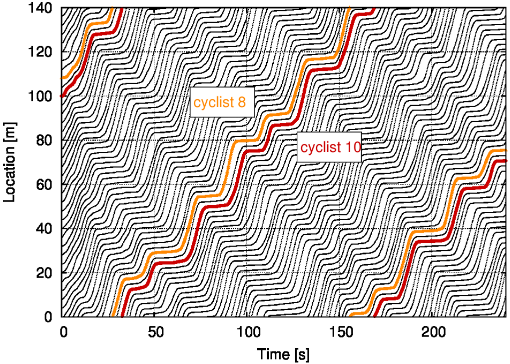
IDM for bicycle traffic? Single-file bicycle traffic experiment

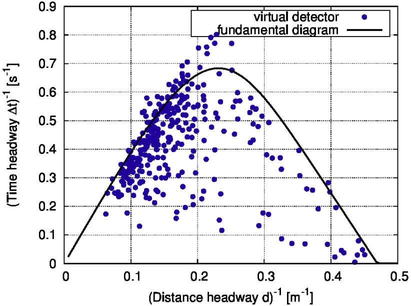


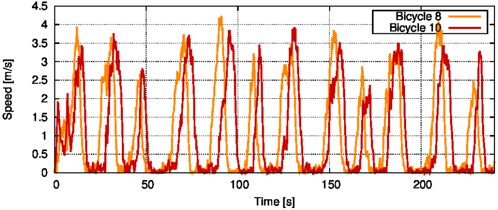


simulate ...

Simulating single-file bicycle traffic with the IDM







$l_{\text{bike}} = 1.7\,\text{m},\ v_0 = 4\,\text{m/s},\ T = 0.6\,\text{s},$
$s_0 = 0.4\,\text{m},\ a = 0.8\,\text{m/s}^2,\ b = 1.5\,\text{m/s}^2$

9.4 Derivatives of the Intelligent Driver Model

For a realistic driving feeling or for use as the core of an ACC controller, the IDM still has several deficiencies:

- When reaching the desired speed, the steady-state time gap

$$T = \frac{s_e(v) - s_0}{v} = \frac{T}{\sqrt{1 - (v/v_0)^4}}$$

  becomes significantly larger than T leading to a somewhat unrealistic platoon behaviour in the *city with traffic lights* situation.

- The IDM reacts too sensitively if the gap is too low, even if there is no real danger. This happens regularly if the leader changes (active and passive lane changes)

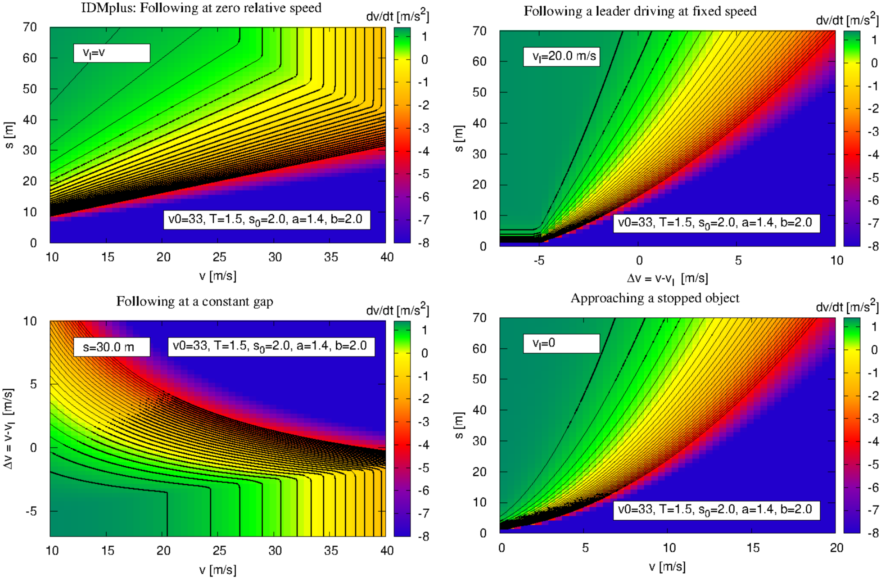
IDM with triangular fundamental diagram: IDM+

The time-gap deficiency can be solved by following modification:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \min\left[a\left(1-(v/v_0)^4\right), a\left(1-(s^*/s)^2\right)\right] \qquad \textbf{IDM+}$$

- The acceleration function is no longer smooth but still continuous
- Instead of the continuous transition of the IDM, the IDM+ has two distinct regimes: free acceleration (the first expression of the min function is relevant), and interacting (the second expression matters)
- Steady-state time gap: $\frac{\mathrm{d}v}{\mathrm{d}t} = 0 \Rightarrow$ if $v < v_0$, the second expression in the min-condition matters
  $\Rightarrow s = s^*(v, v) = s_0 + vT \Rightarrow$ constant time gap and triangular FD
- The *intelligent* braking strategy is not affected (see the following plots)

IDM+ acceleration function

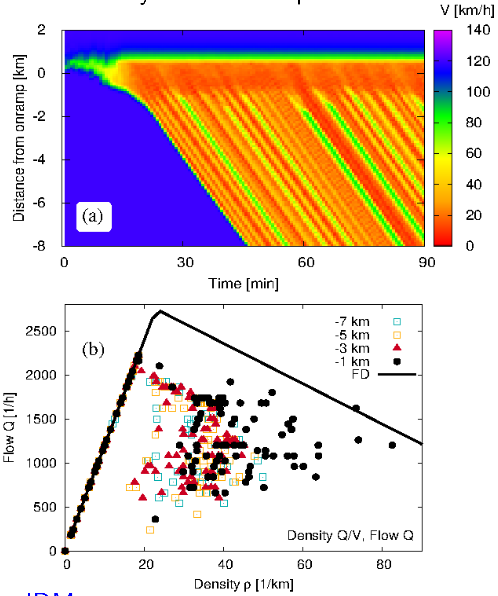


⇐ accIDM

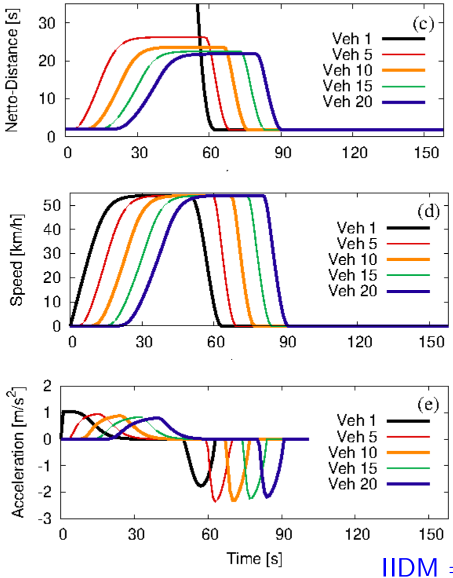
⇒ accACC

Factsheet of the Improved IDM (IDM+)

freeway with on-ramp



city with traffic lights



⇐ IDM

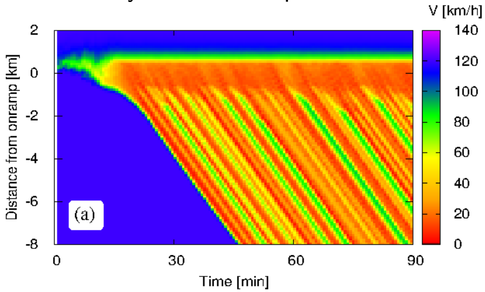
IIDM ⇒

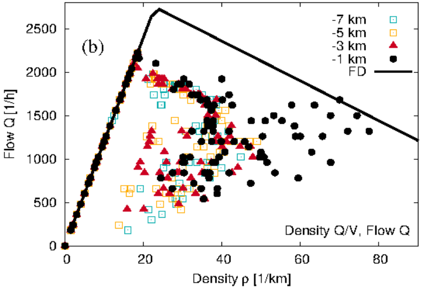
Another IDM with triangular fundamental diagram: IIDM

- Another possibility to obtain an IDM-like model with a triangular FD and the intelligent brakingstrategy unaffected
- In contrast to the IIDM, the acceleration function is smooth
- However, this implies a more complicated formulation (not shown)

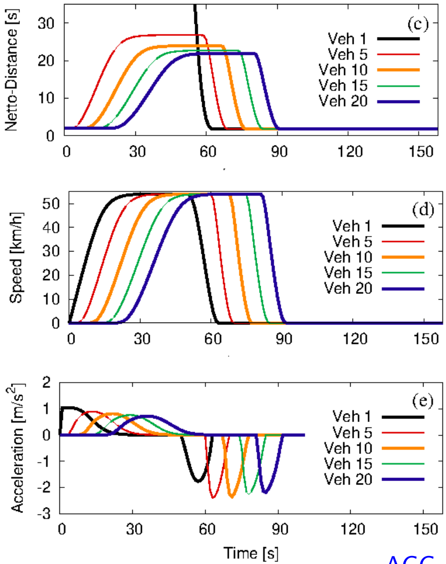
Factsheet of the Improved IDM (IIDM)

freeway with on-ramp





city with traffic lights



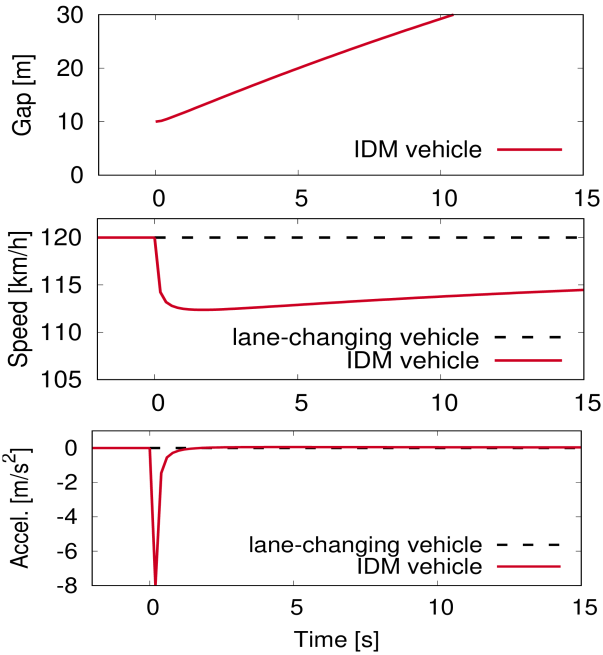
⇐ IDMplus

ACC ⇒

9.5 Models for Adaptive Cruise Control

- Besides a triangular FD (i.e., constant time gaps in the following regime), an ACC model needs to be robust against changing leading objects caused, e.g., by active or passive lane changes
- This is realized by replacing the worst-case heuristics of the IDM by a more realistic "constant acceleration heuristics": Human drivers also do not expect a full braking maneuver to the stop *out of the blue* (and would not be able to handle it)
- In contrast, because the ACC model does only have insignificant reaction delays (all IDM variants presented in this lecture have zero reaction time!), the ACC controller could even handle this
- The actual model is not shown, just the results

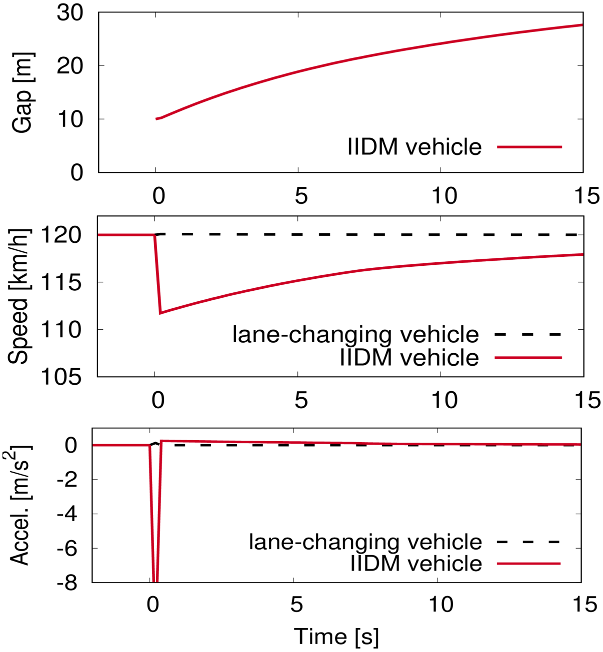
Response to a close cut-in maneuver at same speed: IDM



Lane-changing vehicle: same speed 120 km/h as the follower, cuts in leaving a gap of 10 m

IDM responds too panically

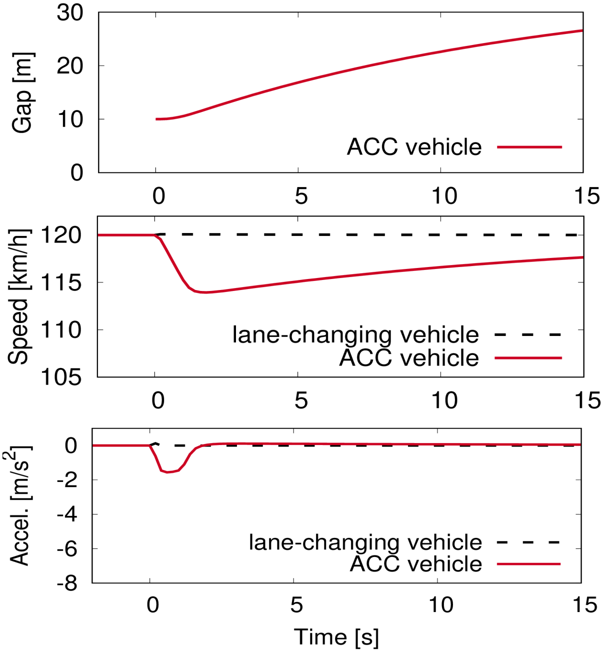
Response to a close cut-in maneuver at same speed: IIDM



Lane-changing vehicle: same speed 120 km/h as the follower, cuts in leaving a gap of 10 m

IIDM response similarly panically but has a better following behaviour afterwards

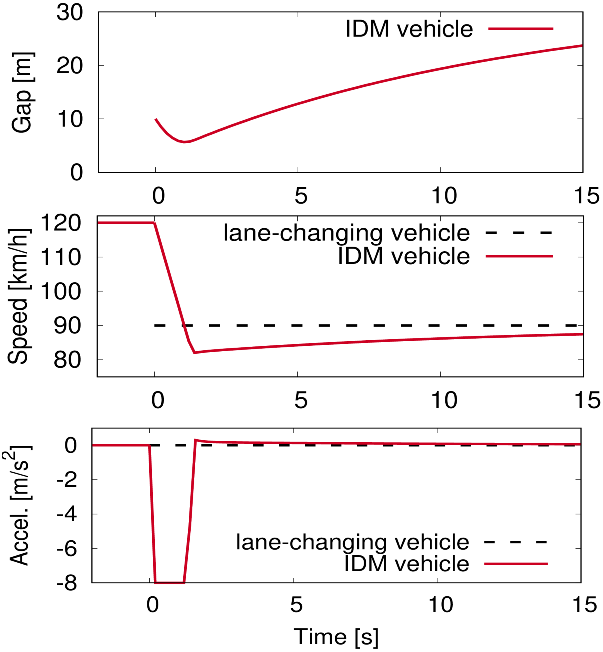
Response to a close cut-in maneuver at same speed: ACC



Lane-changing vehicle: same speed 120 km/h as the follower, cuts in leaving a gap of 10 m

The (IDM+-based) ACC model has a *cool* immediate response and a plausible following behaviour afterwards

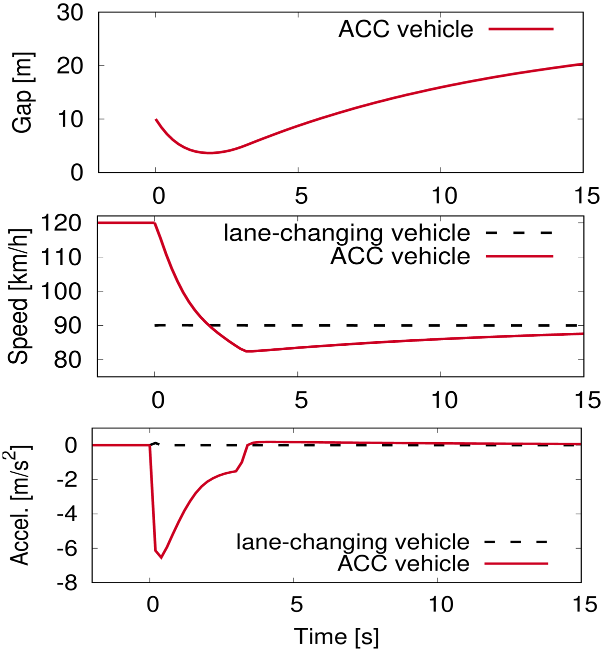
Response to a critical cut-in maneuver: IDM



Lane-changing vehicle: 30 km/h slower than the follower, cuts in leaving a gap of just 10 m

IDM switches to emergency mode which is right in this situation

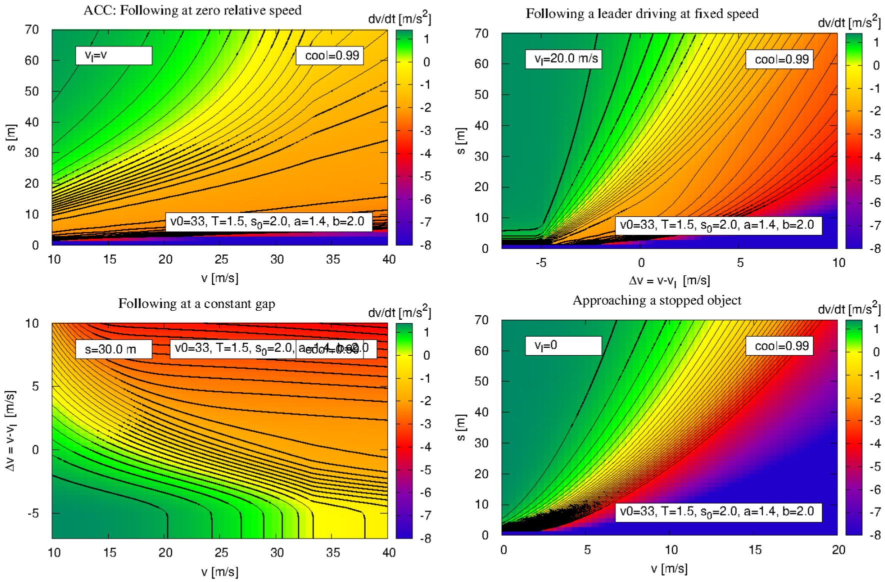
Response to a critical cut-in maneuver: ACC model



Lane-changing vehicle: 30 km/h slower than the follower, cuts in leaving a gap of just 10 m

Also the ACC model looses its *coolness* which is completely justified in this situation

ACC model acceleration function

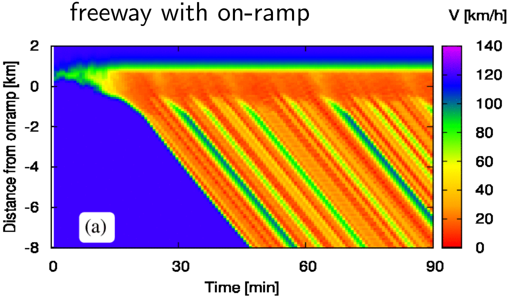


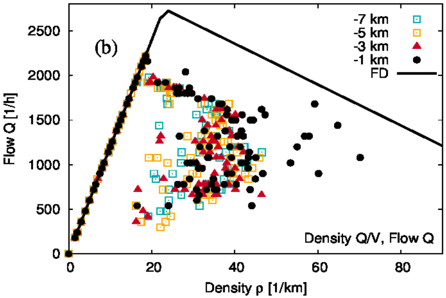
⇐ accIDMplus

⇒ accPT

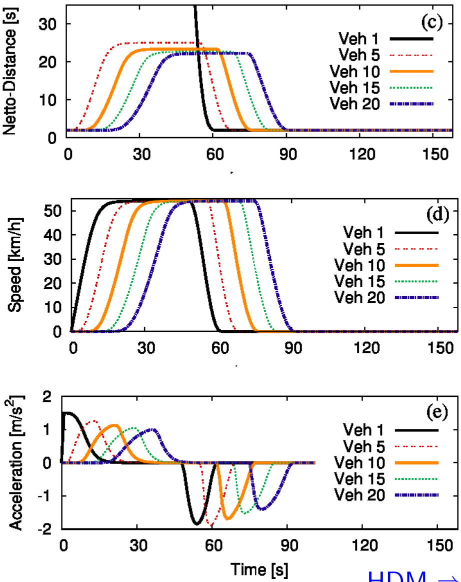
Factsheet of the ACC model

freeway with on-ramp



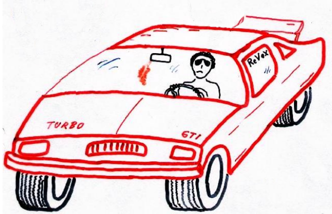


city with traffic lights

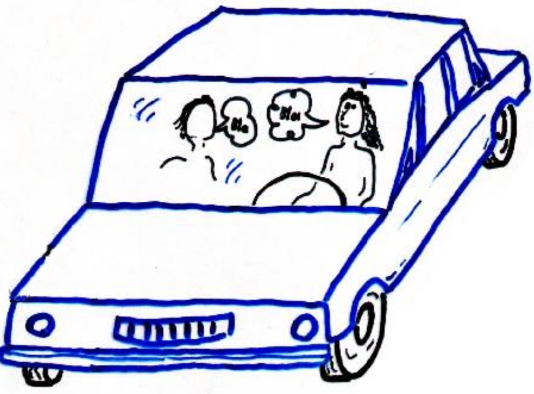


⇐ IDM+ HDM ⇒

9.6 Human-Driver Car-Following Models



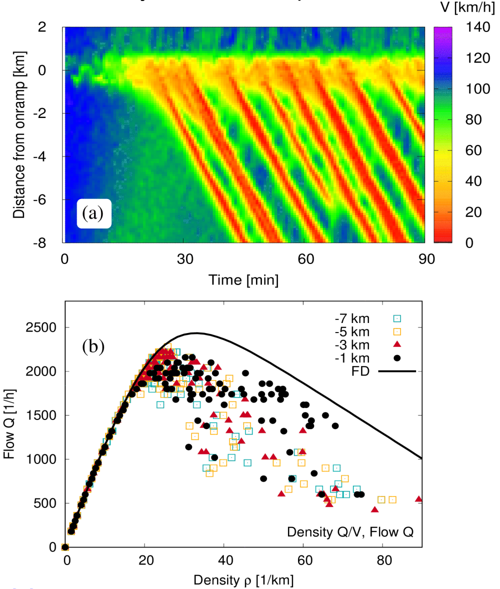
In contrast to ACC controllers, humans have ...



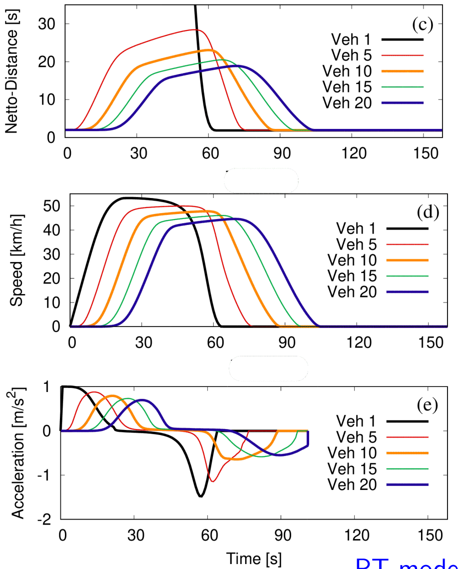
- Significant **reaction times** $\Rightarrow$ state $s(t-T_r)$, $v_l(t-T_r)$
- Response **thresholds** ($\Rightarrow$ Wiedemann)
- Risk **attitude** ($\Rightarrow$ **Prospect Theory**)
- Correlated **estimation errors** in s, v, and v_l and general acceleration noise
- Temporal **anticipation**: $s(t+T_a) = s(t) + T_a\,(v_l(t) - v(t))$
- Spatial anticipation: **multi-anticipation** to next-nearest leaders
- Response to braking lights, winkers, ...

Factsheet of the IDM-based Human Driver Model

freeway with on-ramp



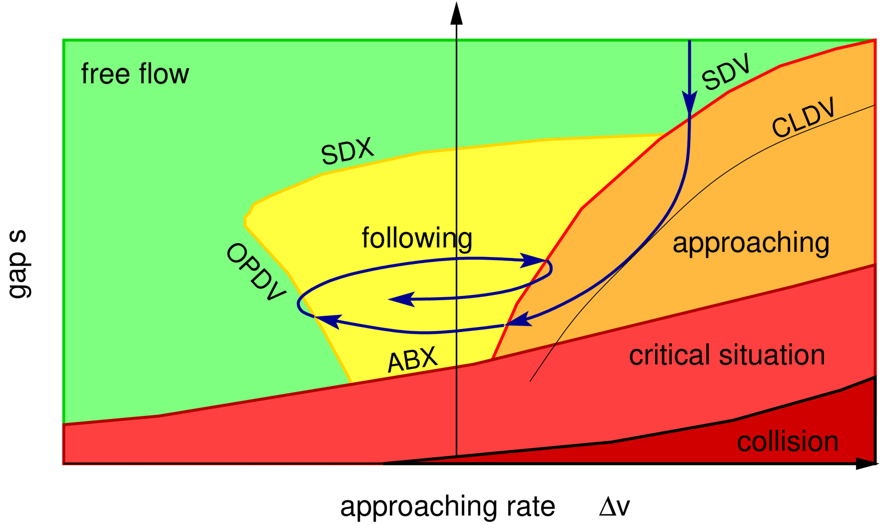
city with traffic lights



⇐ ACC model

PT model ⇒

Response thresholds:
Wiedemann trajectories in space-relative speed space



Base model in VISSIM

CF models based on risk perception: Prospect Theory of Kahneman and Twersky

Prospect theory is a variant of **Expected Utility Theory (EUT)**:

- Given is a decision situation where, depending on the action a, a discrete set of outcomes $k \in \mathcal{K}(a)$ with utilities $U_k(a)$ can happen with probabilities $P_k(a)$
- The *Homo Oeconomicus'* action a tries to maximize the **expected utility**

$$E(U) = \sum_{k \in \mathcal{K}(a)} P_k(a) U_k(a) \stackrel{!}{=} \max_a$$

- The actions a can be discrete such as accepting an offer or not, or continuous such as deciding on an acceleration
- In **Prospect Theory**, both the probabilities and the utilities get a subjective bias and the outcome weighted in this way is called a *prospect*:
  - Small probabilities are overestimated (for probabilities > 0.5, the complement probability is considered)
  - At a certain **framing reference**, the sensitivity to utility changes is at its maximum
  - Losses with respect to the reference are weighted more than wins: **loss aversion**

Examples

1. Taking part in a lottery: a lot costs 1 €, the probability of winning 95 € (outcome 1) is 1 %:
   - Action "Y": $P_1 = 0.01, U_1 = 95 - 1 = 94$, $P_2 = 0.99, U_2 = -1$,
     Action "N": Only outcome $k = 2$ with certainty $(P_2 = 1, U_2 = 0)$
   - EUT: E("Y")=0.01*94+0.99*(-1)=-0.05, E("N")=0 $\Rightarrow$ decision "N"
   - PT: The loss aversion and the reference effect shift the decision towards "N", the positively biased probability P_1 towards it $\Rightarrow$ depends on the person
2. Signing an insurance contract. The insurance costs 1 € and protects from a damage of 95 € (outcome 1) occurring at a probabilty of 1 %
   - Action "Y": $P_1 = 0.01, U_1 = -1$, $P_2 = 0.99, U_2 = -1$,
     Action "N": $P_1 = 0.01, U_1 = -95$, $P_2 = 0.99, U_2 = 0$
   - EUT: E("Y")=-1, E("N")=-0.95 $\Rightarrow$ decision "N"
   - PT: Here, the loss aversion and the subjective increase of P_1 probably prevails over the reference effect and the insurance is taken ("Y")
3. Sitting in a vehicle and deciding on the acceleration (continuous-valued action) a. Outcomes $k = 1$: "crash" and $k = 2$: "no crash" where $P_1(a) = 1 - P_2(a)$ increases with a

Formulation of a CF model based on Prospect Theory

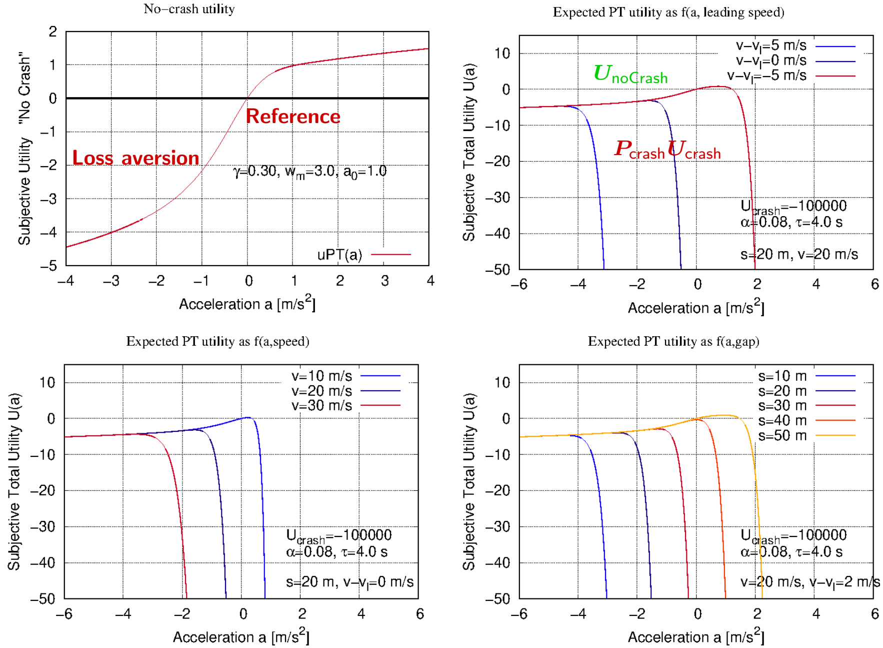
General observations:

- The probability P_1 of the outcome $k = 1$: "crash" increases with the acceleration a (because of the future speed increasing and the future gaps decreasing with a)
- The probability $P_2 = 1 - P_1$ of the outcome "no crash" decreases accordingly but its utility U_2 increases: "due to my higher future speed, I will need less time"

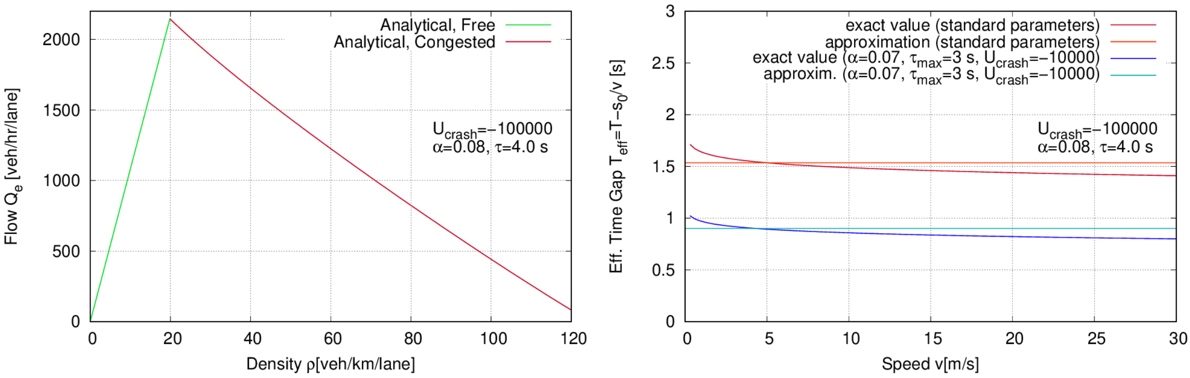
Assumptions of the presented PT model:

- Anticipation time horizon τ_a (e.g., $\tau_a = 5\,\mathrm{s}$) for assessing the crash risk $P_1 \ll 1$
- The leader's speed is unchanged and the uncertainty of assessing the relative speed Δv increases with the speed: $\Delta \hat{v} \sim N(\Delta v, \sigma)$ with $\sigma = \alpha v$ (e.g., $\alpha = 0.2$).
- The utility $U_2(a)$ with the slope $U_2'(a)$ of the order of 1 (scaling of U) reflects the reference at $a = 0$ and loss aversion
- The subjective crash utility U_1 is a very negative constant (e.g., $U_1 = -10^5$)
- Minimum of free and PT acceleration is taken

Prospect-theoretic utilities

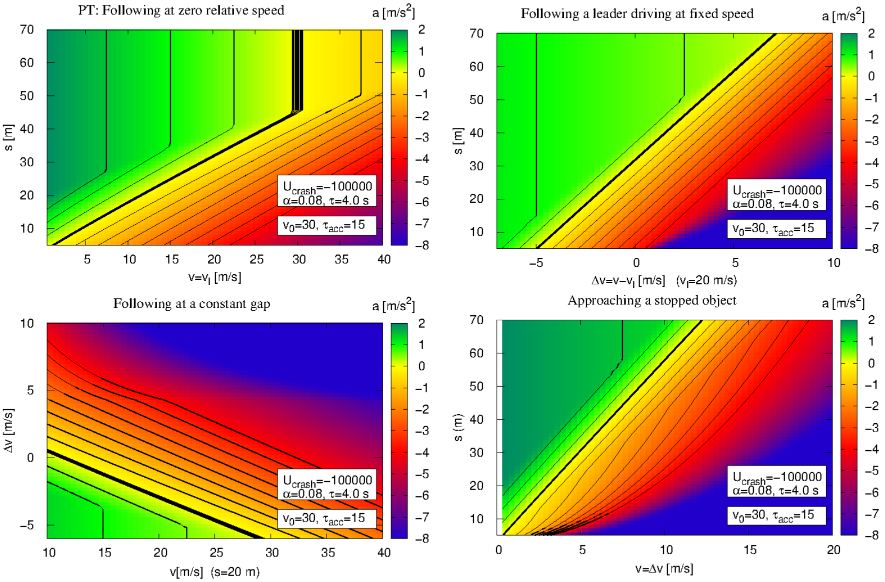


Fundamental diagram and steady-state gap



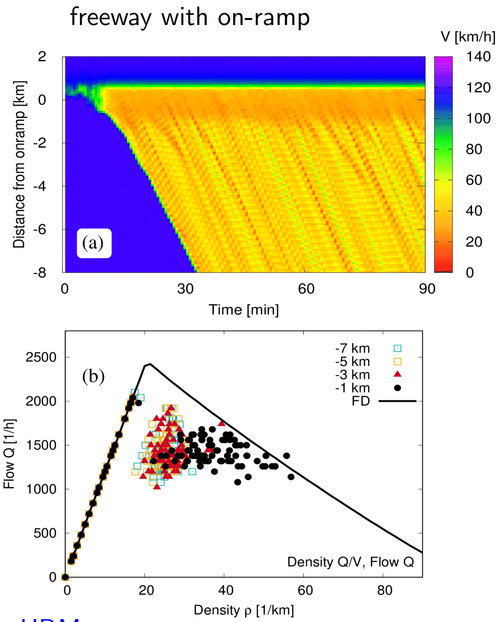
$$T_{\text{eff}} \approx \alpha \tau_a \sqrt{2 \ln(-P_{\text{crash}})}$$

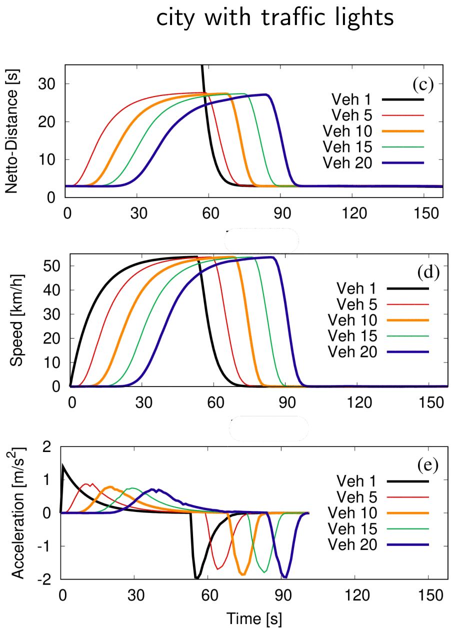
PT model acceleration function



⇐ accACC

Factsheet of the PT model based on Kahneman and Twersky





⇐ HDM