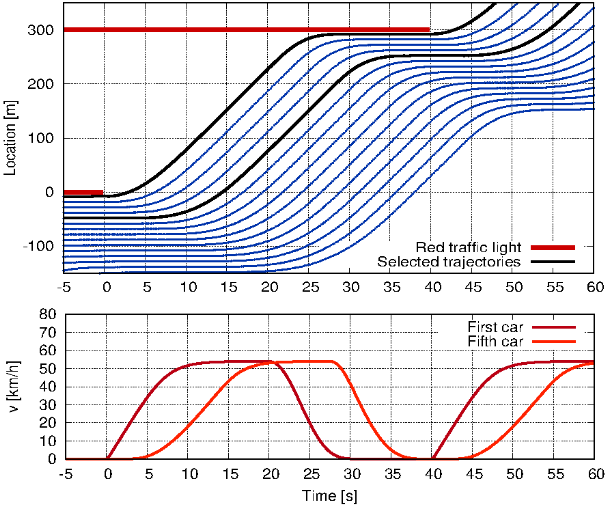
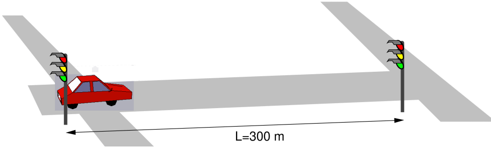
Lecture 08: Microscopic Models I

Elementary Car-Following Models

- 8.1 Difference between Micro and Macromodels
- 8.2 Types and Mathematical Forms
- 8.3 Car-Following Models
- 8.4 Optimal Velocity Model
- 8.5 Full Velocity Difference Model
- 8.6 Newell's Car-Following Model
- 8.7 Car-Following Cellular Automata

8.1 Difference between Micro and Macromodels





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describes the trajectories or FC time series

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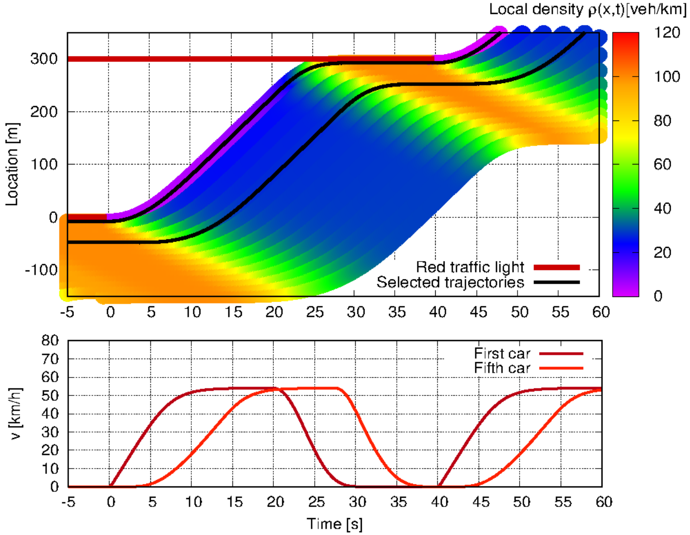

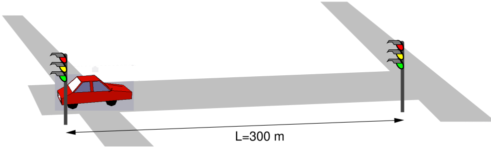

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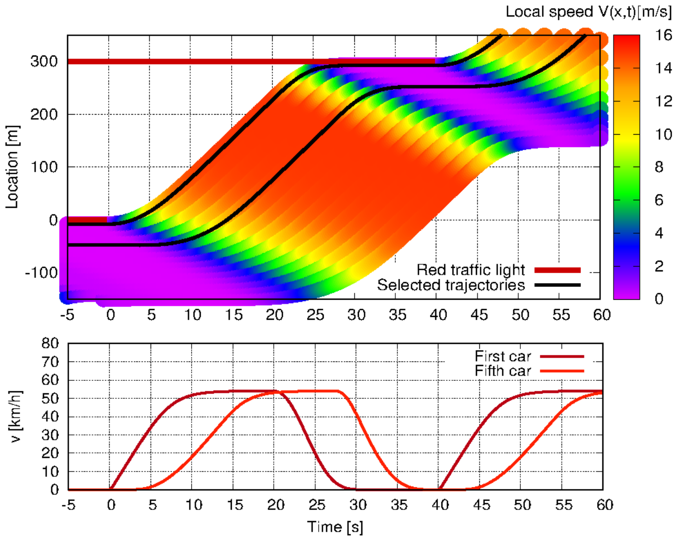

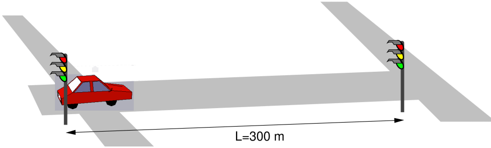

Microscopic:
describes the trajectories or FC time series

Macroscopic:
describes the inverse of the local distance of the lines (**density**) or the local gradient of the trajectories (**local speed**)

Characterisation of Microscopic Models

- Generally, **microscopic models** consider the smallest objects that make sense/play a role in the given context, e.g., molecules/atoms/elementary particles in physics or individual decision makers in economics.
- In traffic flow, this smallest object usually is the **driver-vehicle unit** (why vehicle *and* driver?) but it can also be a cyclist, a pedestrian, or others.
- Microscopic models are more detailled than the macroscopic models discussed in the previous sections which locally aggregate the microscopic quantities.
- Microscopic models are less detailled than models for the **vehicle dynamics** ("submicroscopic models") treating aspects such as brake and engine control path, slip, or stability control

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Where micromodels play out their advantages: heterogeneous traffic

Microscopic models play out their advantages when describing different **driver-vehicle units**, i.e., **heterogeneous traffic**. They are also called **self-driven particles** or **agents** (no stirring or shaking involved!).

There are four conceptual levels for heterogeneity that all can be tackled:

- *Same model, same vehicle category, same driving style*: Since drivers are no machines, some **acceleration noise** is plausible.
- *Same model, same vehicle category (e.g., only cars or only trucks), different driving styles (e.g. considerate or aggressive)*: every agent gets its individual parameter set drawn from a distribution
- *Same model, different vehicle categories, different styles*: The agents of each category get their parameters from separate distributions
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8.2 Microscopic Traffic Flow Models: Types and Mathematical Forms

- Generally, microscopic traffic flow models can describe any aspect of the dynamics of a driver and his/her vehicle on two levels:
  - **Operative level**: accelerating, braking, steering
  - **Tactical levels**: lane changing, entering a priority road and other discrete-choice tasks
  - **Strategic level**: route choice
- Hence, their are different model categories:
  - **Car-following (CF) models** or more generally models for the longitudinal dynamics are the most important representatives of microscopic traffic flow models
  - **lane-changing models** or **integrated models** (combining longitudinal and lateral dynamics)
  - **non-lane-based models**, e.g., for mixed traffic (India), cross-country skiing and running events,
  - general **discrete-choice models** for situations such as entering or crossing a road, stopping behind a traffic light
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Mathematical forms

- Continuous in space and time: **coupled ordinary differential equations (ODEs)** as in Newtonian dynamics:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = v_i, \quad \frac{\mathrm{d}v_i}{\mathrm{d}t} = f_i(x_i, x_{i-1}, v_i, v_{i-1}, ..)$$

Why $f_i(.)$ instead of $f(.)$? Different driving styles or even models

- Discrete update timesteps: **iterated maps**

$$x_i(t+\Delta t) = f_i^x(x_i(t), v_i(t)), \quad v_i(t+\Delta t) = f_i^v(x_i(t), x_{i-1}(t), v_i(t), v_{i-1}(t), ..)$$

- Space, time, and state are all discrete: **cellular automata**(CA)

$$v(t+1) = f^{\text{CA}}(v(t)), \quad v_k = \begin{cases} -1 & \text{cell } k \text{ empty} \\ 0, 1, \ldots & \text{cell } k \text{ occupied, speed } v_k^{\text{phys}} = v_k \Delta x / \Delta t \end{cases}$$

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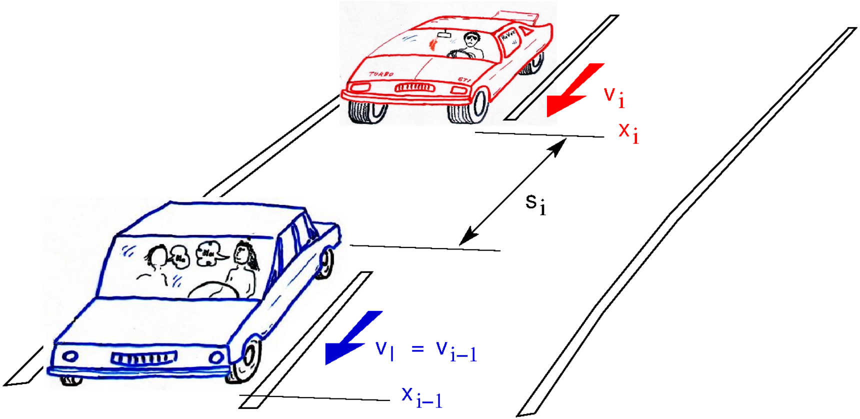

Most car-following models consider just the immediate leader, exactly like an **adaptive-cruise control (ACC)** system:

- Independent variables: speed v_i, gap $s_i = x_{i-1} - x_i - l_{i-1}$, and leading speed $v_{i-1} := v_l$
- Position x_i: front bumper of vehicle i, increasing in driving direction
- Indices i as in a race: the first becomes Number 1, so $x_{i-1} > x_i$

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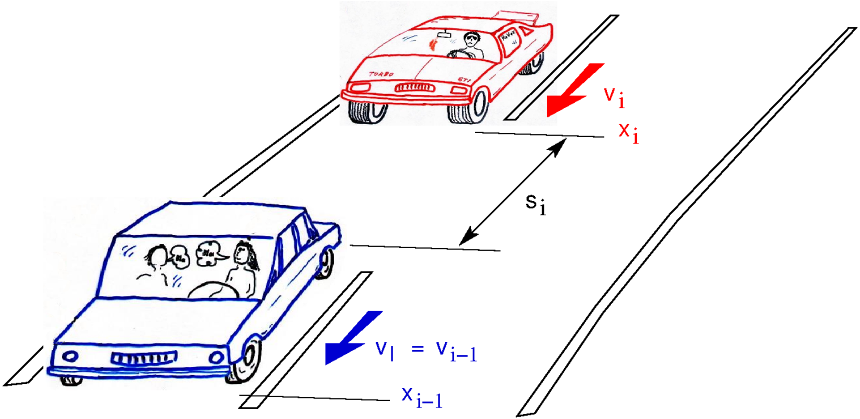

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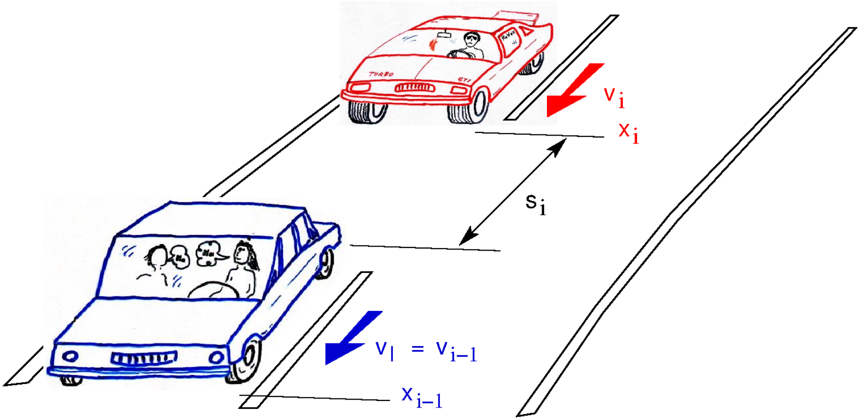

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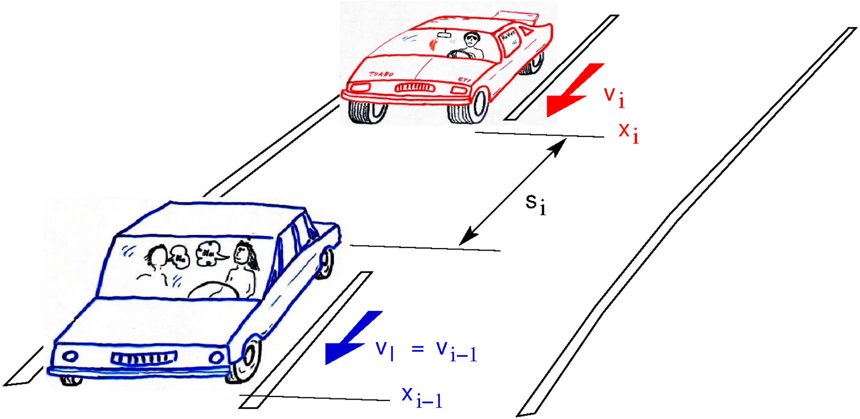

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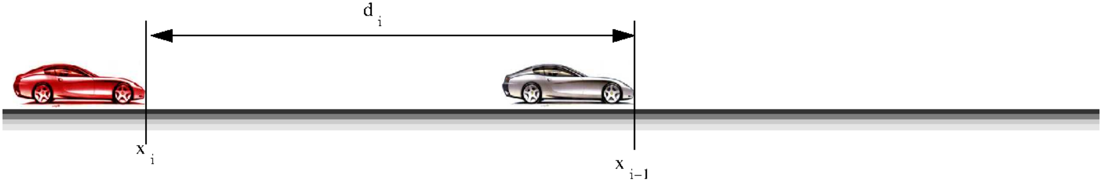
Clarification: headways and gaps



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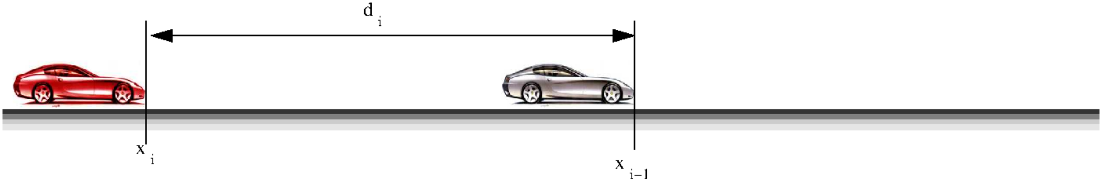

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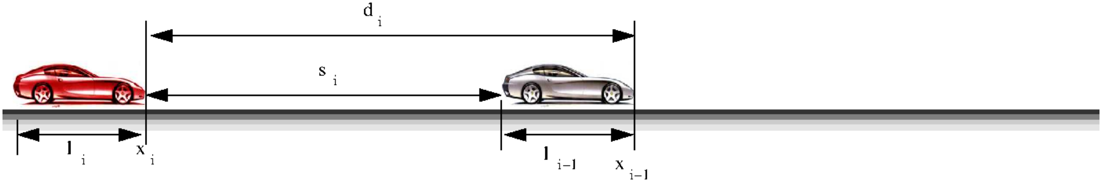

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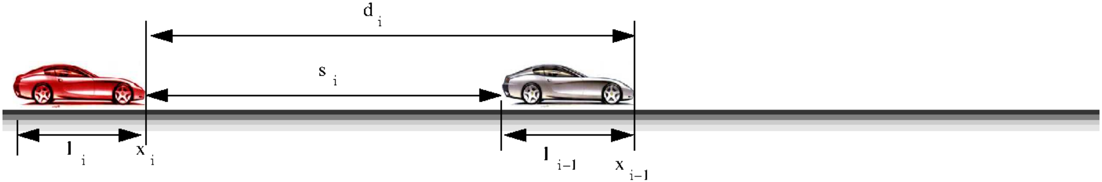

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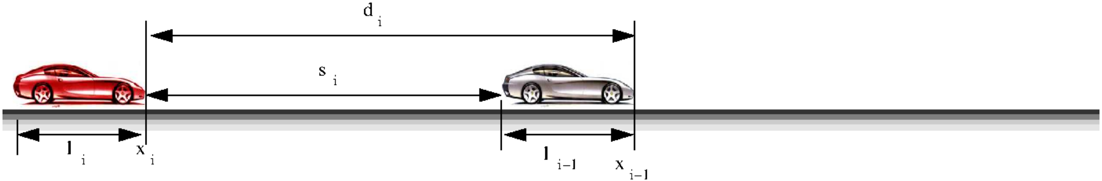

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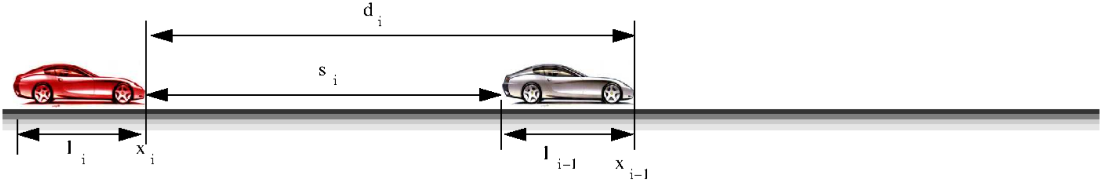

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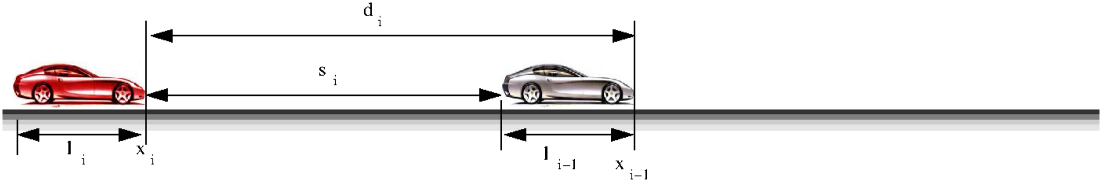

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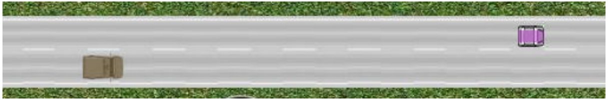

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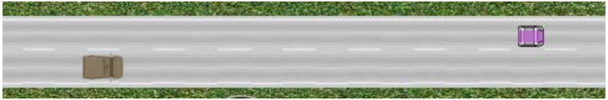

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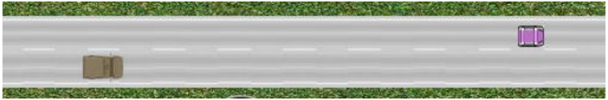

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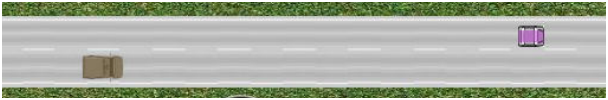

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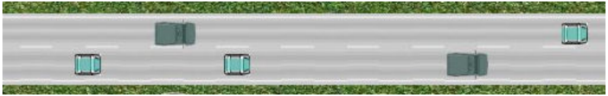

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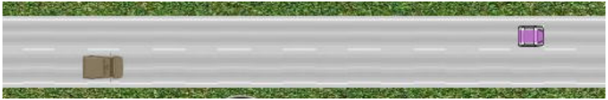

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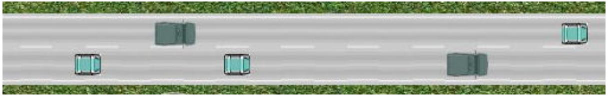

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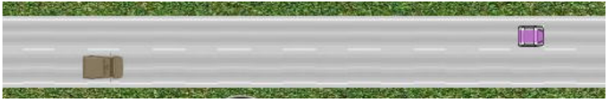

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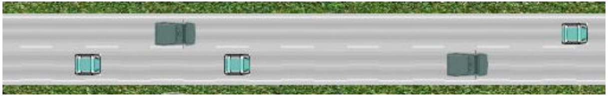

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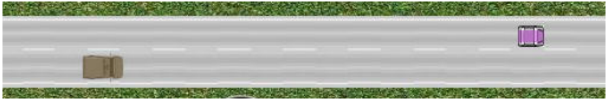

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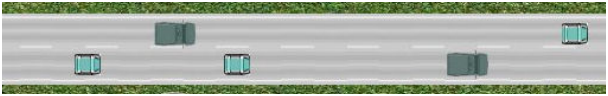

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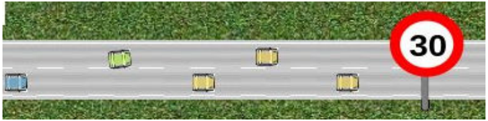

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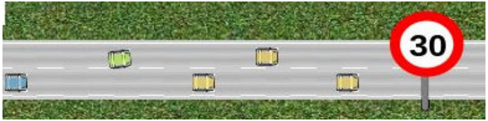

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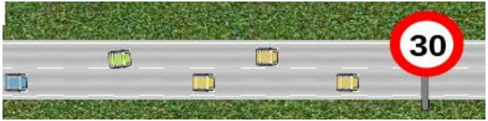

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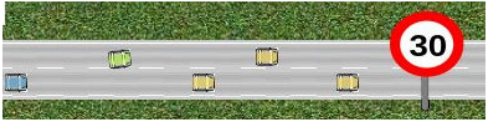

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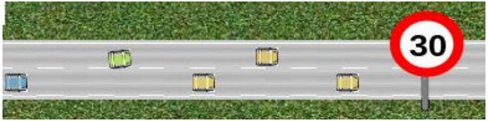

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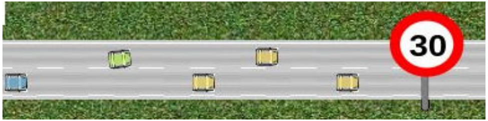

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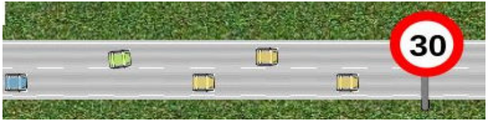

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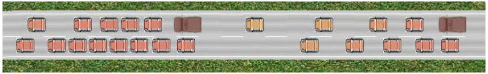

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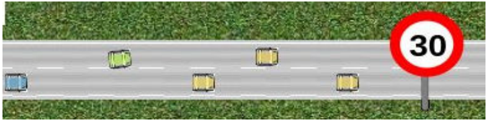

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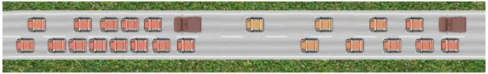

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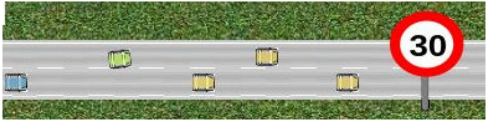

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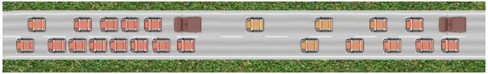

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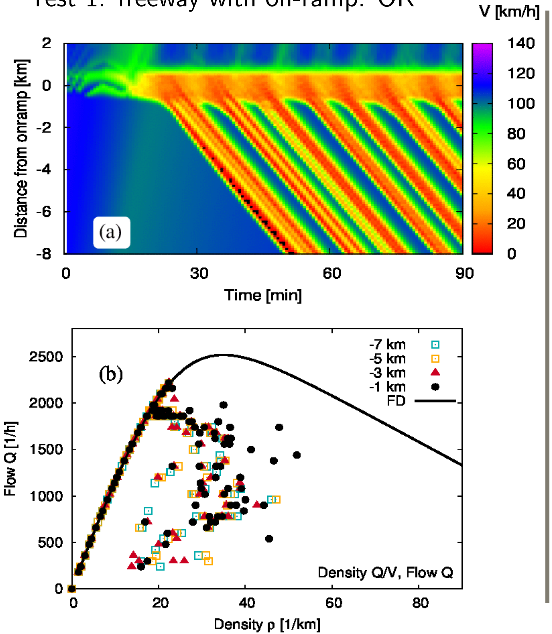

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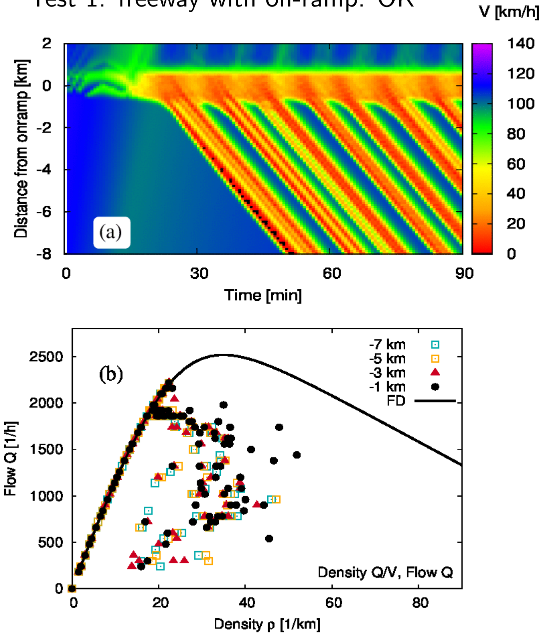
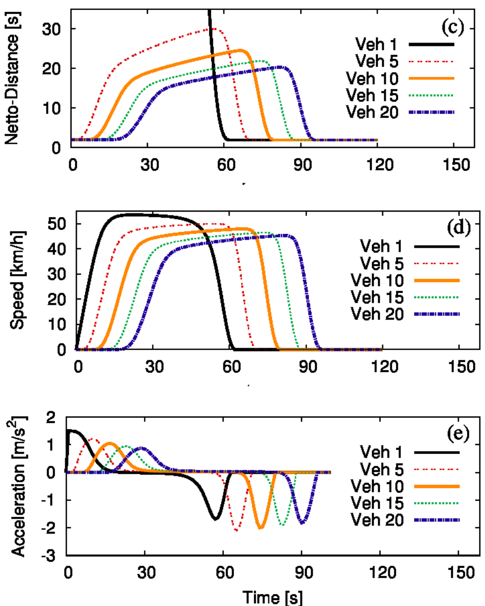


Test 2: traffic lights: OK



Example of an incomplete model: FVDM

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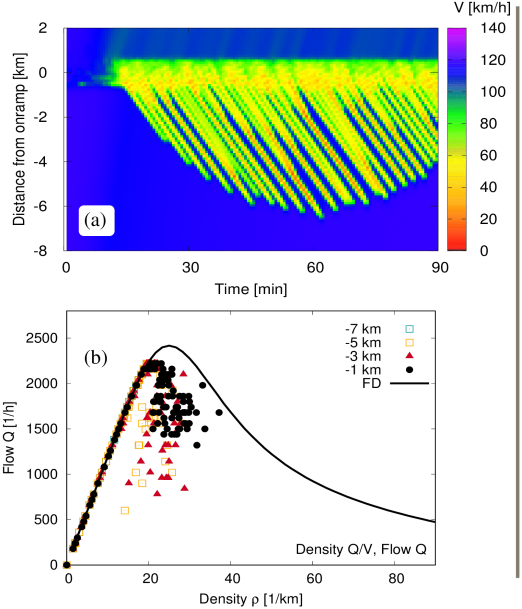

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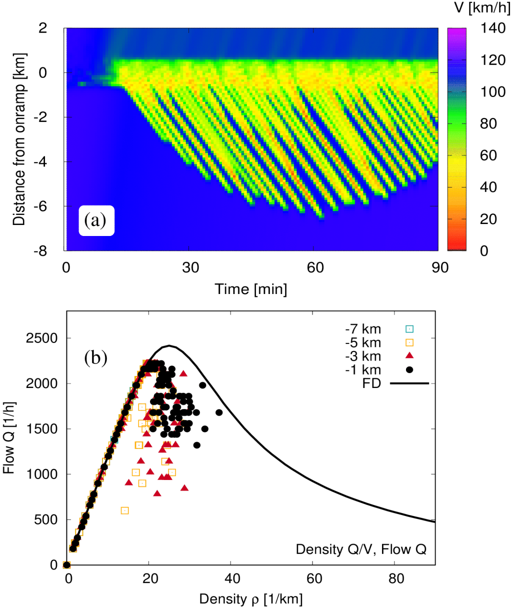
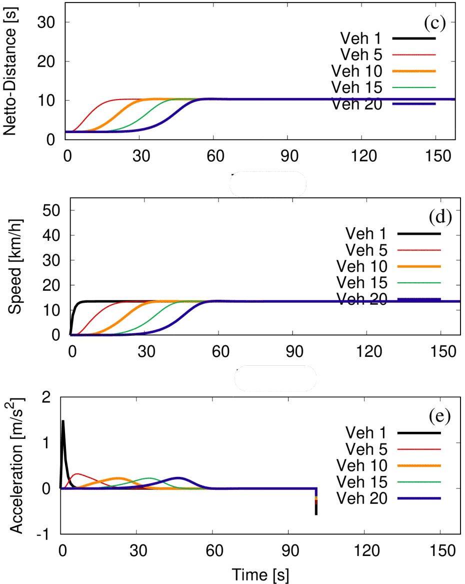


Test 2: traffic lights:
transition to free flow fails($v_0 = 54\,\mathrm{km/h}$)



Plausibility criteria: the acceleration function

Formulate both ODE and iterated map models such that $f(.)$ stands for the acceleration function:

- ODE models:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = v_i, \qquad \frac{\mathrm{d}v_i}{\mathrm{d}t} = f(s_i, v_i, v_{i-1}) \equiv f(s, v, v_l)$$

- Iterated-map models:

$$\begin{aligned} v_i(t+\Delta t) &= v_i(t) + f(s_i(t), v_i(t), v_{i-1}(t))\,\Delta t, \\ x_i(t+\Delta t) &= x_i(t) + \frac{1}{2}\left[v_i(t) + v_i(t+\Delta t)\right]\Delta t \end{aligned}$$

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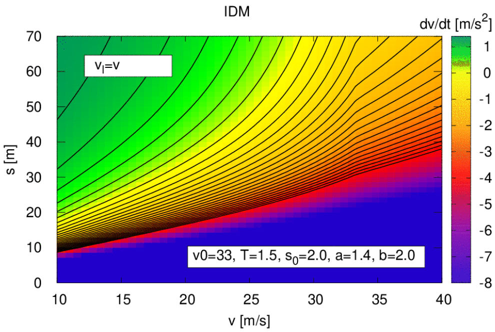

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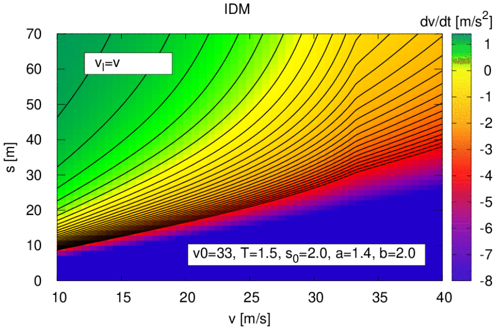

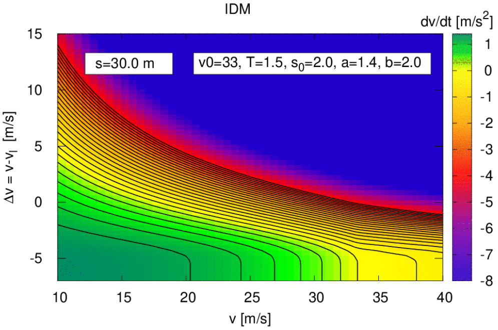

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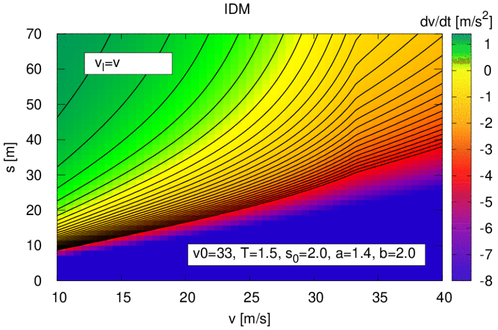

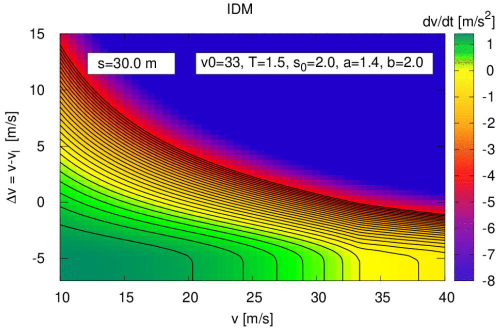

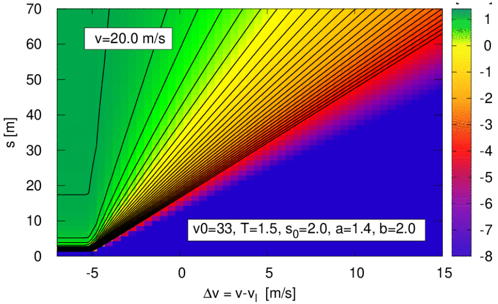

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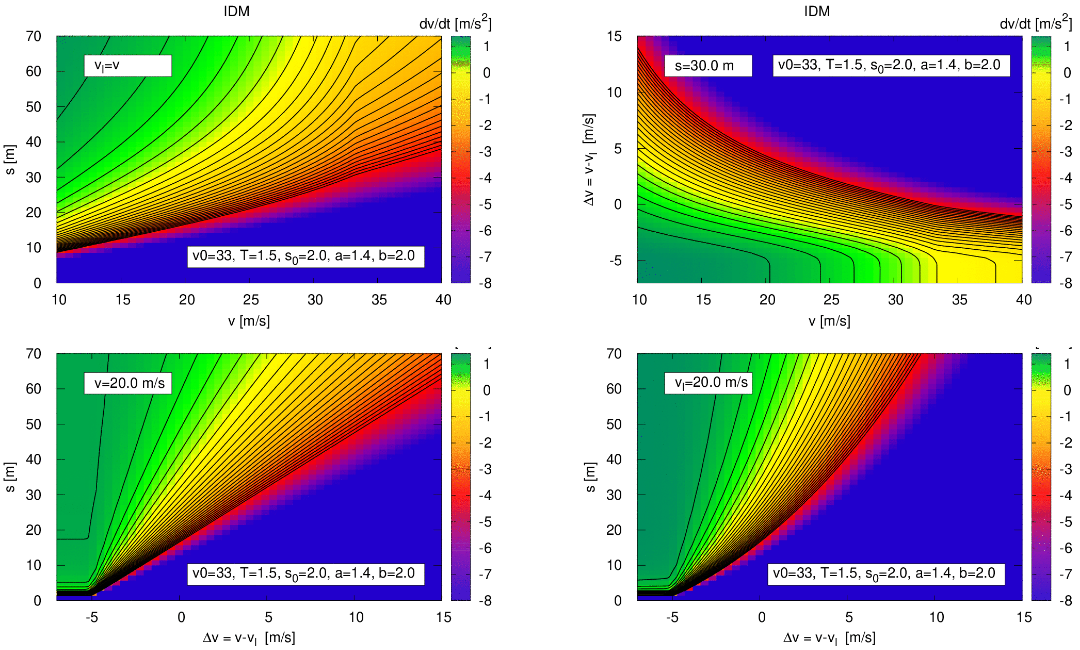
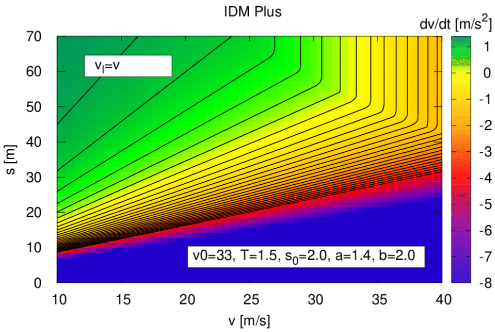


Plausibility criteria: the IDMplus acceleration function



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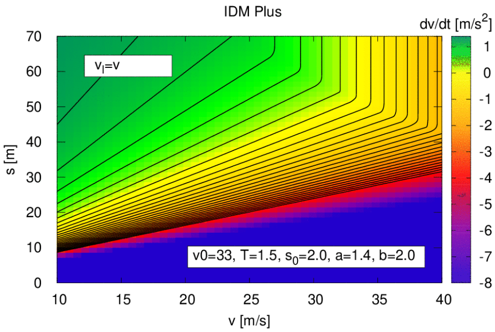

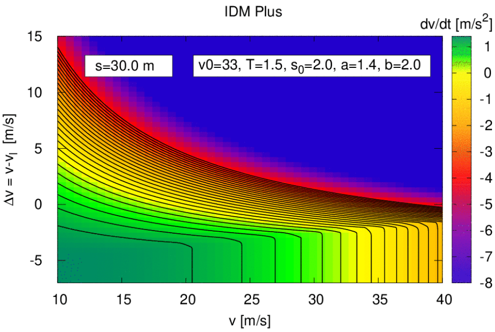

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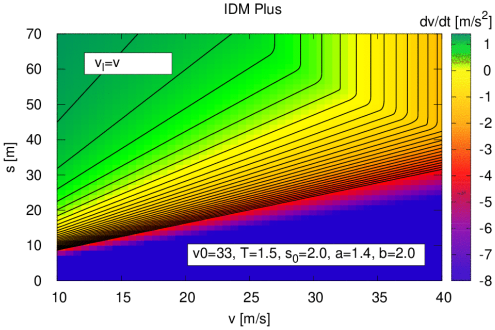

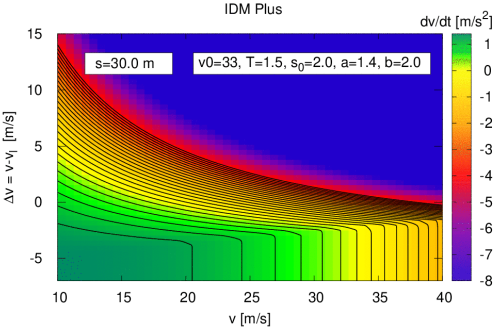

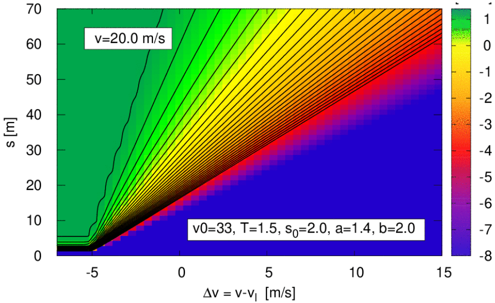

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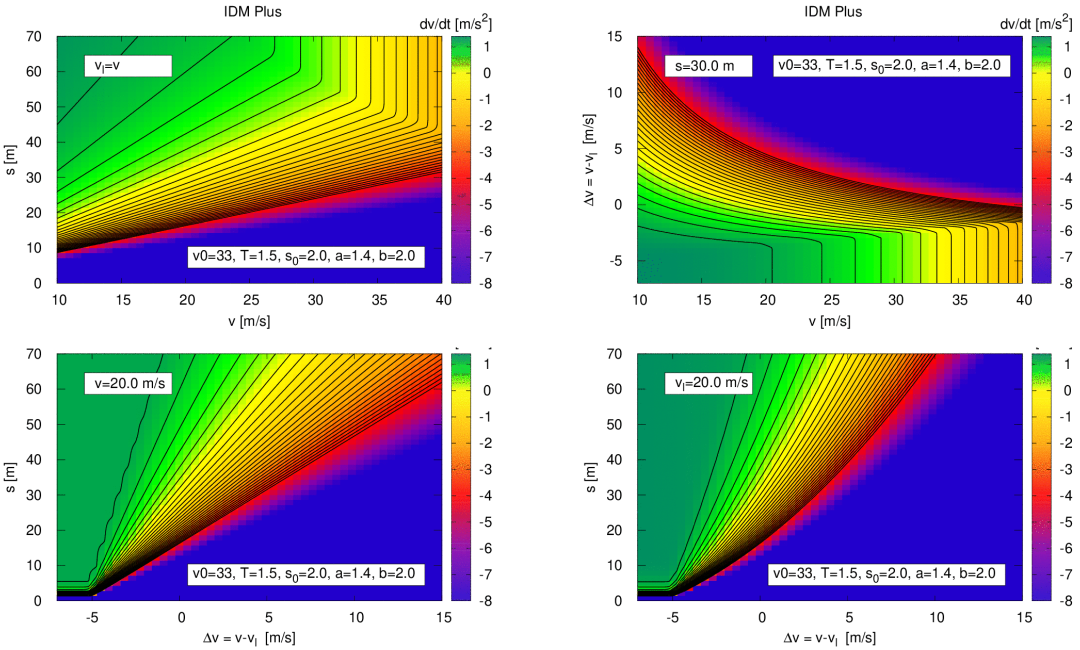

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A necessary condition for completeness is that the following **plausibility conditions** are satisfied:

(1) Dependence of the acceleration on the own speed and existence of a desired speed v_0:

$$\frac{\partial f(s,v,v_l)}{\partial v} < 0, \quad \lim_{s\to\infty} f(s,v_0,v_l) = 0$$

(2) Dependence on the gap with limiting case of no interaction:

$$\frac{\partial f(s,v,v_l)}{\partial s} \geq 0, \quad \lim_{s\to\infty} \frac{\partial f(s,v,v_l)}{\partial s} = 0$$

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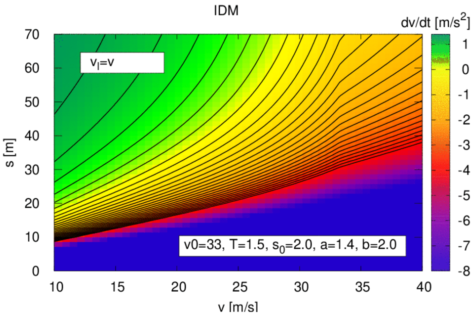
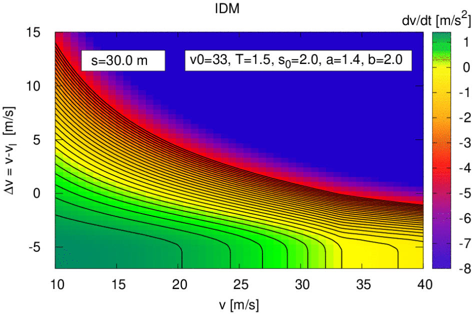
Plausibility criteria II: Steady-state relation





Steady-state speed-gap relation and existence of a minimum gap:

The steady-state speed $v_e(s)$ defined by $f(s, v_e(s), v_e(s)) = 0$ satisfies

$$v_e'(s) \ge 0, \ \lim_{s \to \infty} v_e(s) = v_0, \ v_e(s_0) = 0 \text{ for some } s_0 > 0$$

Express $v_e'(s)$ in terms of $\frac{\partial f}{\partial s}$, $\frac{\partial f}{\partial v}$, and $\frac{\partial f}{\partial v_l}$ and show that this condition follows from (1) and (2)

$f(s_e, v, v) = 0$

$$\begin{aligned} \Rightarrow 0 &= \mathrm{d}f \\ &= \frac{\partial f}{\partial s}\,\mathrm{d}s + \frac{\partial f}{\partial v}\,\mathrm{d}v + \frac{\partial f}{\partial v_l}\,\mathrm{d}v \\ &= \left(\frac{\partial f}{\partial s} + \frac{\partial f}{\partial v}v_e'(s) + \frac{\partial f}{\partial v_l}v_e'(s)\right)\,\mathrm{d}s \end{aligned}$$

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≥ 0 since $\frac{\partial f}{\partial s} \ge 0$, $\frac{\partial f}{\partial v} < 0$, and $\left|\frac{\partial f}{\partial v_l}\right| \le \left|\frac{\partial f}{\partial v}\right|$

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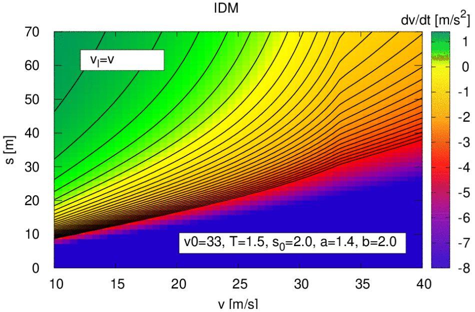

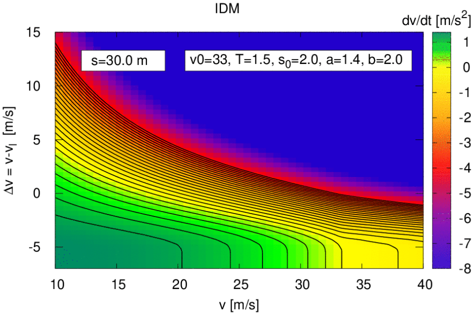

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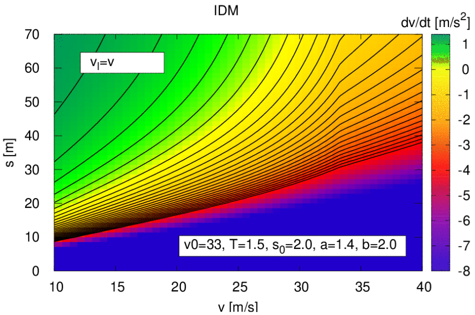

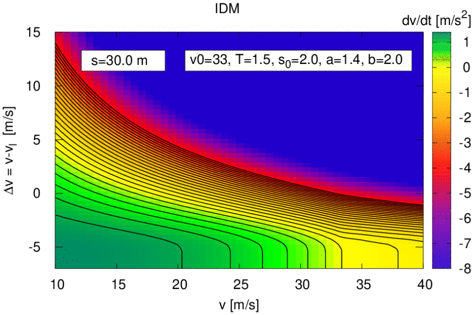

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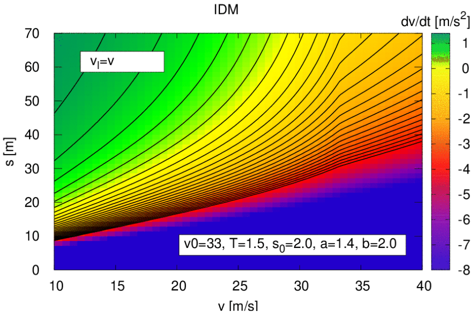

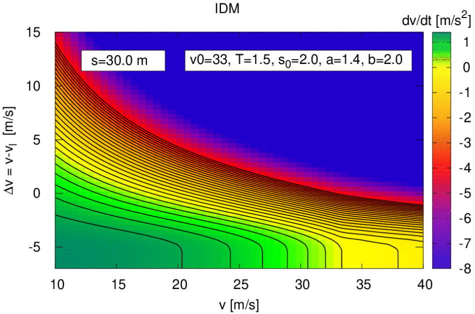

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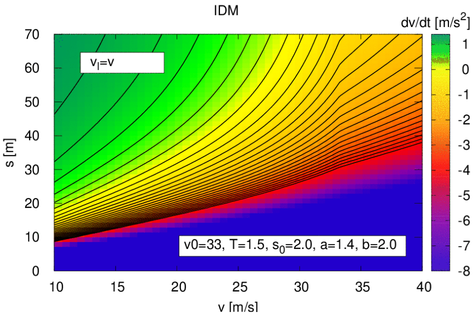

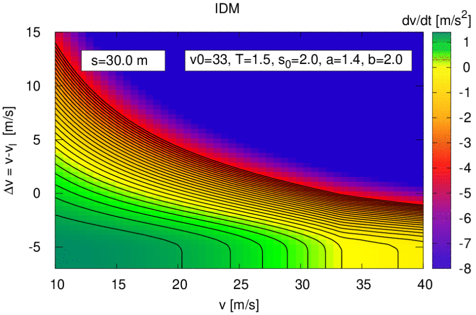

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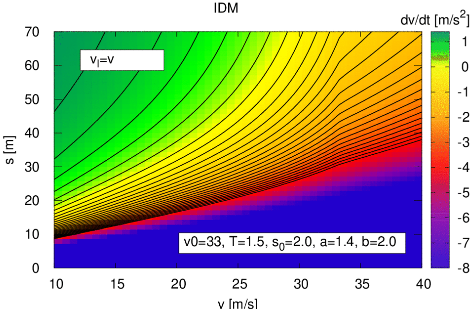

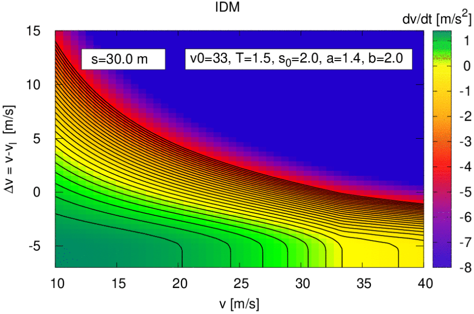

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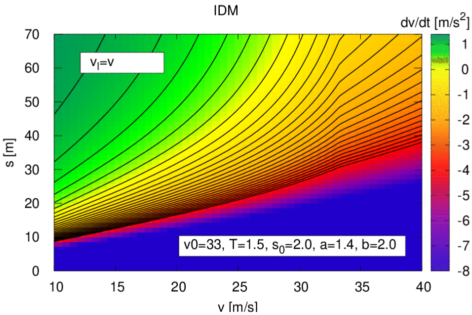

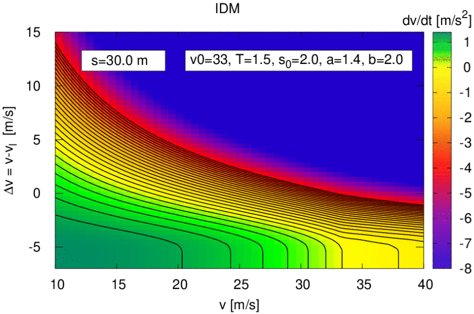

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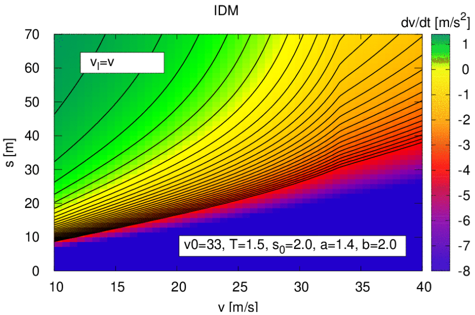
$$\begin{aligned} \Rightarrow 0 &= \mathrm{d}f \\ &= \frac{\partial f}{\partial s}\,\mathrm{d}s + \frac{\partial f}{\partial v}\,\mathrm{d}v + \frac{\partial f}{\partial v_l}\,\mathrm{d}v \\ &= \left(\frac{\partial f}{\partial s} + \frac{\partial f}{\partial v} v_e'(s) + \frac{\partial f}{\partial v_l} v_e'(s)\right)\,\mathrm{d}s \end{aligned}$$

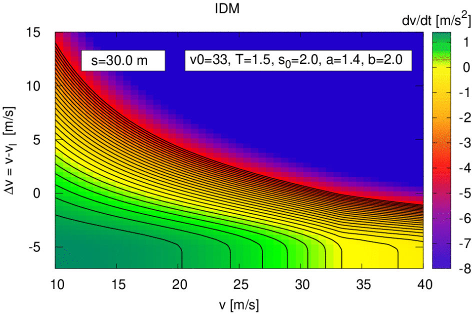
$\Rightarrow v_e'(s) = -\frac{\partial f}{\partial s} / \left(\frac{\partial f}{\partial v} + \frac{\partial f}{\partial v_l}\right)$

≥ 0 since $\frac{\partial f}{\partial s} \geq 0$, $\frac{\partial f}{\partial v} < 0$, and $\left|\frac{\partial f}{\partial v_l}\right| \leq \left|\frac{\partial f}{\partial v}\right|$

and $v_e(s \to \infty) = v_0$ from (1)

Plausibility criteria II: Steady-state relation





Steady-state speed-gap relation and existence of a minimum gap:

The steady-state speed $v_e(s)$ defined by $f(s, v_e(s), v_e(s)) = 0$ satisfies

$$v_e'(s) \geq 0, \lim_{s\to\infty} v_e(s) = v_0, \; v_e(s_0) = 0 \text{ for some } s_0 > 0$$

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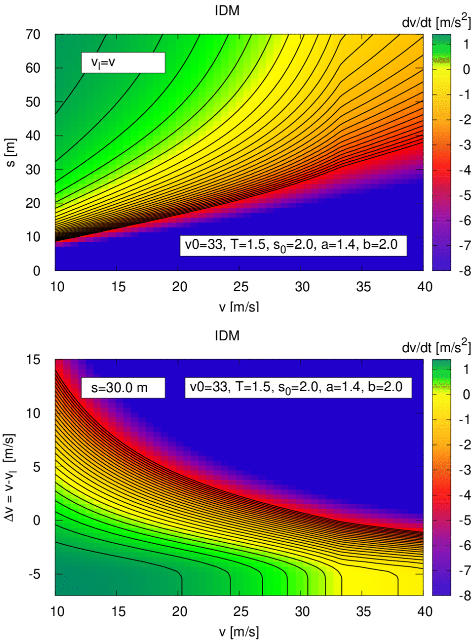

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Some Examples of Elementary Car-Following Models

- Not really useful for actually simulating traffic flow
- but very good for showing the basic principles,
- also serve as basis for the more sophisticated ones

8.4 Optimal Velocity Model (OVM)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v_{\text{opt}}(s) - v}{\tau} \qquad \text{Optimal Velocity Model}$$

Whole model class parameterized by the **optimal-velocity function** $v_{\text{opt}}(s)$, e.g.,

- Original OVM function by Bando et al:

$$v_{\text{opt}}(s) = v_0 \frac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh\beta}{1 + \tanh\beta}$$

- OVM function corresponding to the triangular FD:

$$v_{\text{opt}}(s) = \max\left[0,\ \min\left(v_0, \frac{s - s_0}{T}\right)\right]$$

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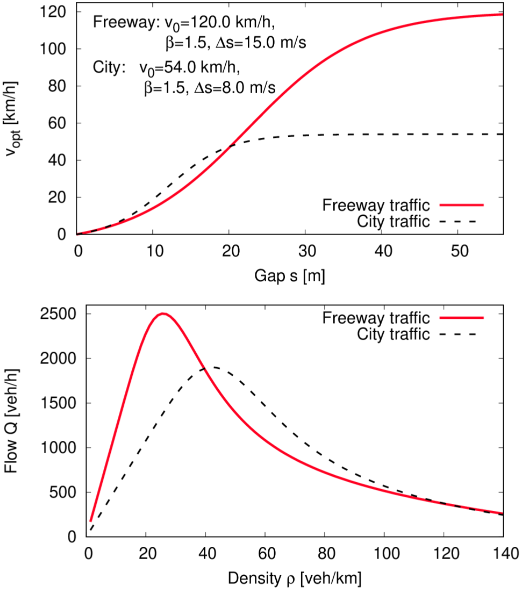
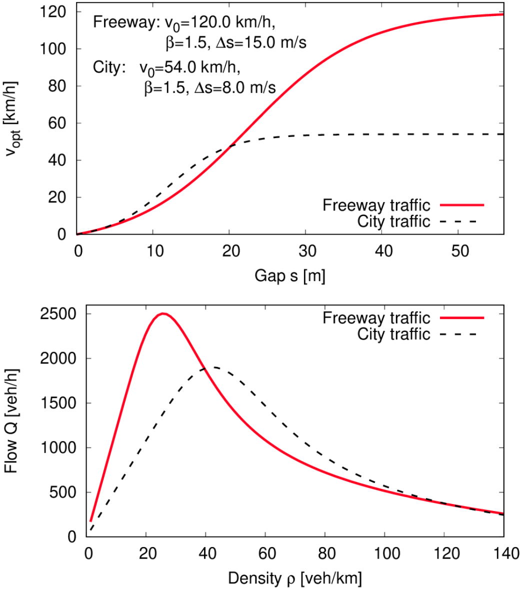


Bando OV function

OV functions



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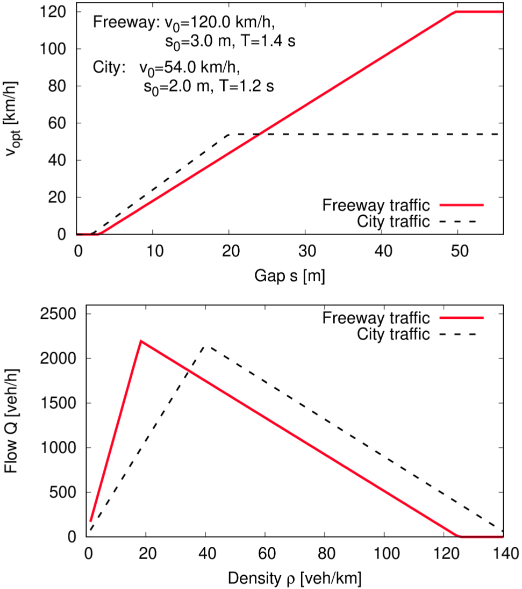

triangular OV function

Properties of the Optimal Velocity Model (OVM)

- The homogeneous-steady-state speed $v_e(s)$ is given by the OV function
- Technically, the model *marginally* satisfies all plausibility conditions (no sensitivity to the leader's speed) but results in unrealistic accelerations, or crashes, or both
- Besides the parameters of the OV function, the OVM has the **speed relaxation time** τ as additional parameter:
  - The more responsive the driver, the lower τ,
  - the higher τ, the more instabilities

Parameter	Typical Value Highway	Typical Value City Traffic
Adaptation time τ	0.65 s	0.65 s
Desired speed v_0	120 km/h	54 km/h
Transition width Δs (Bando FD)	15 m	8 m
Form factor β (Bando FD)	1.5	1.5
Time gap T (triangular FD)	1.4 s	1.2 s
Minimum distance gap s_0 (triangular FD)	3 m	2 m

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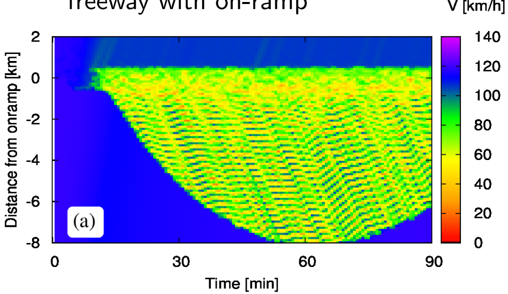
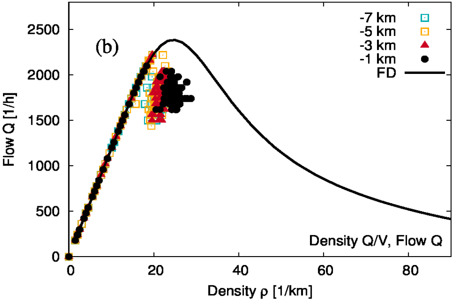
Factsheet of the Optimal Velocity Model (OVM)

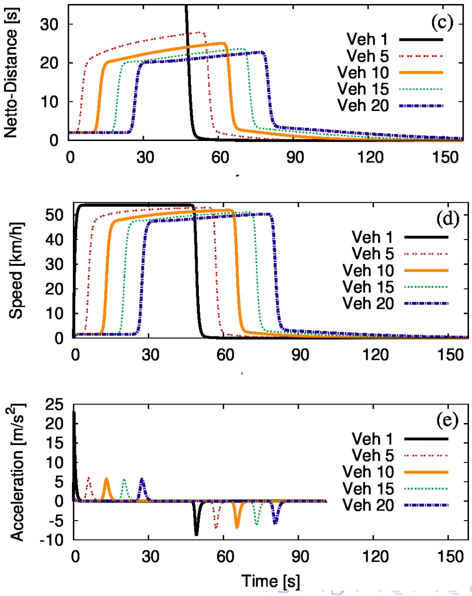
freeway with on-ramp





city with traffic lights

extreme accelerations!



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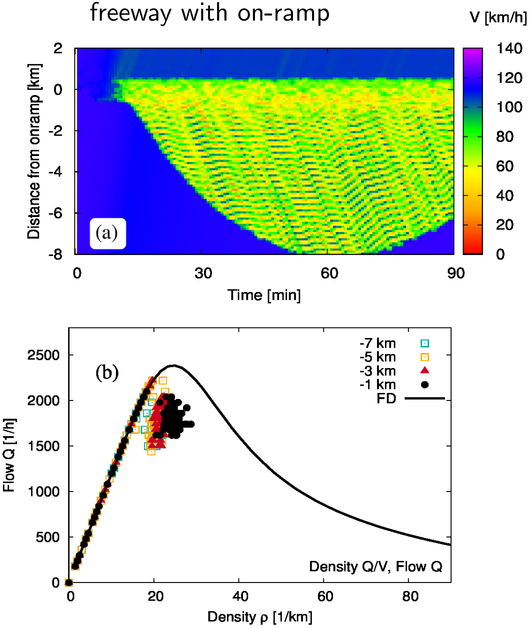

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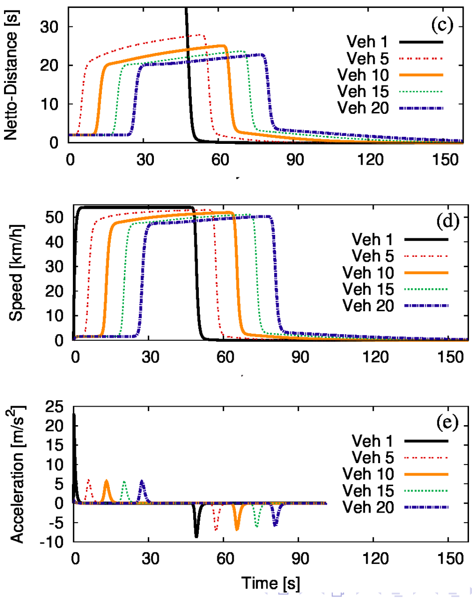

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! Steady State $v = v_l$, $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$: $0 = (v_{\text{opt}}(s) - v)/\tau$. Since the speed adaptation time $\tau > 0$, we have $v = v_e(s) = v_{\text{opt}}(s)$

? Check the plausibility conditions

! (1) $\frac{\mathrm{d}f}{\mathrm{d}v} = -1/\tau < 0$ OK
(2) $\frac{\mathrm{d}f}{\mathrm{d}s} = v_e'(s)/\tau \geq 0$ if $v_e'(s) \geq 0$ OK
(3) $\frac{\mathrm{d}f}{\mathrm{d}v_l} = 0$ marginally OK
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(4b) triangular OV function: $v_{\text{opt}}'(s) = 1/T$ or $=0$, $v_{\text{opt}}(s \to \infty) = v_0$, $v_{\text{opt}}(s_0) = 0$ OK

? show that the "triangular" OV function in fact leads to the triangular FD

! triangular FD: $Q(\rho) = \rho v_{\text{opt}}(1/\rho - l - s_0) = \rho \max[0, \min(v_0, (1/\rho - l)/T)] = \max[0, \min(v_0\rho, 1/T(1 - \rho l))]$
$= \max[0, \min(v_0\rho, 1/T(1 - \rho/\rho_{\text{max}}))]$

OVM questions $f_{\text{OVM}}(s, v, v_l) = (v_{\text{opt}}(s) - v)/\tau$

OV functions: $v_{\text{opt}}^{\text{Bando}} = v_0 \dfrac{\tanh\left(\frac{s}{\Delta s} - \beta\right) + \tanh\beta}{1 + \tanh\beta}$, $v_{\text{opt}}^{\text{triang}} = \max\left[0, \min\left(v_0, \dfrac{s - s_0}{T}\right)\right]$

? Show that the steady state speed $v_e(s)$ is given by the optimal speed.

! Steady State $v = v_l$, $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$: $0 = (v_{\text{opt}}(s) - v)/\tau$. Since the speed adaptation time $\tau > 0$, we have $v = v_e(s) = v_{\text{opt}}(s)$

? Check the plausibility conditions

! (1) $\frac{\mathrm{d}f}{\mathrm{d}v} = -1/\tau < 0$ OK
(2) $\frac{\mathrm{d}f}{\mathrm{d}s} = v_e'(s)/\tau \geq 0$ if $v_e'(s) \geq 0$ OK
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8.5. Full Velocity Difference Model (FVDM)

$$\frac{dv}{dt} = \frac{v_{\text{opt}}(s) - v}{\tau} + \gamma(v_l - v) \quad \text{Full Velocity Difference Model}$$

- The FVDM is the optimal-velocity model with an additional sensitivity to the relative speed $v - v_l$ to the leader
- The additional sensitivity parameter γ has values of the order of $0.5\,\mathrm{s}^{-1}$
- As in the OVM, the homogeneous steady state speed $v_e(s) = v_{\text{opt}}(s)$
- As a pure car-following model, the FVDM behaves more realistically. However, in contrast to the OVM, it is *not* complete **Why?** For $s \to \infty$, the FVDM acceleration still depends strongly on v_l thereby violating plausibility requirement (3b) $\lim_{s\to\infty} \frac{\partial f}{\partial v_l} = 0$: There is no transition from car-following to free traffic

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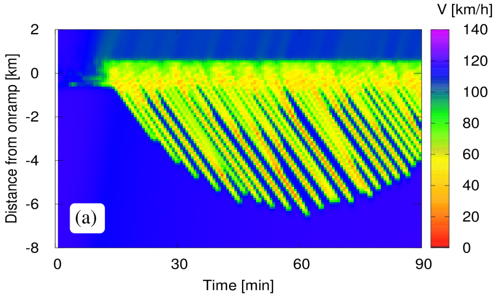
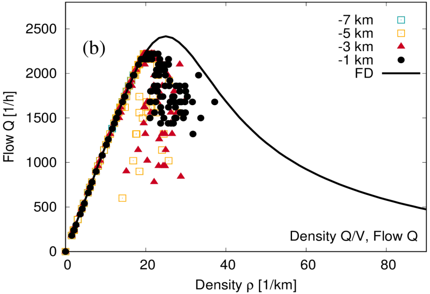
Factsheet of Bando's Full Velocity Difference Model (FVDM)

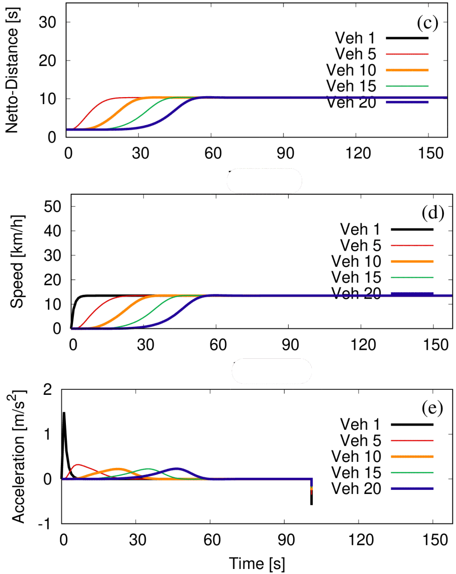
freeway with on-ramp





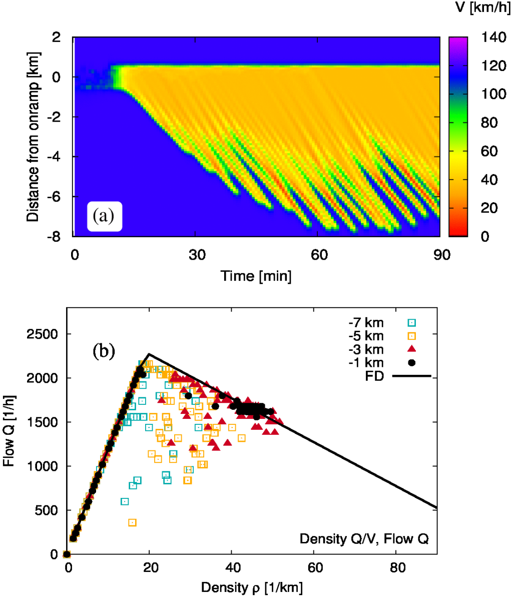
city with traffic lights

spot and explain the unrealistic behaviour!

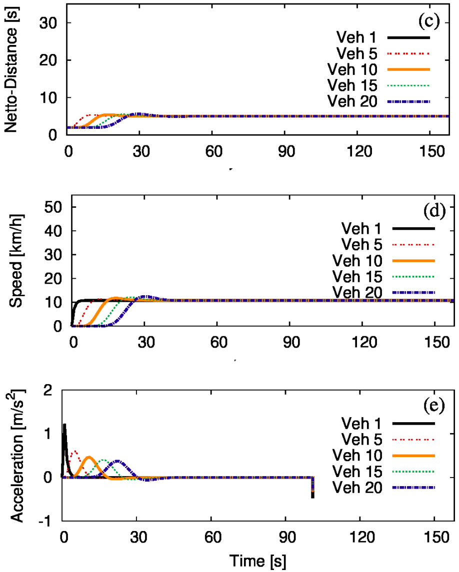


Factsheet of the FVDM with triangular FD

freeway with on-ramp



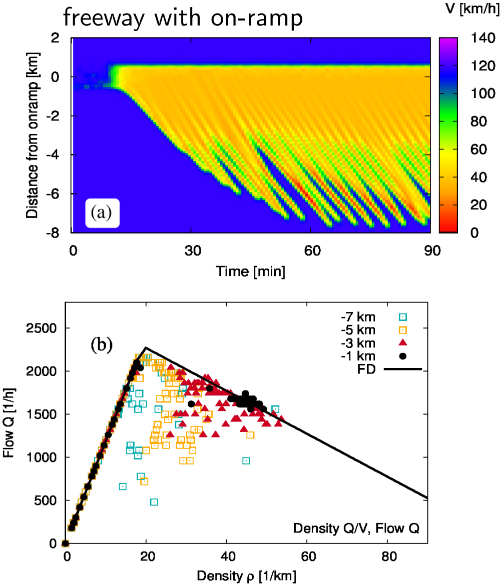
city with traffic lights



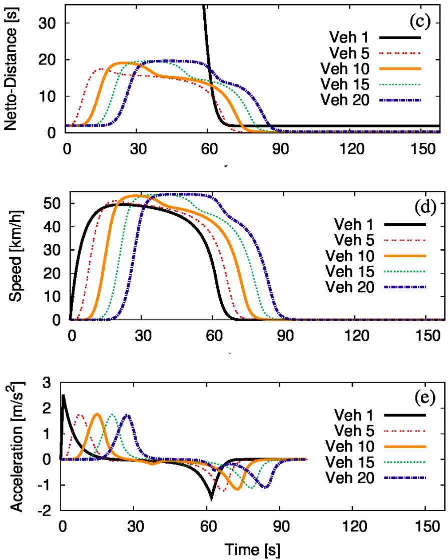
Factsheet of the modified FVDM with triangular FD

$$f(s, v, v_l) = (v_{\text{opt}}^{\text{triang}} - v)/\tau + \gamma(v_l - v)\ \min(1, v_0 T/s)$$

freeway with on-ramp



city with traffic lights



8.6 Newell's Car-Following Model

$$v(t+T) = v_{\text{opt}}(s(t)), \quad v_{\text{opt}}(s) = \min\left(v_0, \frac{s}{T}\right) \qquad \text{Newell's Model}$$

- The OV relation can also be written in terms of the distance headway $\tilde{v}_{\text{opt}}(d) = v_{\text{opt}}(s + l_{\text{eff}})$ and represents the triangular FD (**check!**)

$$Q(\rho) = \min\left[V_0\rho, \frac{1}{T}(1 - \rho l_{\text{eff}})\right]$$

- Three parameters: effective vehicle length l_{eff} (incl minimum gap s_0), reaction time T, and desired speed v_0
- T is not only the reaction time but also the time gap, the speed adaptation time, and the numerical update timestep (**check!**)

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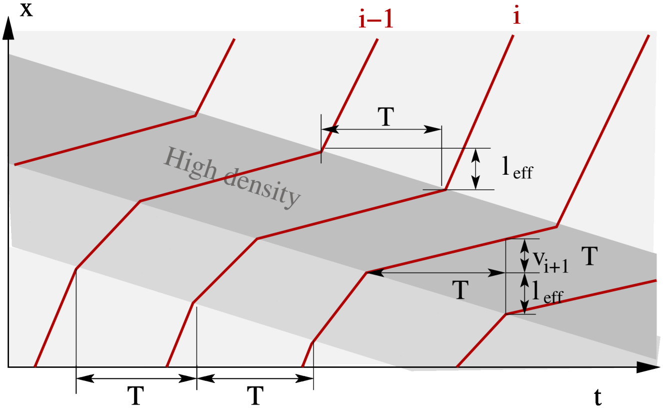
Newell's car-following model: properties



- Constant **wave speed** w by considering the start of a queue of standing vehicles (distance headway $d = l_{\text{eff}}$) or simply by the general expression $w = Q'_{\text{cong}}(\rho)$ from the congested part of the FD:

$$w = -l_{\text{eff}}/T$$

- This means that, in the car-following regime $(s/T < v_0)$, the follower adopts the leader's speed one "reaction time" T ago and proceeds by the gap value one "reaction time" T ago:

$$v(t+T) = v_l(t), \quad x(t+T) = x_l(t) - l_{\text{eff}}$$

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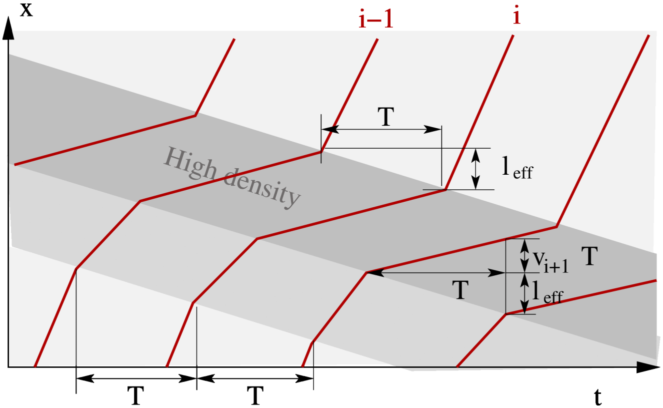

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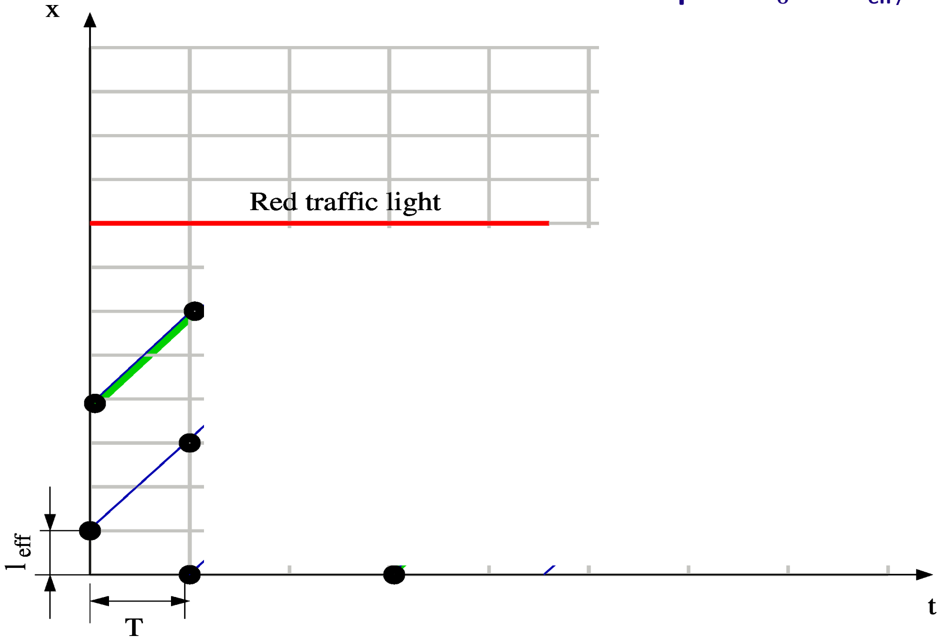
Numerics of Newell's micromodel: iterated map for $v_0 = 2l_{\text{eff}}/T$



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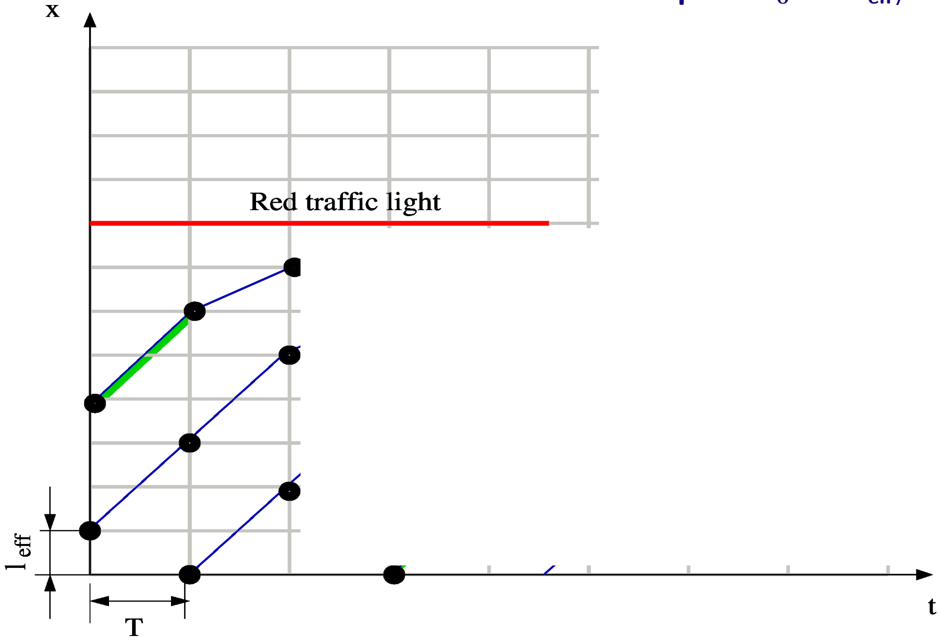

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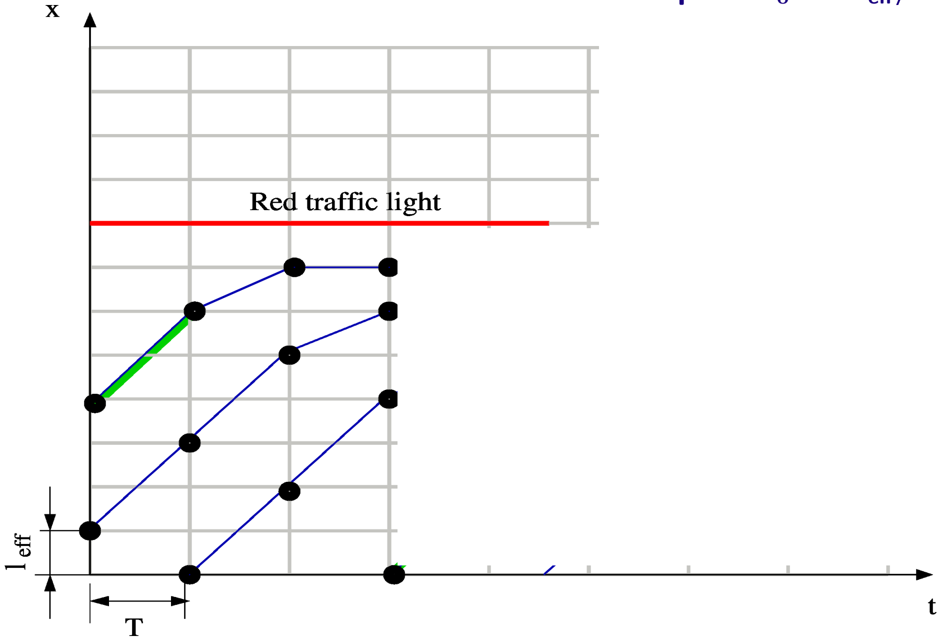

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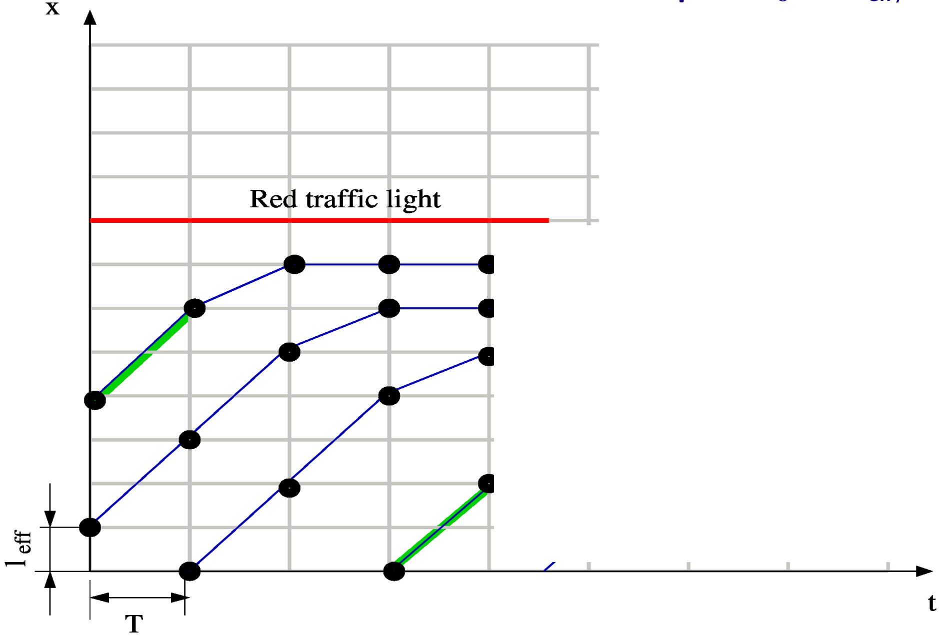

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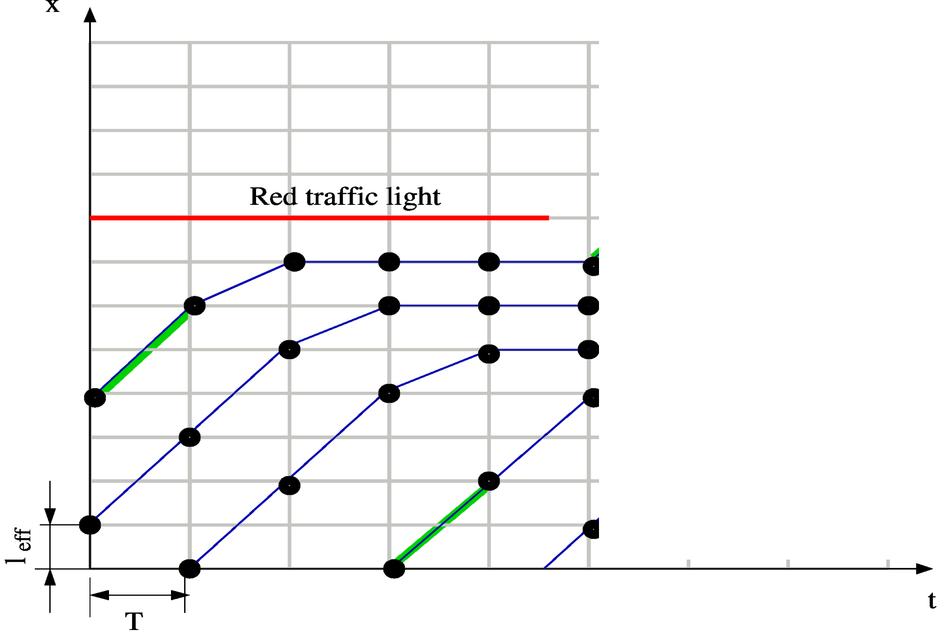

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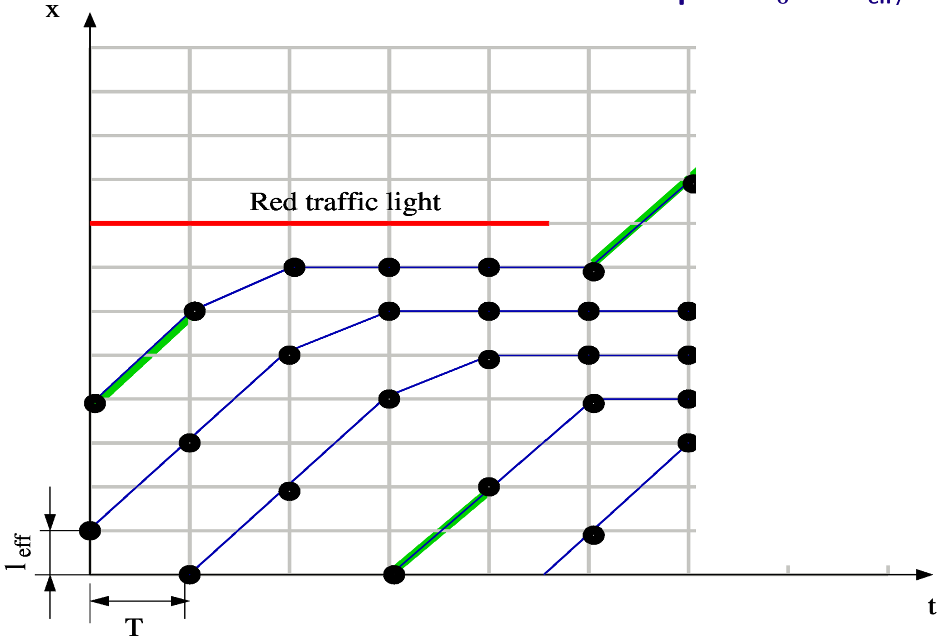

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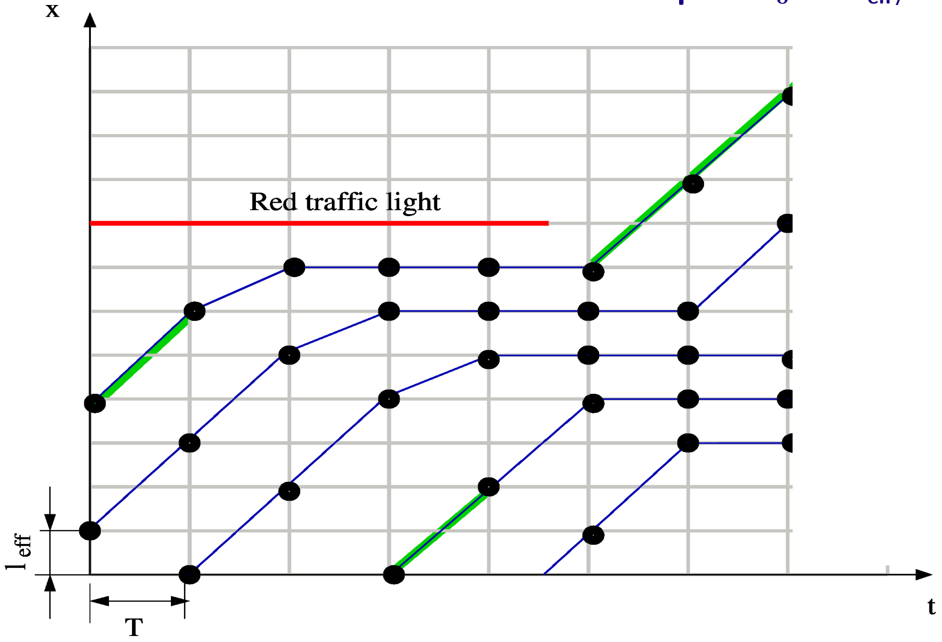

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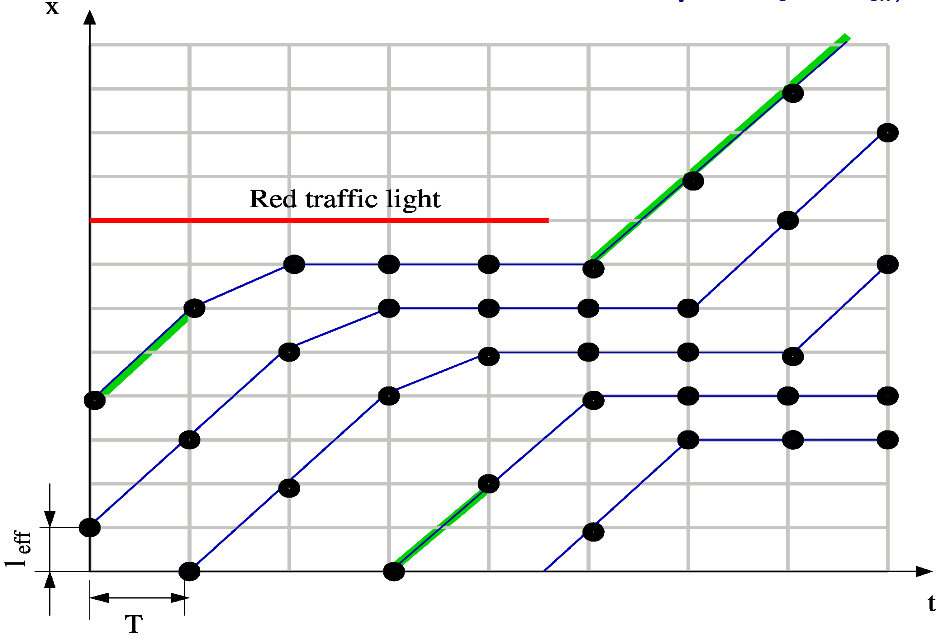

8.7. Car-Following Cellular Automata (CA)

Cellular automata (CA) describe all aspects of dynamical systems by using (generally small) integers:

- Space is subdicided into cells
- Time is subdivided into time steps Δt
- State variables are multiples of the natural unit, e.g., speed in cells/Δt and accelerations in cells/$(\Delta t)^2$
- In the **Euler** or occupation number representation the dynamical unit is a cell that can be occupied (1) or not (0) [here, the maximum speed $v_0 = 1$ and we have redefined the state $-1 \to 0$ for empty, and 0 or $1 \to 1$ for occupied with speed 0 or 1 to match the historic example] such as in the famous Rule 184 $(= 2^7 + 2^5 + 2^4 + 2^3)$ (try to understand it):

current local pattern	7=111	6=110	5=101	4=100	3=011	2=010	1=001	0=000
new state of the center cell	1	0	1	1	1	0	0	0

- In the **Lagrange** representation a CA looks like a discretized car-following model such as the **Nagel-Schreckenberg Model** below

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- In the **Euler** or occupation number representation the dynamical unit is a cell that can be occupied (1) or not (0) [here, the maximum speed $v_0 = 1$ and we have redefined the state $-1 \to 0$ for empty, and 0 or 1 $\to 1$ for occupied with speed 0 or 1 to match the historic example] such as in the famous Rule 184 $(= 2^7 + 2^5 + 2^4 + 2^3)$ (try to understand it):

current local pattern	7= 111	6= 110	5= 101	4= 100	3= 011	2= 010	1= 001	0= 000
new state of the center cell	1	0	1	1	1	0	0	0

- In the **Lagrange** representation a CA looks like a discretized car-following model such as the **Nagel-Schreckenberg Model** below

8.7. Car-Following Cellular Automata (CA)

Cellular automata (CA) describe all aspects of dynamical systems by using (generally small) integers:

- Space is subdicided into cells
- Time is subdivided into time steps Δt
- State variables are multiples of the natural unit, e.g., speed in cells/Δt and accelerations in cells/$(\Delta t)^2$
- In the **Euler** or occupation number representation the dynamical unit is a cell that can be occupied (1) or not (0) [here, the maximum speed $v_0 = 1$ and we have redefined the state $-1 \to 0$ for empty, and 0 or 1 $\to 1$ for occupied with speed 0 or 1 to match the historic example] such as in the famous Rule 184 $(= 2^7 + 2^5 + 2^4 + 2^3)$ (try to understand it):

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- In the **Lagrange** representation a CA looks like a discretized car-following model such as the **Nagel-Schreckenberg Model** below

Nagel-Schreckenberg Model (NSM) and the Barlovic Model

These are **Stochastic** CAs in the Lagrange representation, i.e., the relevant unit is a vehicle i rather than a cell k:

1. *Deterministic acceleration* as a function of the speed v_i, desired speed v_0 and gap (number of empty cells) g_i:

$$v_i^*(t+1) = \min\left(v_i(t)+1,\ v_0,\ g_i\right)$$

2. *Dawdling* by not accelerating, or braking more than necessary, with a certain dawdling probability p:

$$v_i(t+1) = \begin{cases} \max\left(v_i^*(t+1)-1,\ 0\right) & \text{with probability } p, \\ v_i^*(t+1) & \text{otherwise.} \end{cases}$$

In the Barlovic model, the "slow-to-start" rule applies, i.e., the probability p_0 for standing vehicles $(v_i(t)=0)$ is higher than p for driving vehicles

- *Driving* by moving $v_i(t+1)$ cells forward:

$$x_i(t+1) = x_i(t) + v_i(t+1).$$

Verify that *Rule 184* corresponds to the deterministic NSM with $v_0 = 1$.
Then, a car moves by one cell whenever the new cell is free. Compare with the Rule-184 table.

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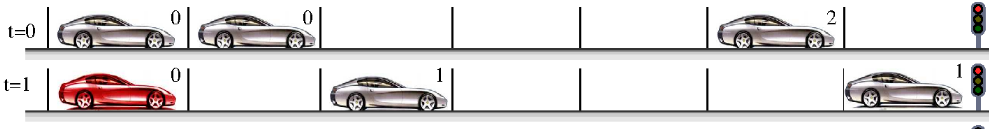
How the NSM works ($v_0 = 2$)

Parameter	Typ. Value Highway	Typ. Value City
Cell length $\Delta x_{\text{phys}} = l_{\text{eff}}$	7.5 m	7.5 m
Time step Δt_{phys}	1 s	1 s
Desired speed v_0	5	2
Dawdling probability p	0.2	0.1
Prob. p_0 when stopped (Barlovic)	0.4	0.2

How the NSM works ($v_0 = 2$)



Parameter	Typ. Value Highway	Typ. Value City
Cell length $\Delta x_{\text{phys}} = l_{\text{eff}}$	7.5 m	7.5 m
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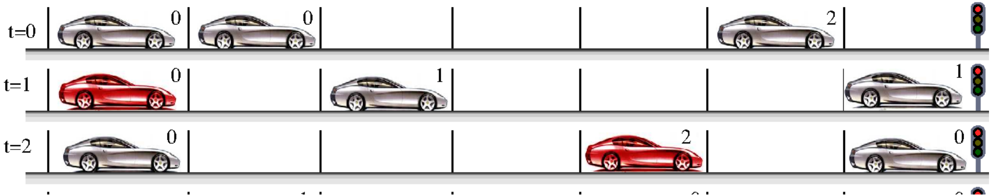

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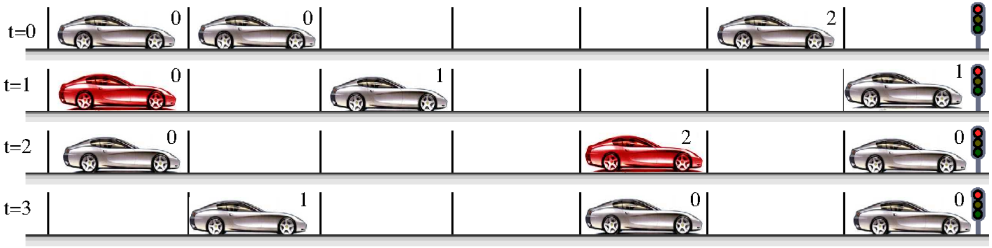

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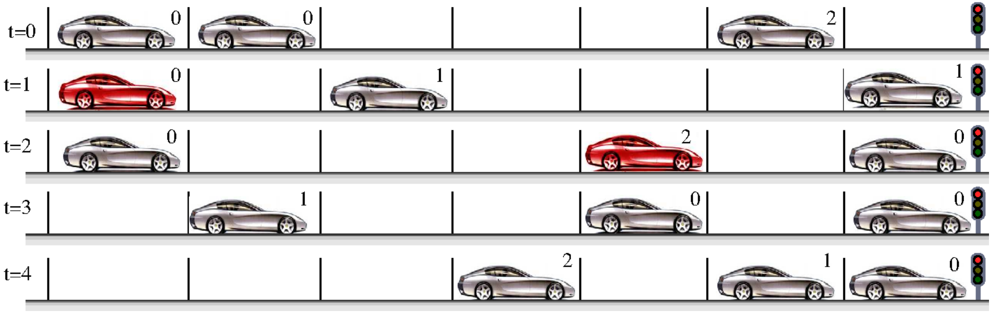

Parameter	Typ. Value Highway	Typ. Value City
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Time step Δt_{phys}	1 s	1 s
Desired speed v_0	5	2
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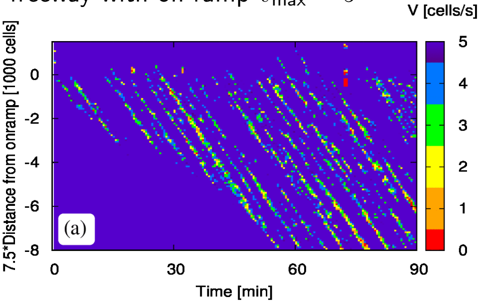
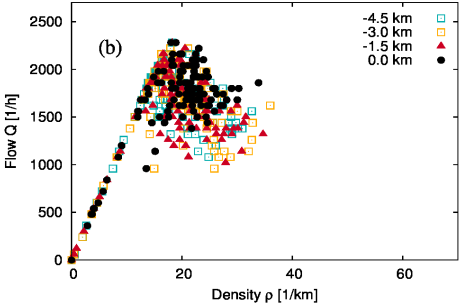
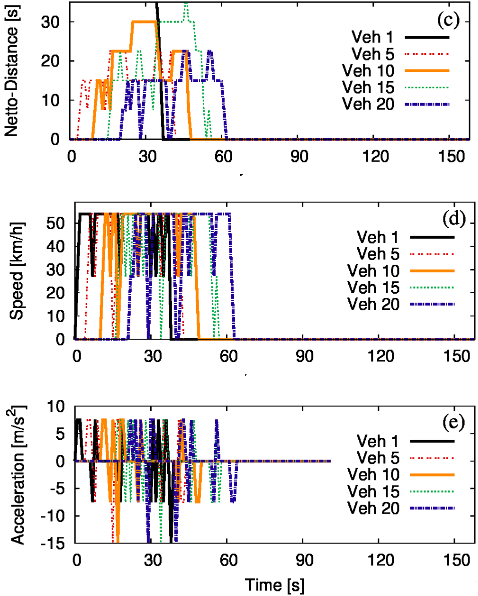
Factsheet of the Nagel-Schreckenberg Model (NSM)

freeway with on-ramp $v_{\text{max}} = 5$



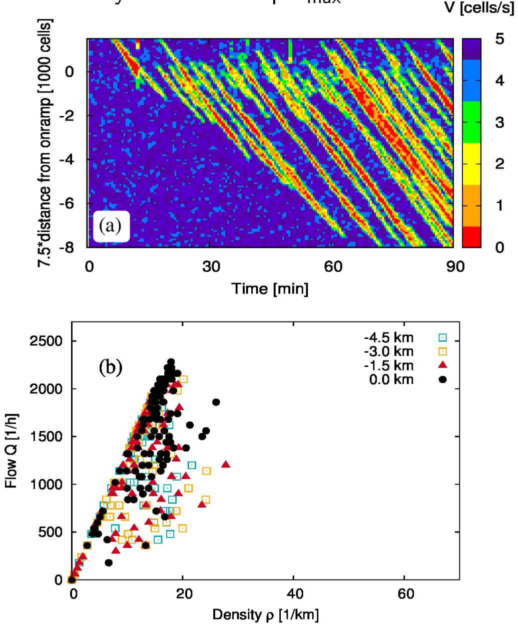


city with traffic lights $v_{\text{max}} = 2$

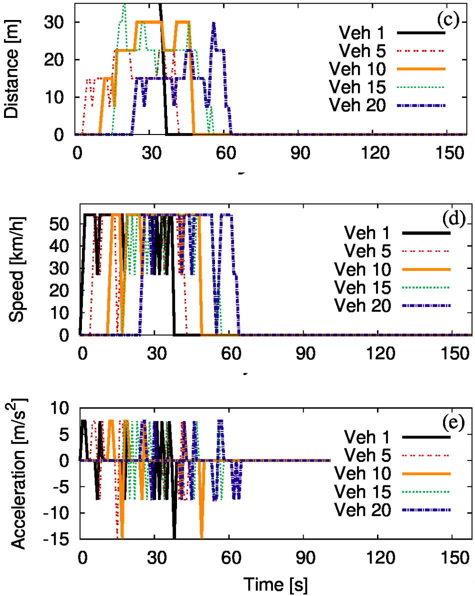


Factsheet of the CA model of Barlovic

freeway with on-ramp $v_{\text{max}} = 5$



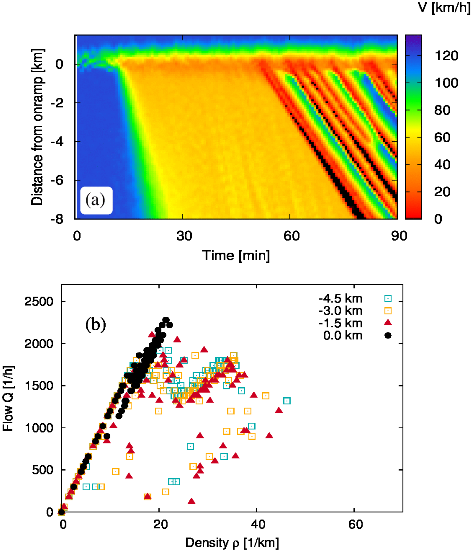
city with traffic lights $v_{\text{max}} = 2$



Factsheet of the CA model of Kerner

There are many more “refined” CAs, e.g., the KCA with a cell size of only 0.5 m

freeway with on-ramp $v_{\text{max}} = 56$



city with traffic lights $v_{\text{max}} = 28$