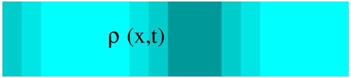
Part II: Traffic Flow Models

Lecture 05: Macroscopic Traffic Flow Models: General

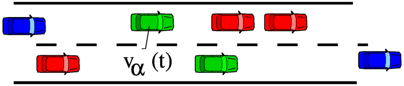
- 5.1. Model Overview
- 5.2. Macroscopic Quantities for lane-based traffic
- 5.3. Macroscopic Quantities for directed 2d traffic
- 5.4. Traffic Stream Relations
- 5.5. Hydrodynamic Relation
- 5.6. Continuity Equation
- 5.7. Eulerian *vs.* Lagrangian view

5.1. Model Overview

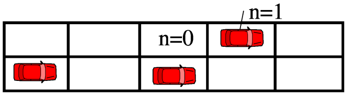
Macroscopic Model



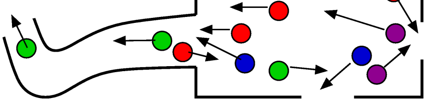
Microscopic Model



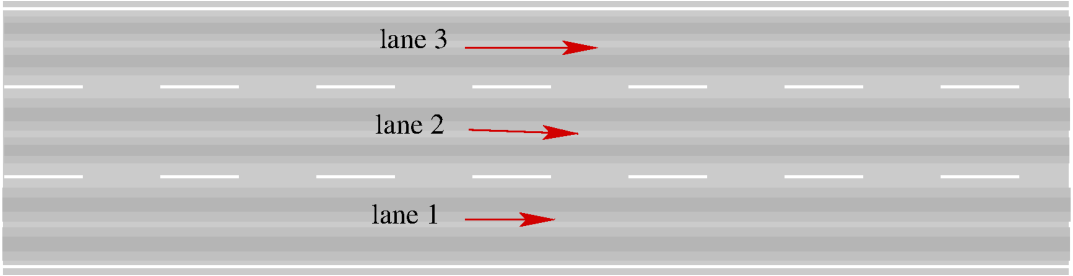
Cellular Automaton (CA)



Pedestrian Model



5.2. Basic Macroscopic Quantities for Lane-Based Traffic



Three categories of macroscopic quantities:

- per lane: ρ_l, Q_l, V_l
- total: ρ^{tot}, Q^{tot}
- effective/average: ρ, Q, V

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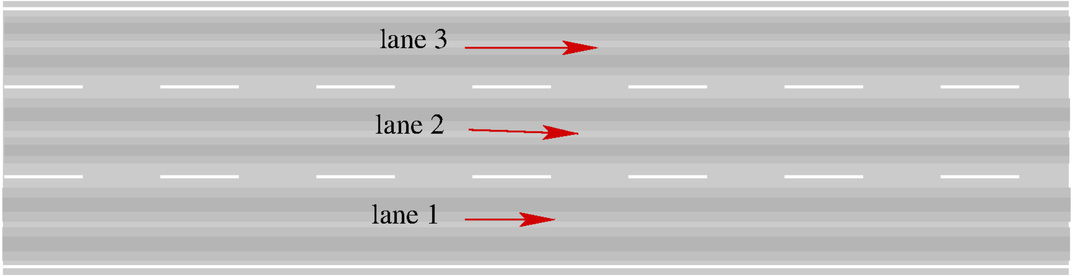

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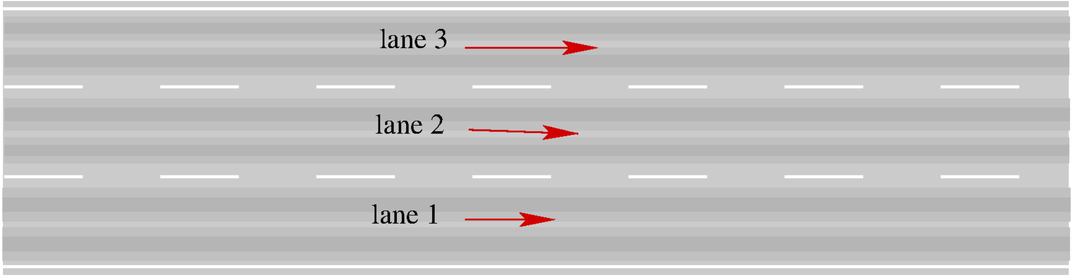

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Extensive and intensive quantities

- **Extensive** quantities (increasing with vehicle number, here ρ and Q) will just added/averaged normally to obtain total and effective values, respectively:

$$\rho^{\text{tot}} = \sum_{l=1}^{L} \rho_l, \quad \rho = \frac{1}{L}\sum_{l=1}^{L} \rho_l = \frac{\rho^{\text{tot}}}{L}, \quad Q \text{ likewise}$$

- **Intensive** quantities such as macroscopic speed V or speed variance cannot be added sensibly $\Rightarrow$ no "total" quantity. The lane averaging may also be more tricky.

? Determine V in two ways such that the macroscopic hydrodynamic relation $Q = \rho V$ and $Q^{\text{tot}} = V\rho^{\text{tot}}$ holds. Identify the results with weighted arithmetic and harmonic averages

! First, because the average and total extensive quantities only differ by the lane number L, we have $Q/\rho = Q^{\text{tot}}/\rho^{\text{tot}}$. We calculate just the ratio of the total quantities

  - $V = \frac{Q^{\text{tot}}}{\rho^{\text{tot}}} = \frac{\sum_l \rho_l V_l}{\rho^{\text{tot}}} = \sum_l w_{l\rho} V_l, \quad \Rightarrow$ **arithmetic** average with weighting $w_{l\rho} = \frac{\rho_l}{\rho^{\text{tot}}}$
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5.3. Basic Directed 2d Traffic

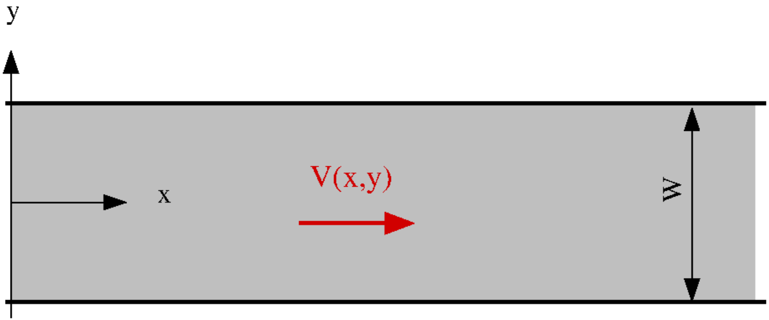
Example II: Hajj in Mekka

Traffic signs at the Hajj

Example III: Loveparade

Example IV: Vasaloppet

Basic macroscopic 2d quantities



- **Density** $\rho(x,y,t) = \rho(\boldsymbol{x},t)$ pedestrians per square meter $[\mathrm{ped/m^2}]$
- **Flow density** $\boldsymbol{J}(\boldsymbol{x},t)$, $J(\boldsymbol{x},t) = |\boldsymbol{J}(\boldsymbol{x},t)|$ pedestrian flow per meter cross section [ped/(ms)] ,
- **Local velocity and speed** $\boldsymbol{V}(\boldsymbol{x},t) = \boldsymbol{J}/\rho$, $V(\boldsymbol{x},t) = J/\rho$ [m/s].

Essentially, the flow density is the limit of the flow per lane divided by the lane width for a multi-lane road with the lane number going to infinity at constant width W: $\sum_l \to \int dy$

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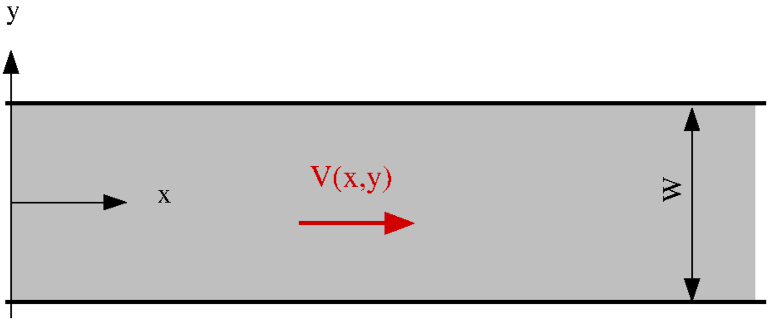

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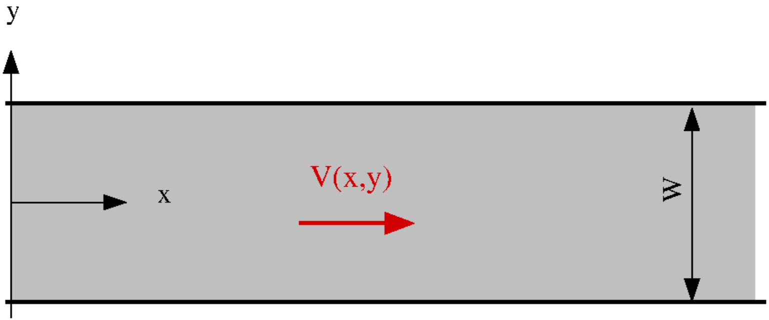

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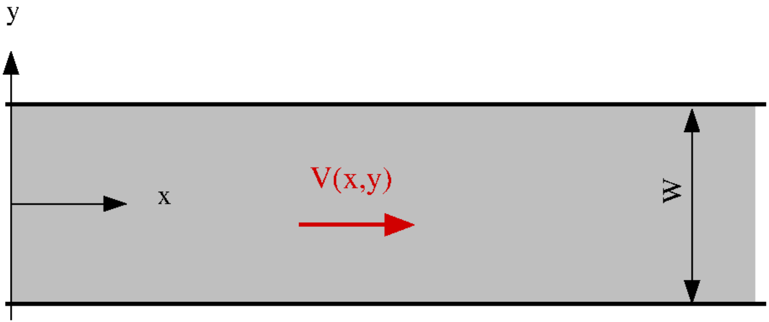
Effective 1d quantities



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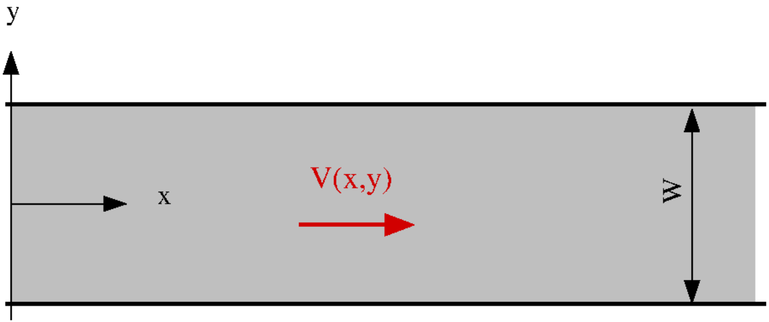

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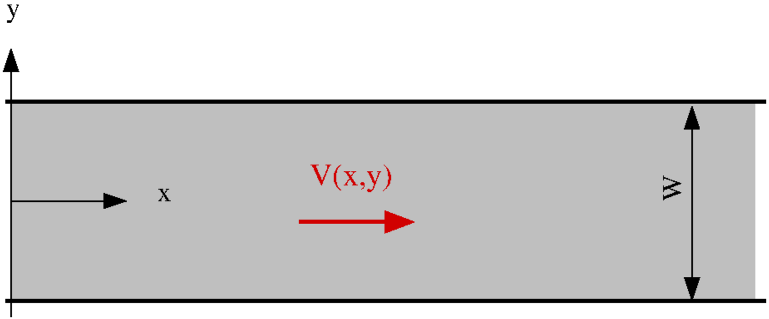

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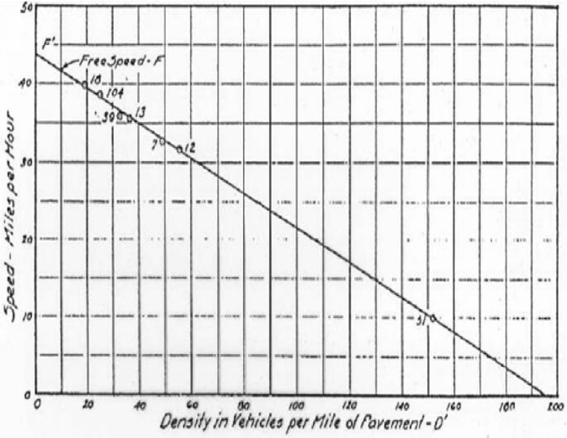

The early days of traffic data: Greenshields (1935)

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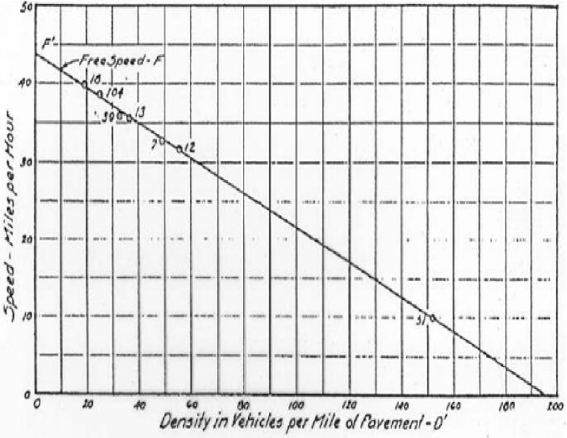
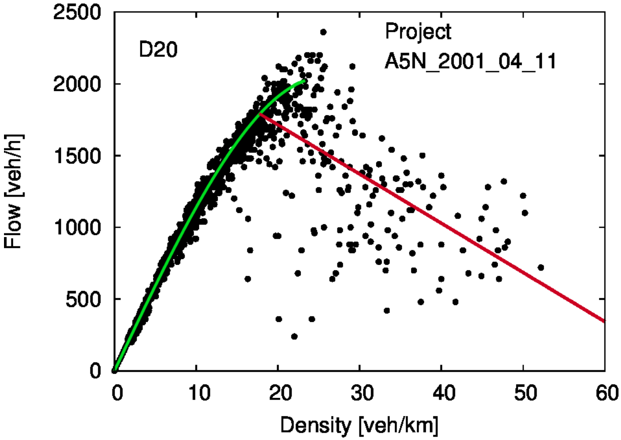


Greenshield's relation: $V(\rho) = V_0 \left(1 - \frac{\rho}{\rho_{\text{max}}}\right)$

The early days of traffic data: Greenshields (1935)

Flow-density data and fundamental diagram



- The traffic-stream relation $Q(\rho)$ is called the **fundamental diagram**
- It can be estimated by flow-density data *taking care of the systematic errors*

? How would the Greenshields fundamental diagram look like? $Q(\rho) = V_0 \rho \left(1 - \frac{\rho}{\rho_{max}}\right)$

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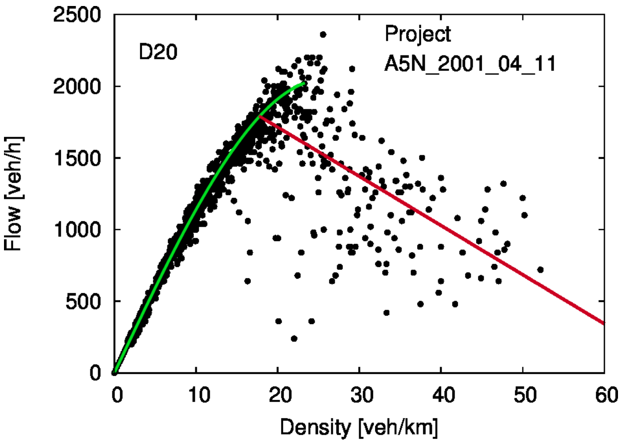

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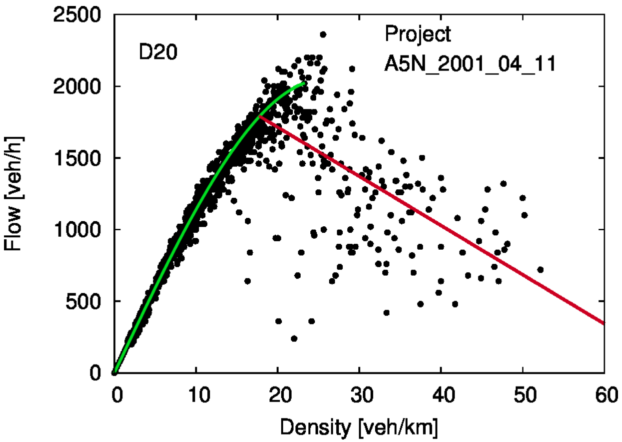

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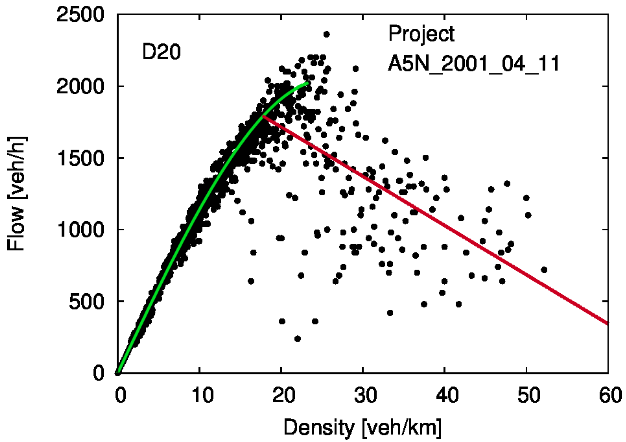

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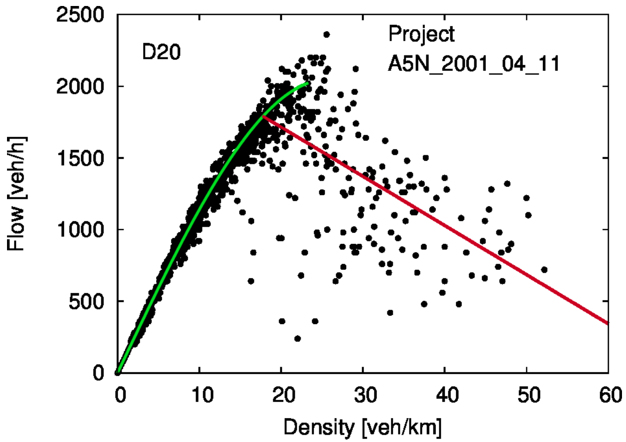

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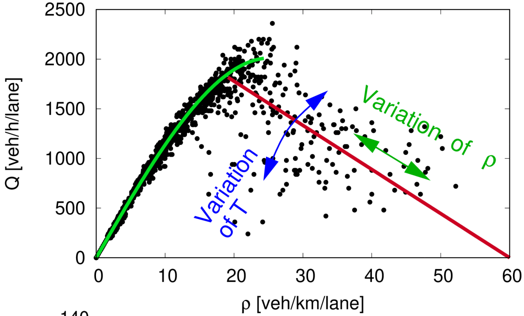
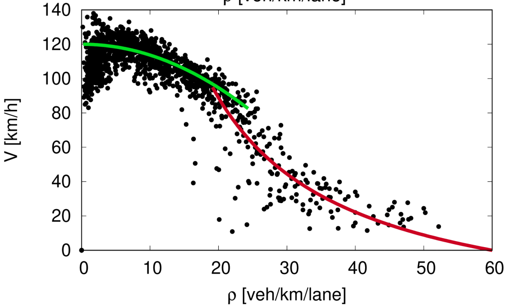
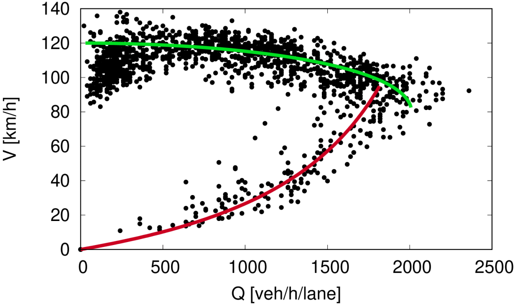
"Two out of three" relations

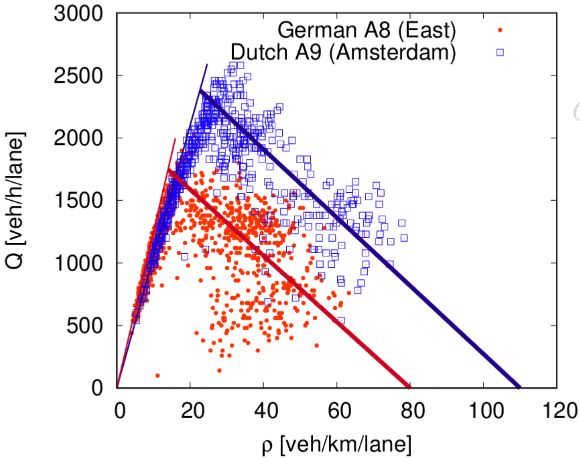


Together with the basic relation $Q = \rho V$, a single traffic stream relation fixes all three relations $Q(\rho)$, $V(\rho)$, and $Q(V)$





Triangular fundamental diagram (FD)



$$Q(\rho) = \min\left[V_0\rho, \quad \frac{1}{T}\left(1 - \frac{\rho}{\rho_{\text{max}}}\right)\right]$$

"free" branch   "congested" branch

? Calculate the theoretical capacity and the density "at capacity"
$Q_{\text{max}} = V_0\rho_c$ at $\rho_c = 1/(V_0T + 1/\rho_{\text{max}})$

? Discuss the model parameters V_0, T, and ρ_{max}
V_0: desired speed, ρ_{max}: maximum density, T: Desired time gap following since gap $s = (1/\rho - 1/\rho_{\text{max}})$

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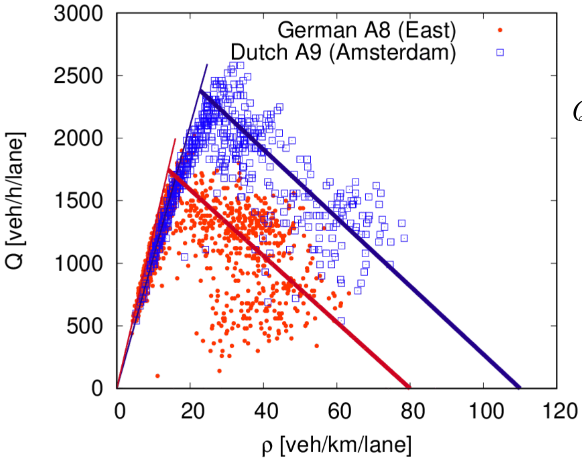

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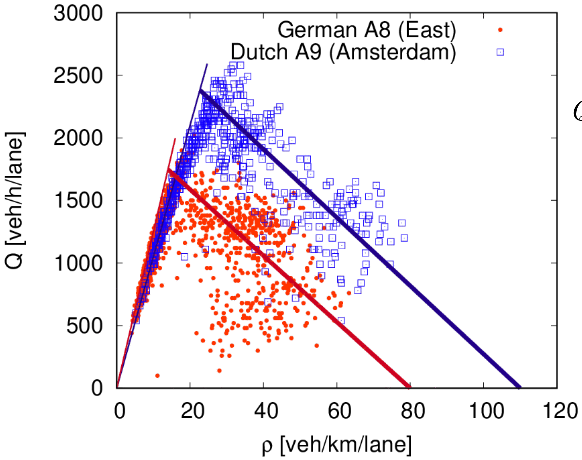

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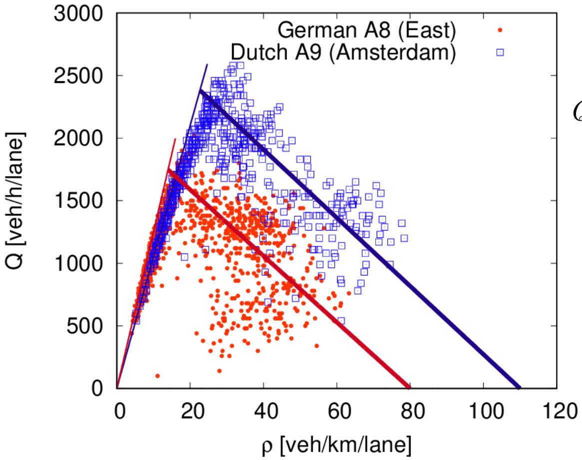
V_0: desired speed, ρ_{max}: maximum density, T: Desired time gap following since gap $s = (1/\rho - 1/\rho_{\text{max}})$

Triangular fundamental diagram (FD)



$$Q(\rho) = \min\left[V_0\rho,\ \frac{1}{T}\left(1 - \frac{\rho}{\rho_{max}}\right)\right]$$

"free" branch — "congested" branch

? Calculate the theoretical capacity and the density "at capacity"

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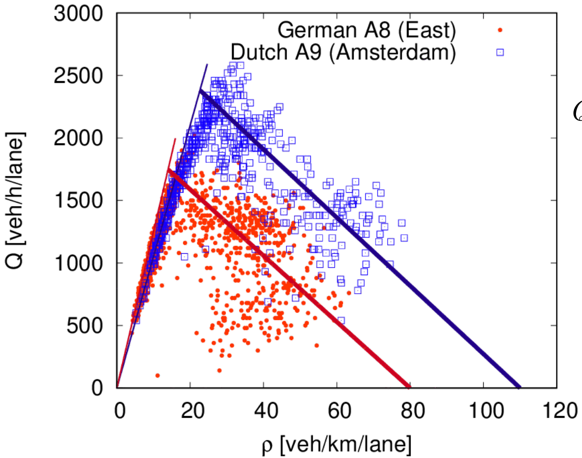

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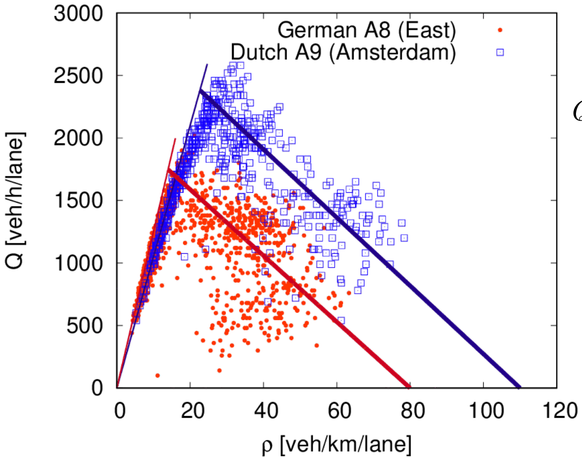

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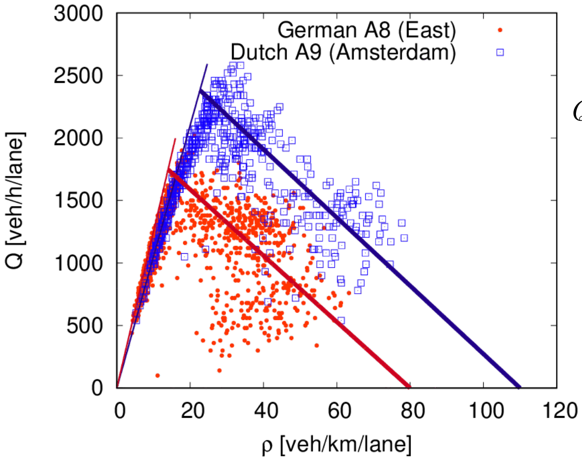

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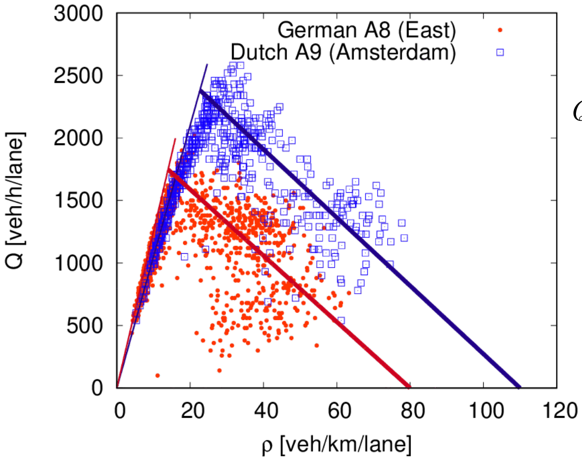

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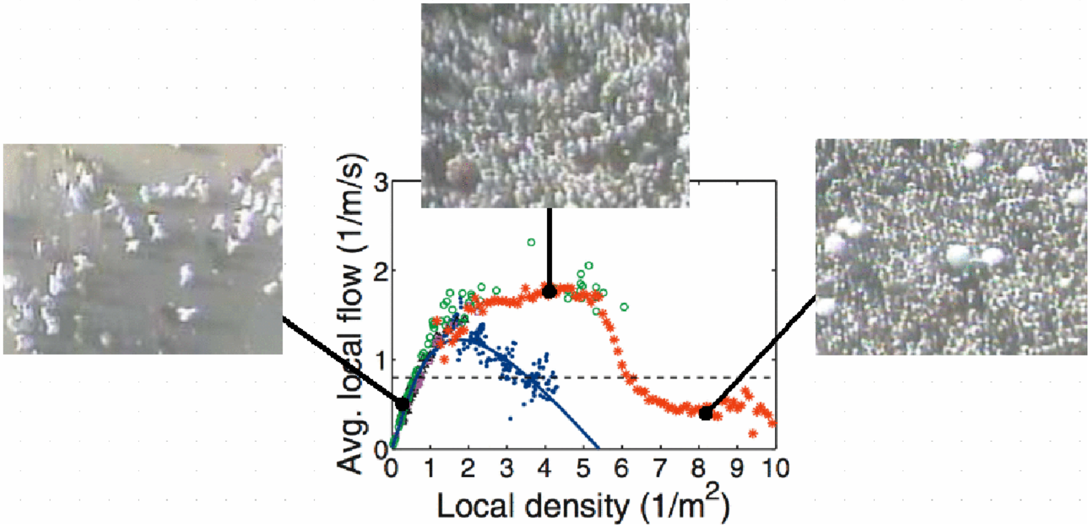
Fundamental diagram for directed 2d traffic



Fundamental diagram for directed 2d traffic

Often, the simplest Greenshields FD for the flow density J as a function of the 2d density ρ is not too bad (only for fast pedestrians such as runners in sporting events, an asymmetric triangular fundamental diagram is better):

$$J(\rho) = V_0 \rho \left(1 - \frac{\rho}{\rho_{\text{max}}}\right)$$

Going from 2d to effective 1d:

Assume a square grid for the pedestrian positions: longitudinal distance Δx_i=lateral "lane width" $\Delta W = \sqrt{1/\rho}$:

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$$Q(\rho^{\text{1d}}) = J((\rho^{\text{1d}})^2)/\rho^{\text{1d}} = V_0 \rho^{\text{1d}} \left(1 - \frac{(\rho^{\text{1d}})^2}{(\rho^{\text{1d}}_{\text{max}})^2}\right)$$

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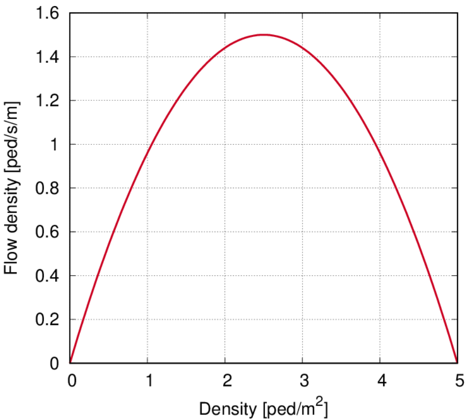

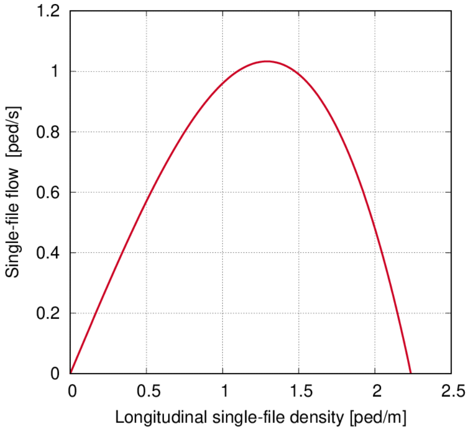

? Discuss the differences of the two FDs

? Give the capacity of a 30 m wide approach corridor assuming unidirectional pedestrian traffic flow and a Greenshields FD with parameters $V_0 = 1.2\,\mathrm{m/s}$ and $\rho_{\text{max}} = 5\,\mathrm{ped/m^2}$ (see the left image)

! Specific capacity $J_{\text{max}} = V_0 \rho_{\text{max}}/4 = 1.5\,\mathrm{ped/m/s}$, capacity $Q_{\text{max}} = W J_{\text{max}} = 45\,\mathrm{ped/s}$ or about 160 000 pedestrians per hour.

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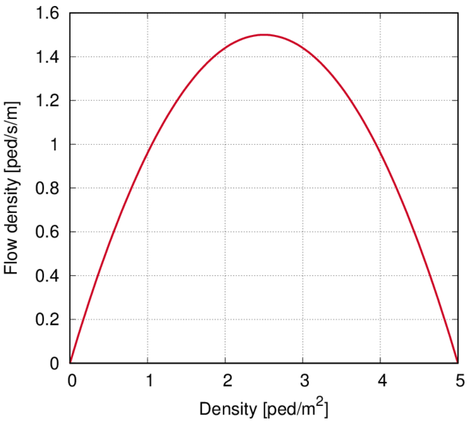

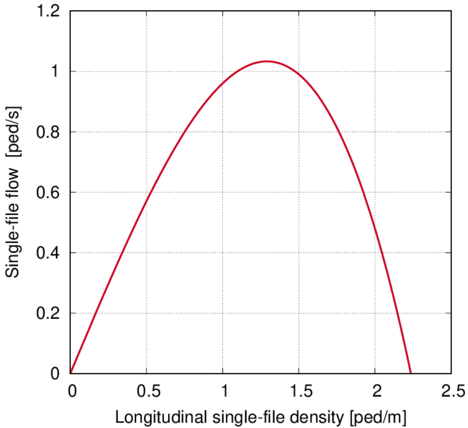

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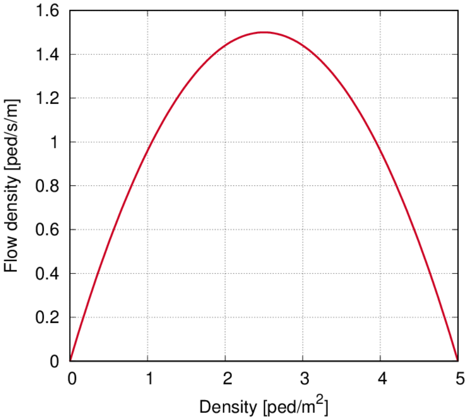

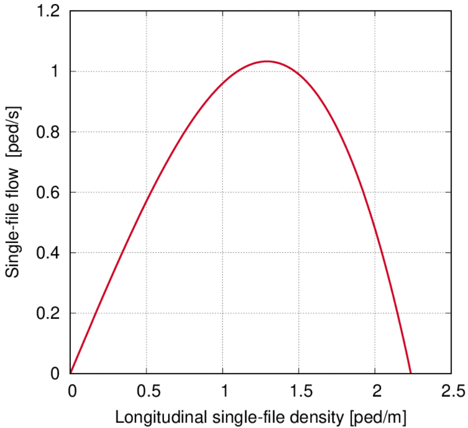

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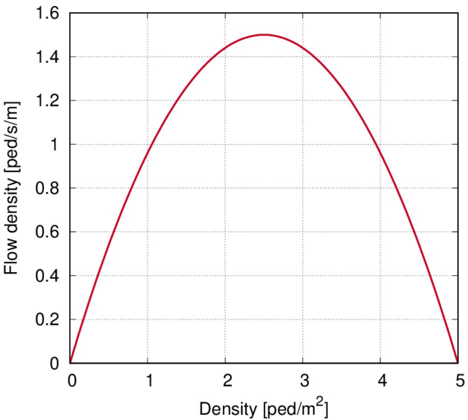

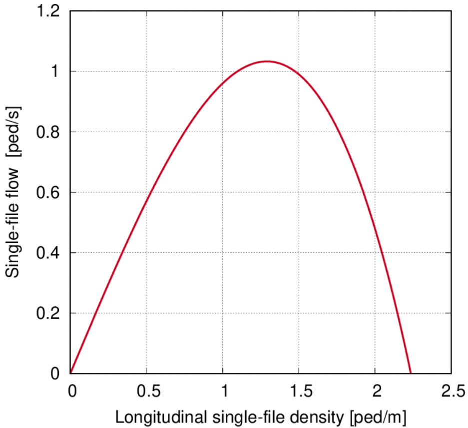

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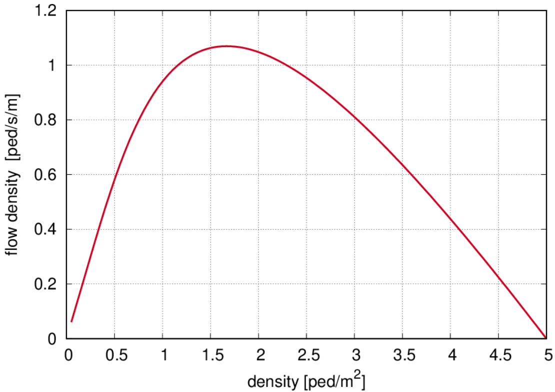
Weidmann FD

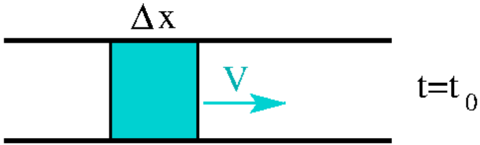


The popular Weidmann FD can be derived from microscopic social-force pedestrian flow models (→ Lecture 11). Its speed-density traffic stream relation reads (with the published parameter $\lambda = -1.913\,\mathrm{m}^{-2}$ and the same V_0 and ρ_{max})

$$J(\rho) = \rho V(\rho), \quad V(\rho) = V_0 \left\{ 1 - \exp\left[-\lambda \left(\frac{1}{\rho} - \frac{1}{\rho_{\text{max}}} \right) \right] \right\}$$

In contrast to the greenshields FD, it is not symmetric

5.5. Hydrodynamic relation



- Number of vehicles in blue-green box: $n \overset{\text{def}}{=} \rho \Delta x$
- hydrodynamic relation: $n = \rho \Delta x = Q \Delta t \;\Rightarrow\; \frac{Q}{\rho} = \frac{\Delta x}{\Delta t} \overset{\text{def}}{=} V$

$Q = \rho V$ hydrodynamic relation

? Give the form for unidirectional 2d traffic: $J = \rho V$

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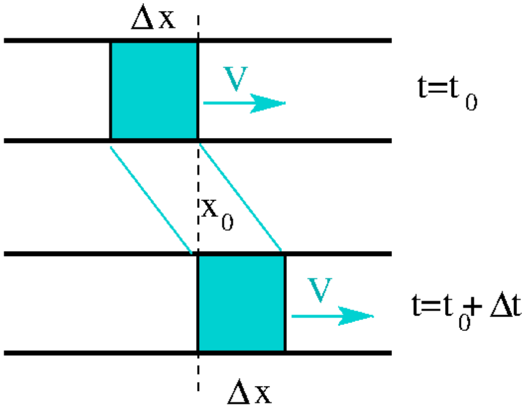

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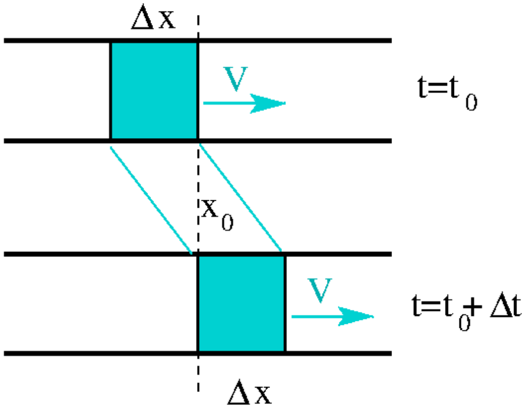

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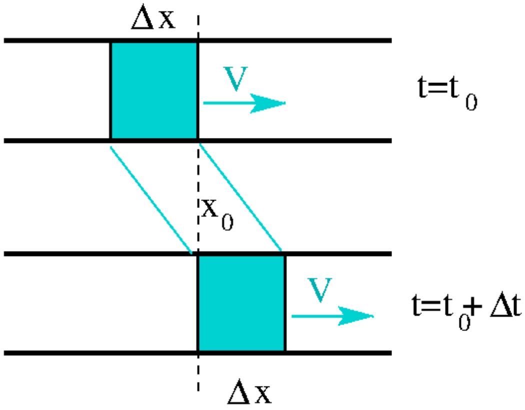

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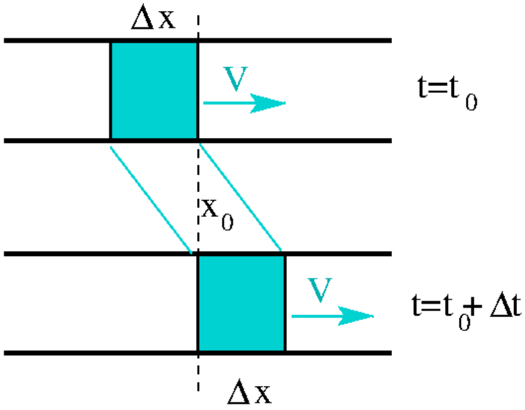

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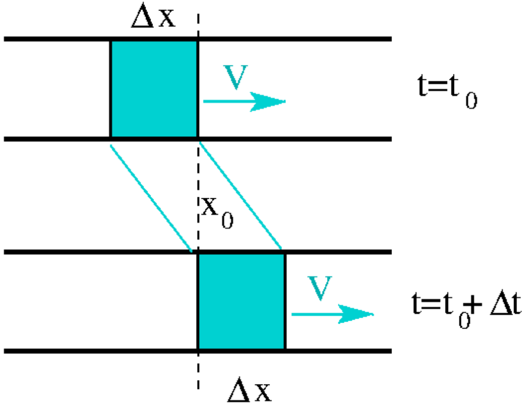

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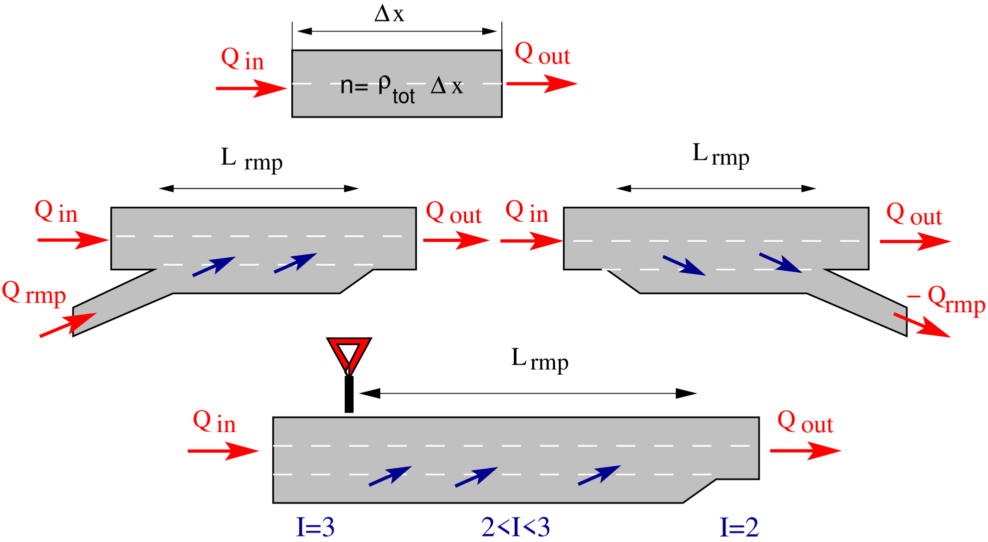
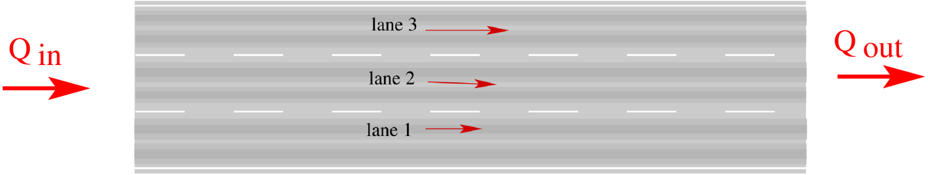
5.6. Continuity Equation



The **continuity equation** just reflects vehicle/pedestrian conservation and is therefore *always* valid

Continuity equation along a homogeneous road



$$\frac{\mathrm{d}n}{\mathrm{d}t} = Q_{\text{in}} - Q_{\text{out}} = Q^{\text{tot}}(x,t) - Q^{\text{tot}}(x+\Delta x, t) \approx -\frac{\partial Q^{\text{tot}}}{\partial x}\Delta x$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\partial}{\partial t}\left(\int \rho^{\text{tot}}\,\mathrm{d}x\right) \approx \frac{\partial \rho^{\text{tot}}}{\partial t}\Delta x$$

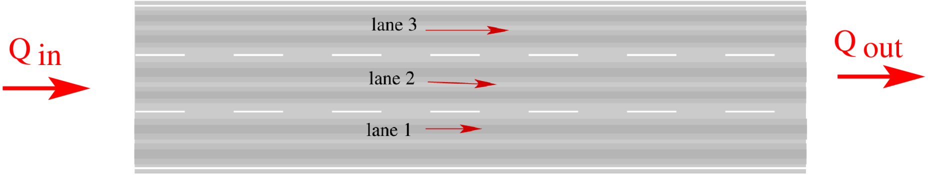
⇒ Total quantities: $\frac{\partial \rho^{\text{tot}}}{\partial t} + \frac{\partial Q^{\text{tot}}}{\partial x} = 0$

Effective quantities: $\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = 0$

Why is this continuity equation not valid for the lane quantities ρ_l, Q_l, V_l?

Because there are source terms due to lane changing

Continuity equation along a homogeneous road



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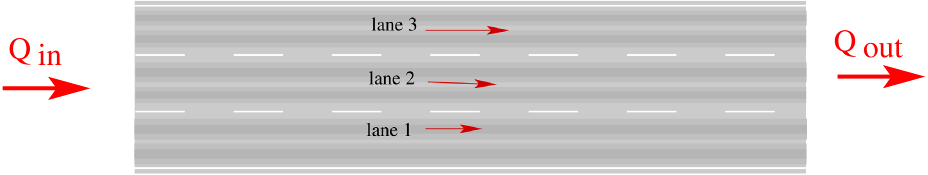

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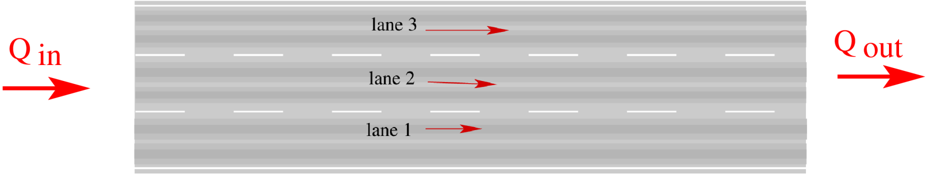
Continuity equation along a homogeneous road



$$\frac{\mathrm{d}n}{\mathrm{d}t} = Q_{\text{in}} - Q_{\text{out}} = Q^{\text{tot}}(x,t) - Q^{\text{tot}}(x+\Delta x, t) \approx -\frac{\partial Q^{\text{tot}}}{\partial x}\Delta x$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\partial}{\partial t}\left(\int \rho^{\text{tot}}\,\mathrm{d}x\right) \approx \frac{\partial \rho^{\text{tot}}}{\partial t}\Delta x$$

⇒ Total quantities: $\frac{\partial \rho^{\text{tot}}}{\partial t} + \frac{\partial Q^{\text{tot}}}{\partial x} = 0$

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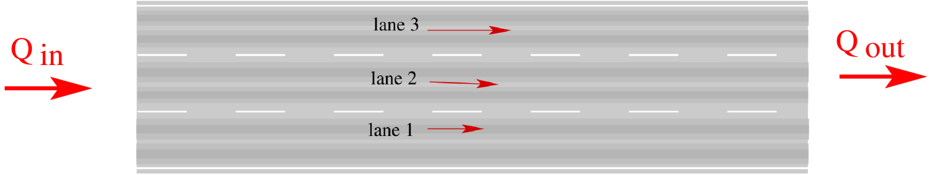

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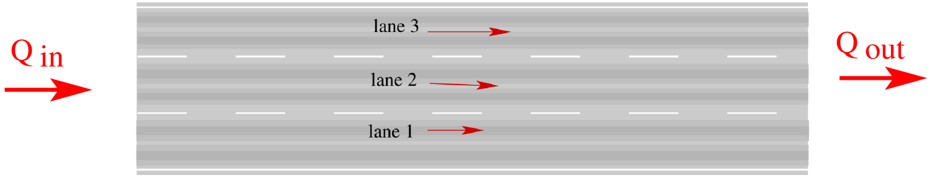

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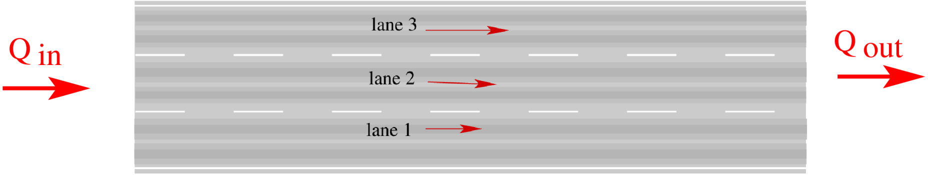

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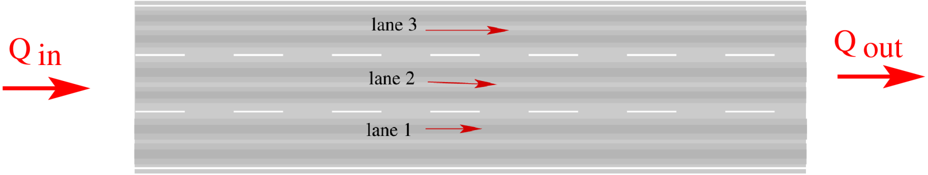

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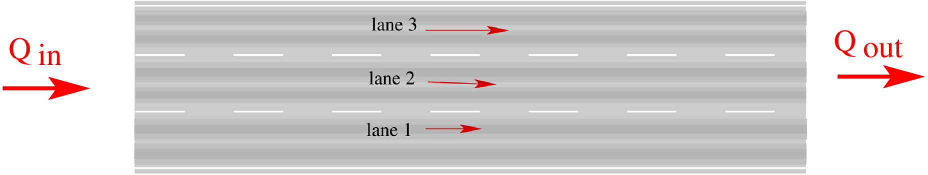

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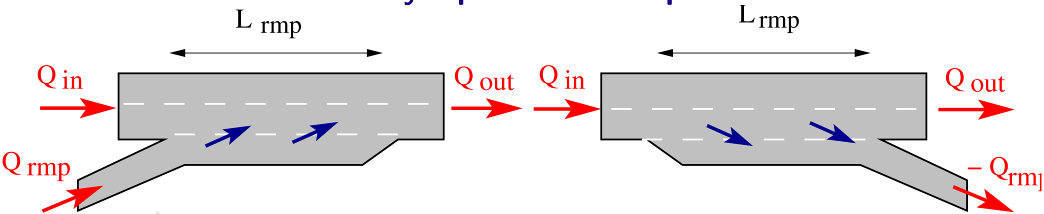
Continuity equation at ramp sections



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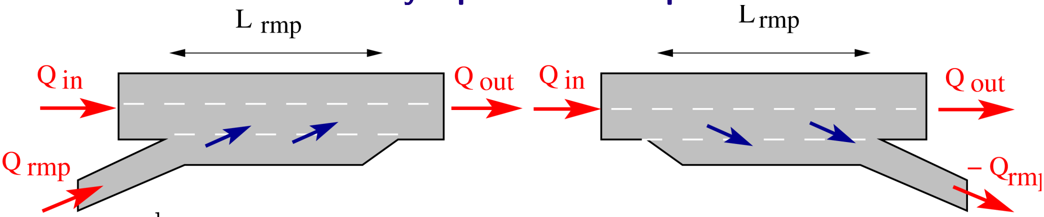
Continuity equation at ramp sections



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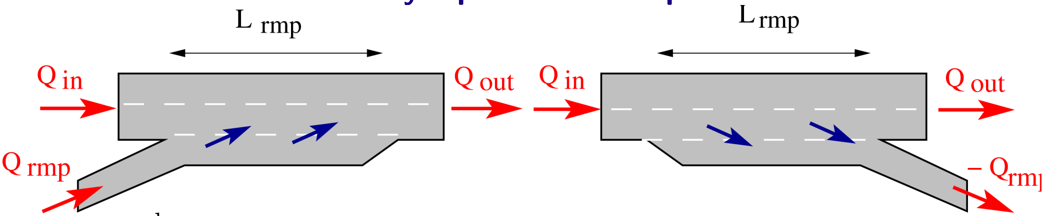

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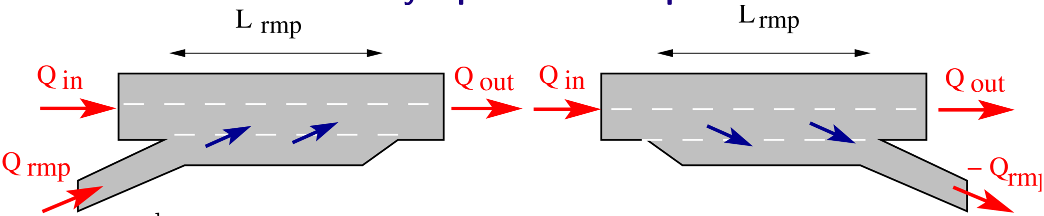

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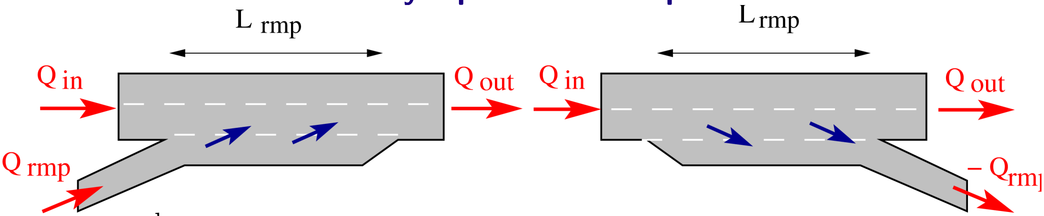

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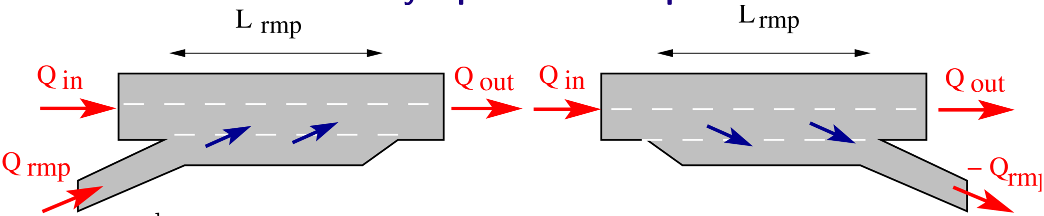

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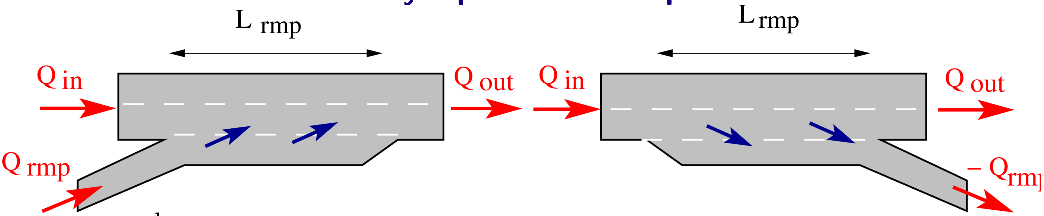

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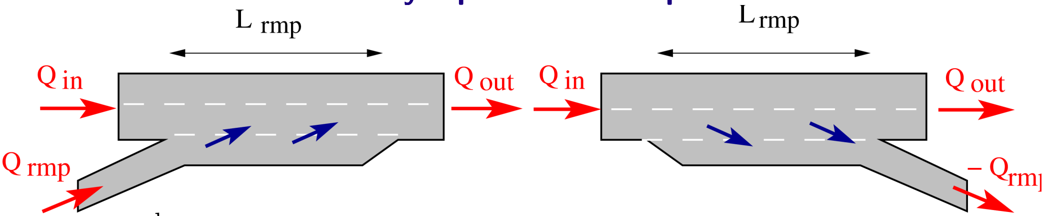

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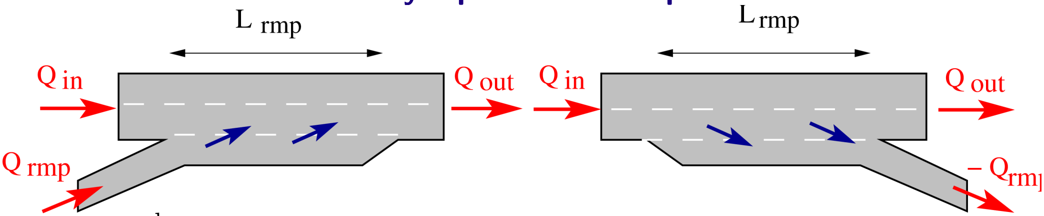

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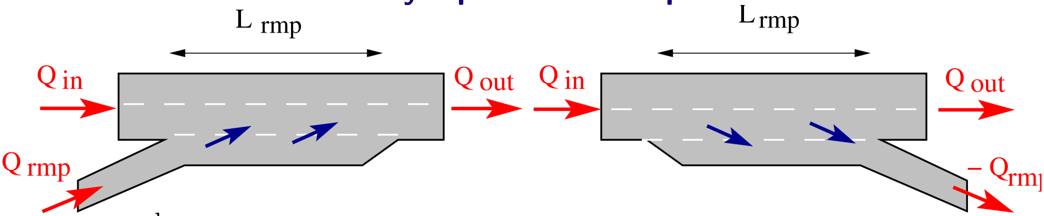

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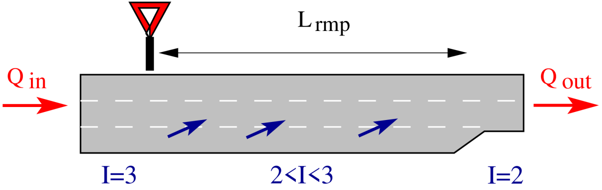
Continuity equation at changes of the lane number



- Variable effective lane number $L(x)$, here from $L = 3 \to 2$ along the merging zone of one or a few hundred meters:
- For the total quantities, the homogeneous continuity equation applies (why?):
  $\frac{\partial \rho^{\text{tot}}}{\partial t} + \frac{\partial Q^{\text{tot}}}{\partial x} = 0$
- For the effective quantities, we get

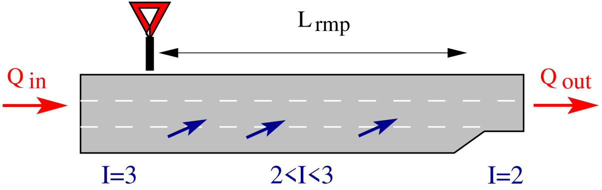
$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = -\frac{Q}{L}\frac{\mathrm{d}L}{\mathrm{d}x}$$

- The source terms of the ramp and lane-closing scenarios can be added

? Why use the more complicated effective continuity equation?

? Try to understand the lane-closing source in terms of the on-ramp source (and the lane opening in terms of an off-ramp)

Continuity equation at changes of the lane number



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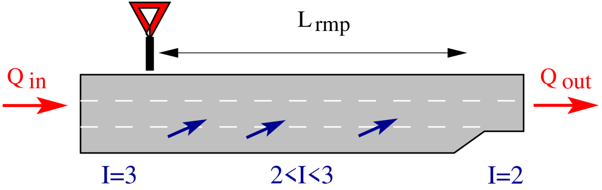

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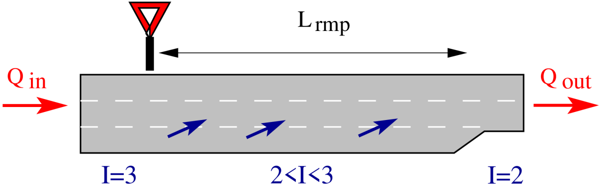

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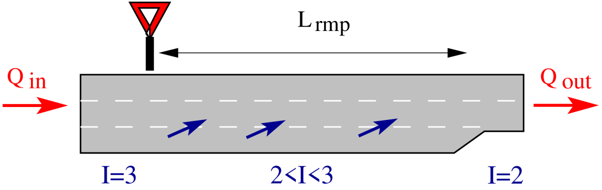

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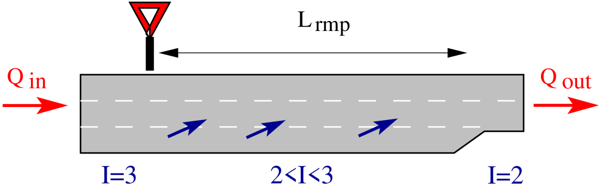

- Variable effective lane number $L(x)$, here from $L = 3 \to 2$ along the merging zone of one or a few hundred meters:
- For the total quantities, the homogeneous continuity equation applies (why?):
  $\frac{\partial \rho^{\text{tot}}}{\partial t} + \frac{\partial Q^{\text{tot}}}{\partial x} = 0$
- For the effective quantities, we get

$$\frac{\partial \rho}{\partial t} + \frac{\partial Q}{\partial x} = -\frac{Q}{L}\frac{\mathrm{d}L}{\mathrm{d}x}$$

- The source terms of the ramp and lane-closing scenarios can be added

? Why use the more complicated effective continuity equation?

? Try to understand the lane-closing source in terms of the on-ramp source (and the lane opening in terms of an off-ramp)

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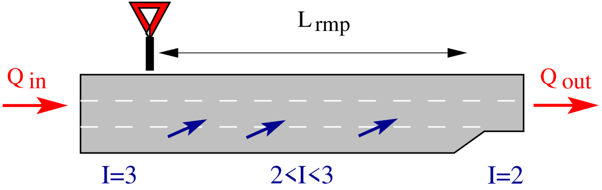

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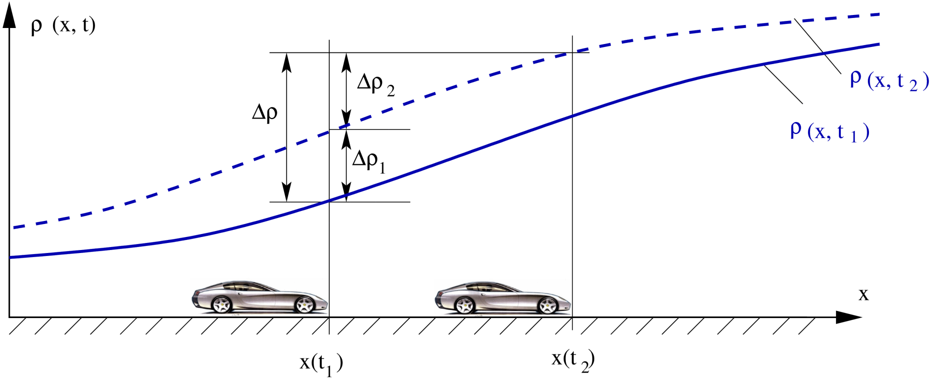
5.7. Coordinate Systems: Eulerian (Fixed Observer's) vs. Lagrangian (Driver's) View



Continuity equation from the floating car (driver's) perspective:

- Change of density: $\Delta\rho \approx \left(\frac{\partial\rho}{\partial t} + V\frac{\partial\rho}{\partial x}\right)\Delta t$ from the driver's perspective leads to the **total** or **convective** time derivative: $\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\partial\rho}{\partial t} + V(x,t)\frac{\partial\rho}{\partial x}$
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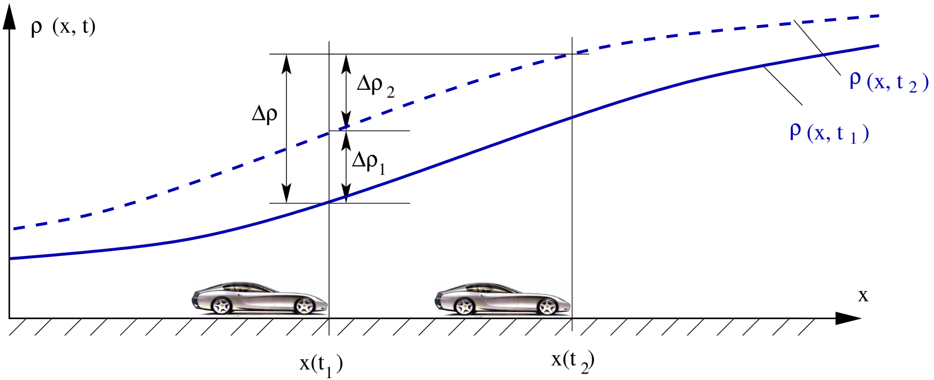

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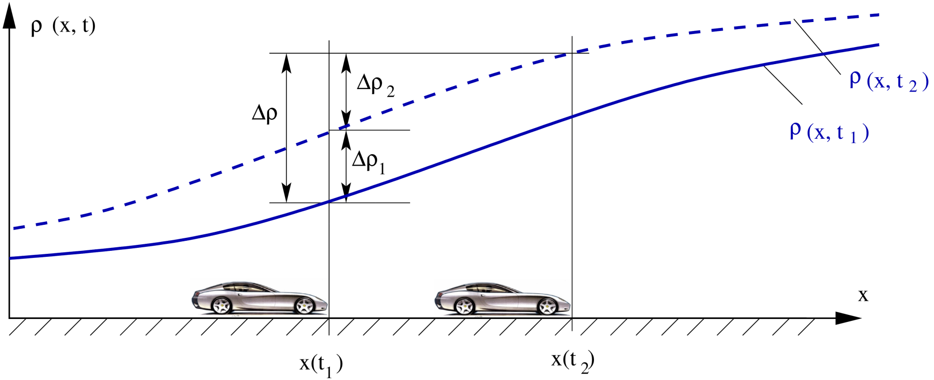

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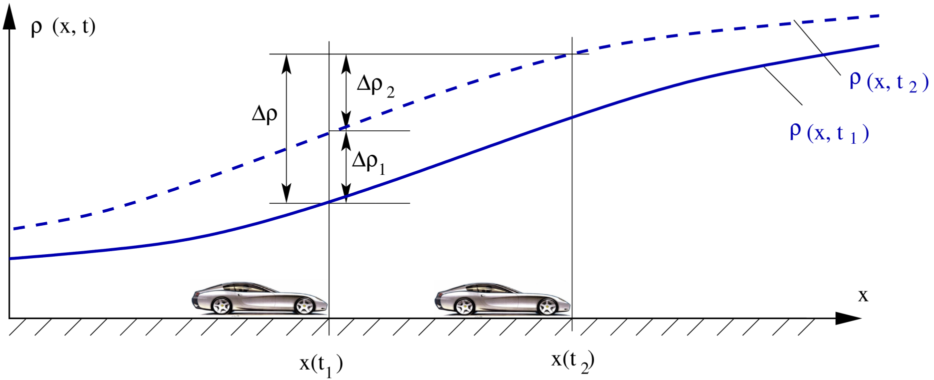

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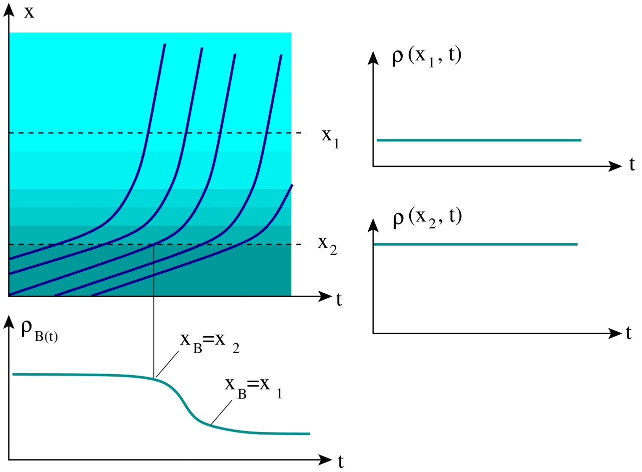
Homogeneous, stationary, and steady state



- Traffic flow is **homogeneous** if $\frac{\partial F}{\partial x} = 0$ where $F = \rho(x,t)$, $V(x,t)$, or any other macroscopic field as a function of x and t
- Traffic flow is **stationary** or in the **steady state** if $\frac{\partial F}{\partial t} = 0$

  Watch out: stationary != standing!

- Traffic flow is in the **homogeneous steady state** if $\frac{\partial F}{\partial x} = \frac{\partial F}{\partial t} = \frac{dF}{dt} = 0$. This is assumed when formulating/deriving the *fundamental diagram*

? Give examples of stationary nonhomogeneous and nonstationary homogeneous states

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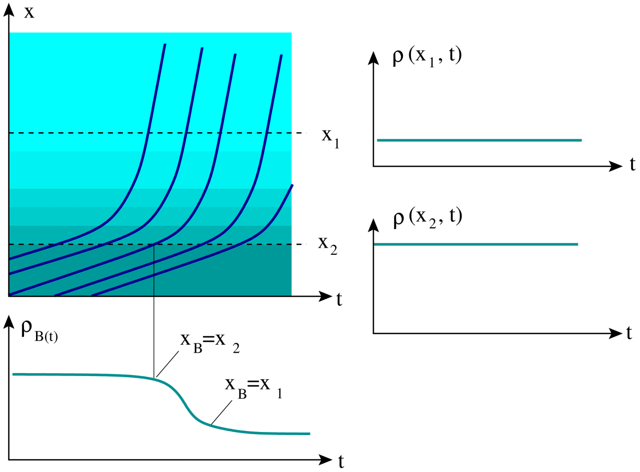

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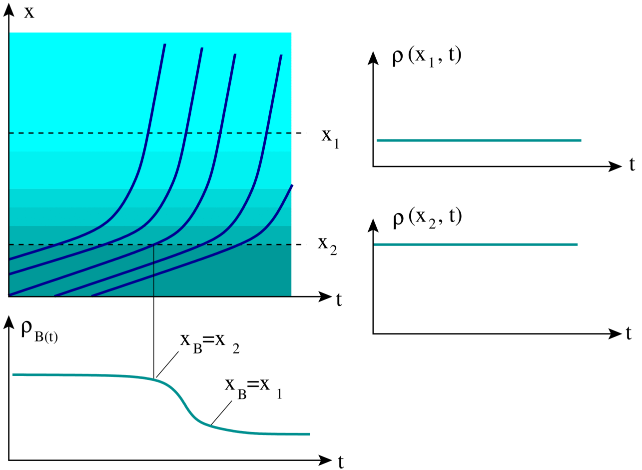

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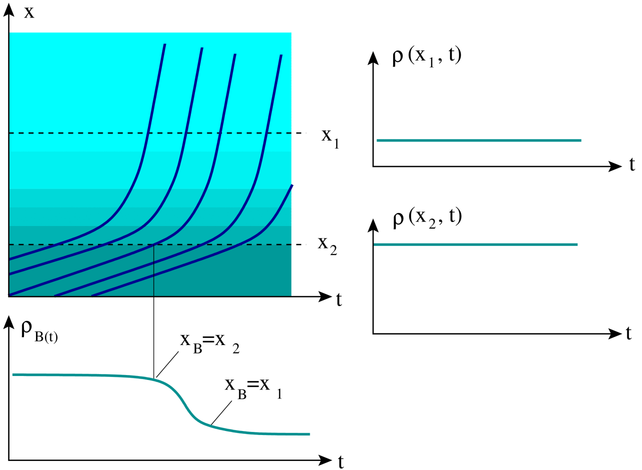

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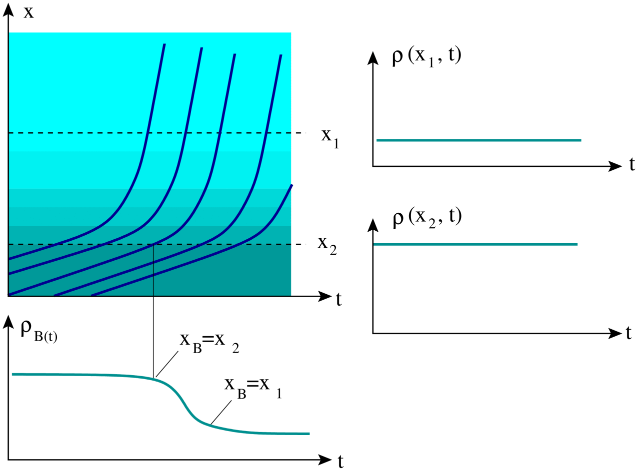

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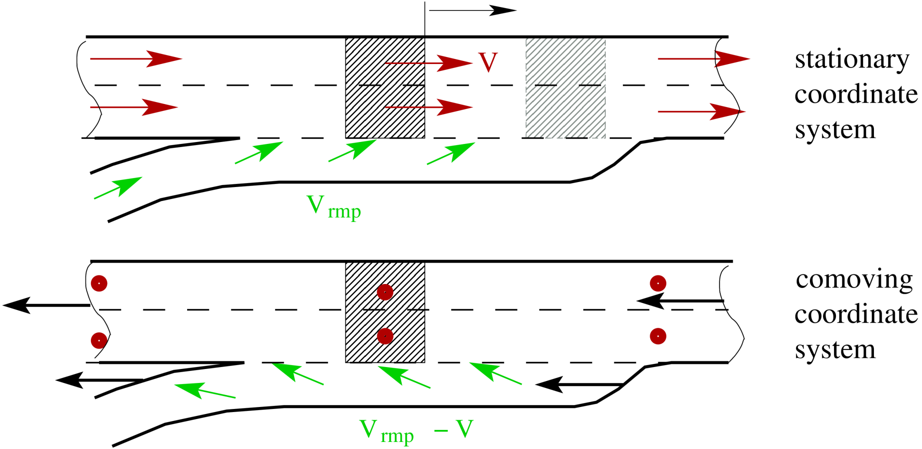

- Advantage: homogeneous systems become easier to describe since the convective term is eliminated
- Disadvantage: inhomogeneous systems become more complicated since ramps and other infrastructure stuff are moving

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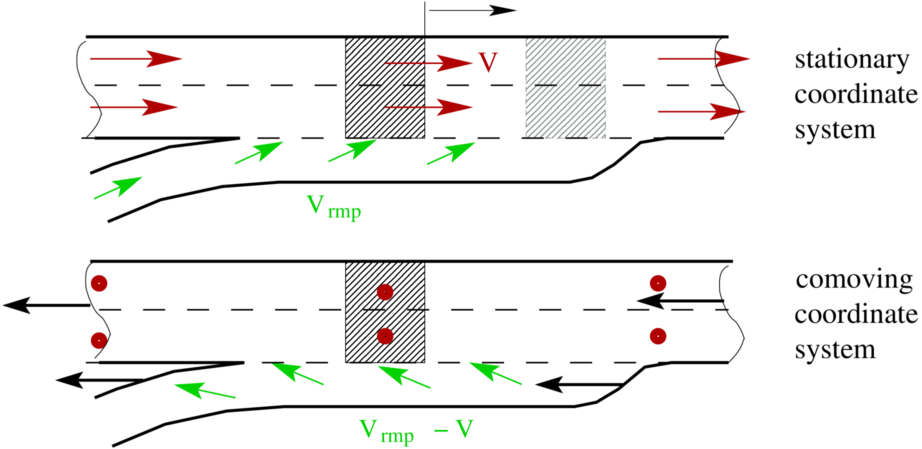

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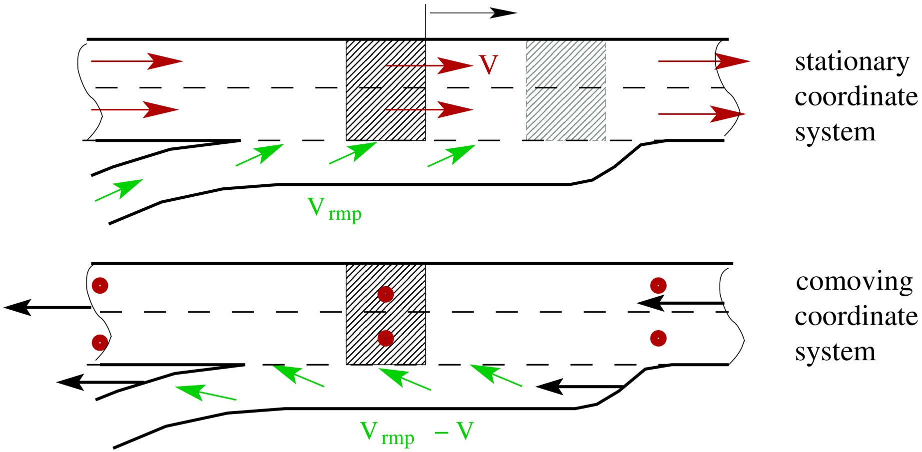

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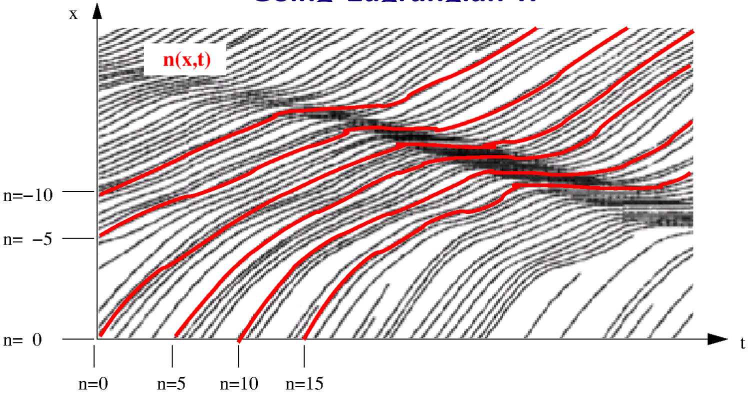

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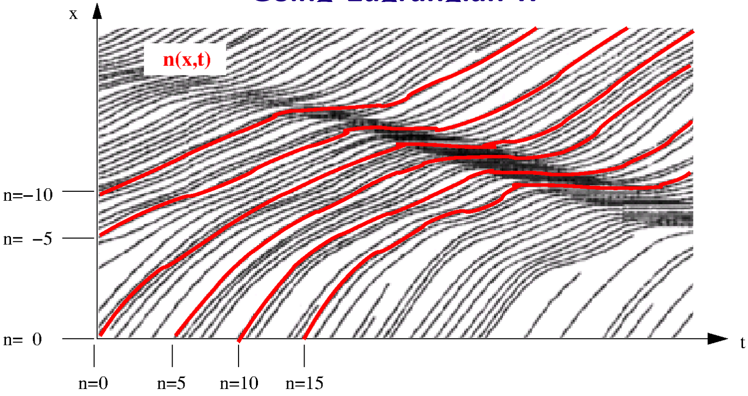

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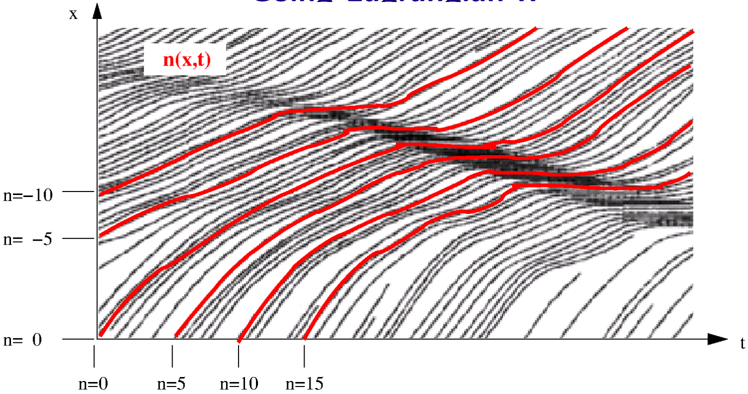

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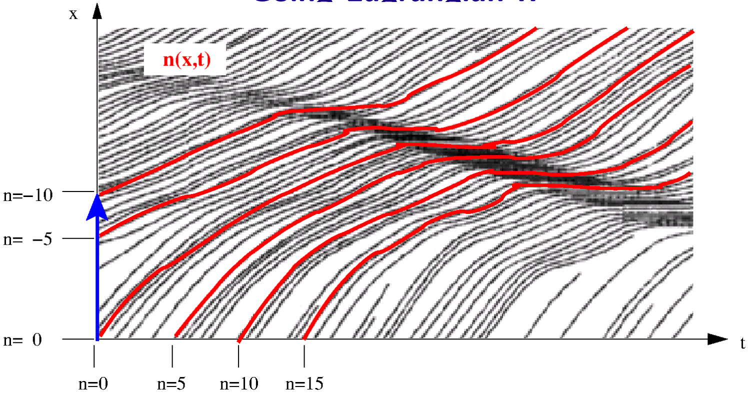

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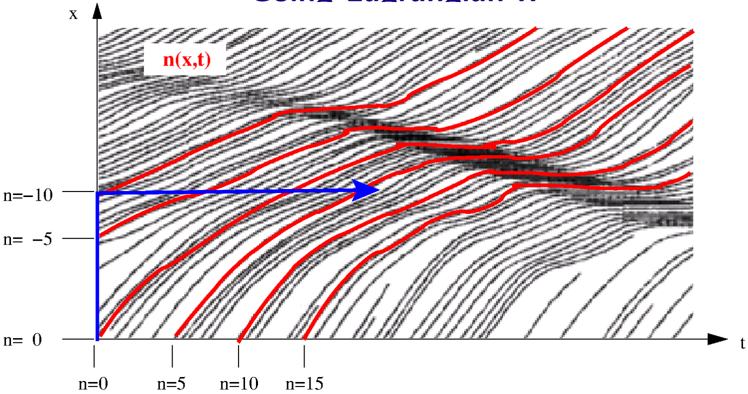

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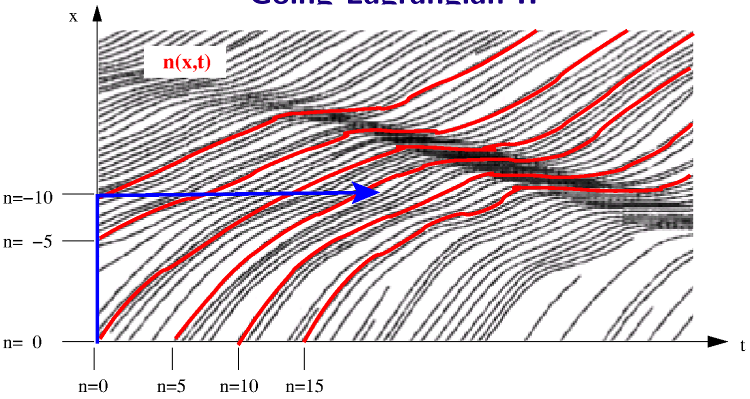

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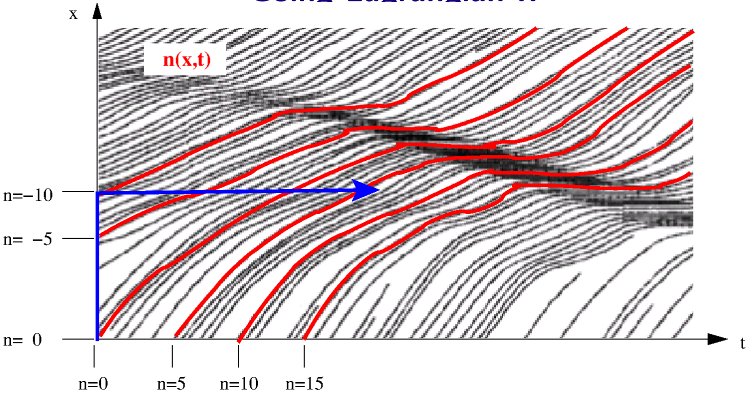

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- dependent variables density becomes **distance headway field** $\rho(x,t) \rightarrow 1/h(n,t)$ (name it h instead of d to avoid confusion with differential operators)

Lagrange Continuity equation for homogeneous roads: derivation

- Lagrangian variables: $\rho(x,t) = \frac{1}{h(n(x,t),t)}, \quad V(x,t) = v(n(x,t),t)$
- The definitions of flow and density directly give

$$\frac{\partial n}{\partial t} = Q = \rho V, \quad \frac{\partial n}{\partial x} = -\rho, \quad h = \frac{1}{\rho}, \quad \frac{\partial}{\partial x} = -\frac{1}{h}\frac{\partial}{\partial n}$$

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&\overset{\frac{\partial n}{\partial t}=\rho V,\ \frac{\partial n}{\partial x}=-\rho}{=} -\frac{1}{h^2}\left(\rho V\frac{\partial h}{\partial n} + \frac{\partial h}{\partial t} - \rho V\frac{\partial h}{\partial n}\right) - \frac{1}{h^2}\frac{\partial v}{\partial n} \\
&= -\frac{1}{h^2}\left(\frac{\partial h}{\partial t} + \frac{\partial v}{\partial n}\right)
\end{aligned}$$

Lagrange Continuity equation for homogeneous roads: derivation

- Lagrangian variables: $\rho(x,t) = \frac{1}{h(n(x,t),t)}, \quad V(x,t) = v(n(x,t),t)$
- The definitions of flow and density directly give

$$\frac{\partial n}{\partial t} = Q = \rho V, \quad \frac{\partial n}{\partial x} = -\rho, \quad h = \frac{1}{\rho}, \quad \frac{\partial}{\partial x} = -\frac{1}{h}\frac{\partial}{\partial n}$$

- Transform the continuity equation (from the driver's view):

$$\begin{aligned}
0 &= \frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\frac{\partial V}{\partial x} \\
&= \left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x}\right)\left[\frac{1}{h(n(x,t),t)}\right] - \frac{1}{h^2}\frac{\partial v}{\partial n} \\
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Lagrange Continuity equation for homogeneous roads: result

$$\frac{\partial h}{\partial t} + \frac{\partial v}{\partial n} = 0$$

Lagrange form of the continuity equation

- This result is plausible by integrating the second term over one unit of the index variable (because n is dimensionless, the lhs. is multiplied by one):

$$\frac{\partial h}{\partial t} + v(n+1,t) - v(n,t) = 0 \quad \Rightarrow \quad \frac{\partial h}{\partial t} = v_{\text{lead}} - v$$

h increases at a rate of the relative speed leader-follower.

? Why is the Lagrange form less efficient if there are bottlenecks?

Because bottlenecks are moving in this view

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