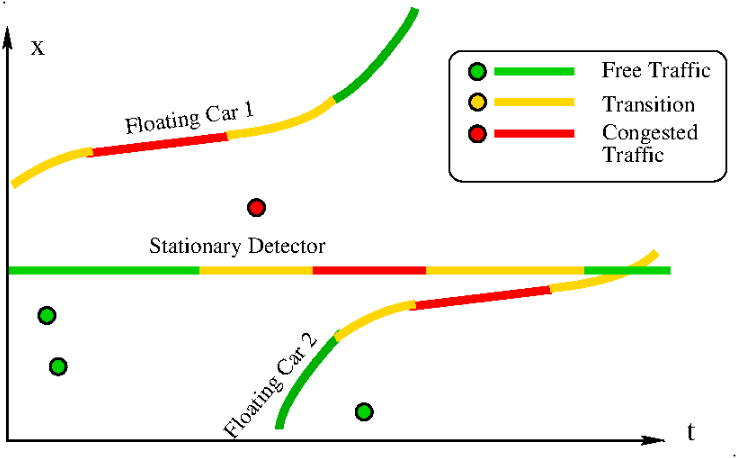
Lecture 4: Data Fusion

4.1. Data Fusion: Problem Statement

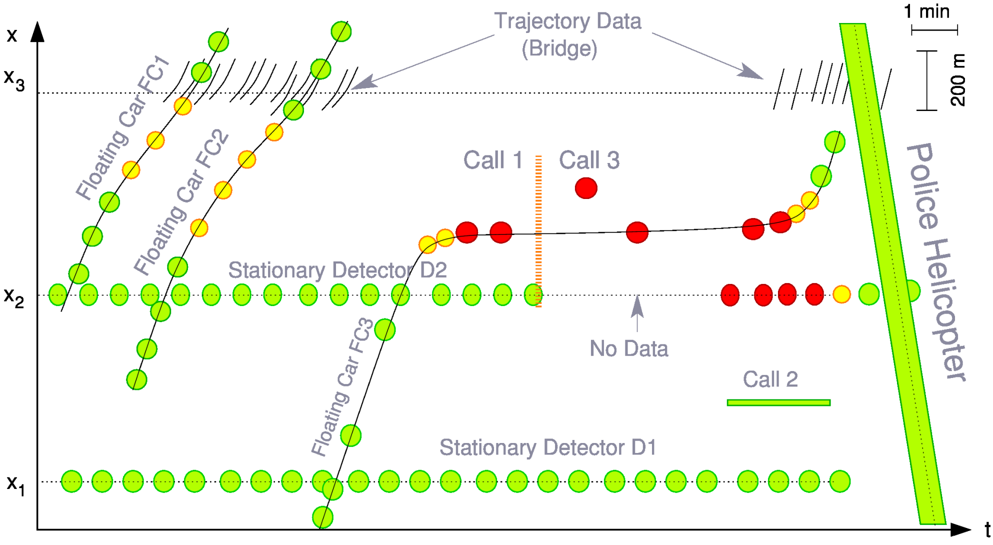


Traffic flow data may come from several sources:

- stationary detectors
- floating cars
- point observations by drivers ("jam reporter") or authorities

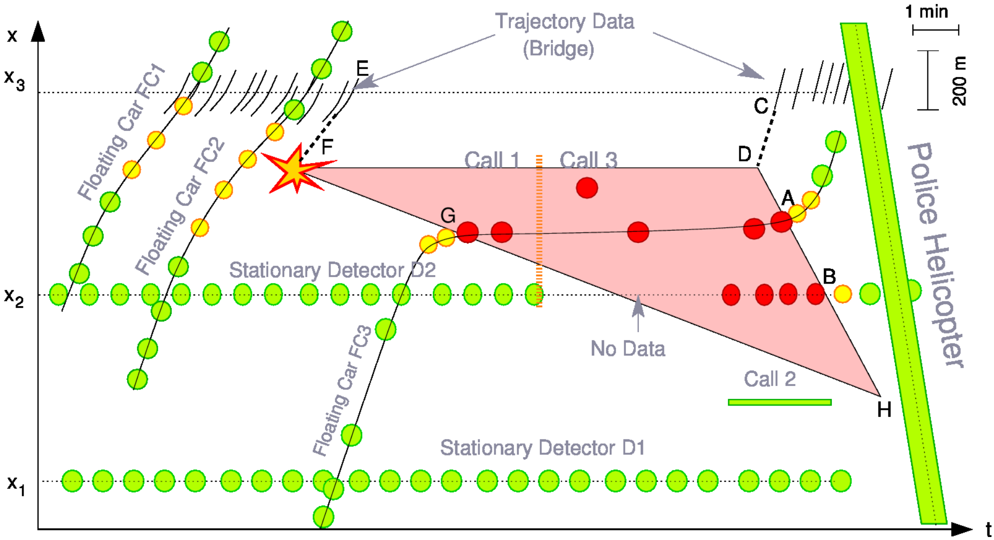
They may also of different reliability and even contradictory **(spot such an inconsistency above!)**.
→ back reliability weighting

4.2. Data Fusion “by Hand”



An accident happened: When and where?

Solution



The accident happened at the spatiotemporal point F

4.3 Reliability weighting

Not all data sources are equally reliable. And may contradict each other. How to weight them optimally, i.e., find optimal weights for $\hat{Y} = \sum_m r_m Y_m$?

- Assume M independent and unbiased measurements $Y_m, m = 1, ..., M$ with error variances σ_m^2. From the unbiasedness and the general variance rule $V(aY_1 + bY_2) = a^2 V(Y_1) + b^2 V(Y_2)$
- $\Rightarrow$ Optimization problem: find the reliability weightings r_i such that the variance

$$\sigma^2_{\hat{Y}}(\boldsymbol{r}) = \sum_m r_m^2 \sigma_m^2 \stackrel{!}{=} \min_{\boldsymbol{r}} \quad \Big| \quad \sum_m r_m = 1$$

  of the weighted average $\hat{Y} = \sum_m r_m y_m$ is minimized subject to the normalisation condition.

? Why we need independence when using this formula? Is it practically fulfilled?

Otherwise, the variance formula will contain additional covariance terms. Independency generally fulfilled.

? Why we need the restraint $\sum_m r_m = 1$?

Otherwise, the estimator is no longer unbiased: $E(\hat{Y}) = \sum_m r_m E(Y) \neq E(Y)$

Solving the restrained minimization problem

The method of **Lagrange multipliers** does the magic! With a single restraint (a generalisation is straightforward, see Tutorial 04), do the following:

1. Formulate the restraint as an "=0" equation: $g(\boldsymbol{r}) = \sum_m r_m - 1 = 0$

2. Define the **Lagrange function** by adding to the function f to be minimized the restraint multiplied by a *Lagrange multiplier* λ:

$$L(\boldsymbol{r}) = f(\boldsymbol{r}) - \lambda g(\boldsymbol{r}) = \sum_{m'} r_{m'}^2 \sigma_{m'}^2 - \lambda \left(\sum_{m'} r'_m - 1 \right)$$

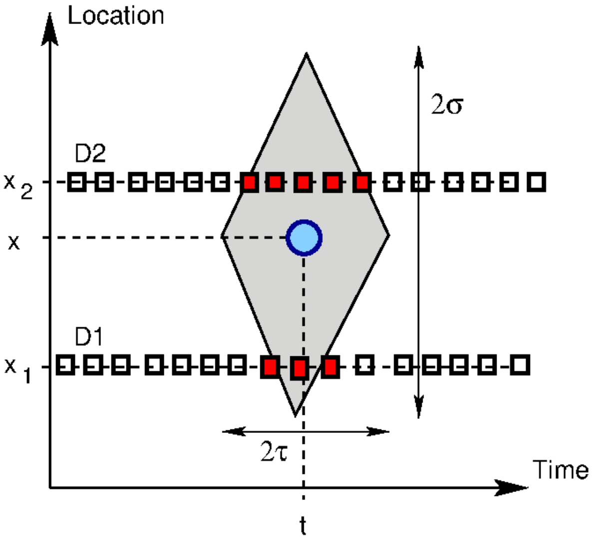
3. Minimize L unconditionally:

$$\frac{\partial L}{\partial r_m} = 2 r_m \sigma_m^2 - \lambda \stackrel{!}{=} 0$$

4. Calculate λ by inserting the result into the restraint: Here, we have $r_m = \lambda / (2\sigma_m^2) \Rightarrow$

$$r_m = \frac{\sigma_m^{-2}}{\sum_{m'} \sigma_{m'}^{-2}}$$

4.4. Adaptive Smoothing Method (ASM)
1. isotropic smoothing



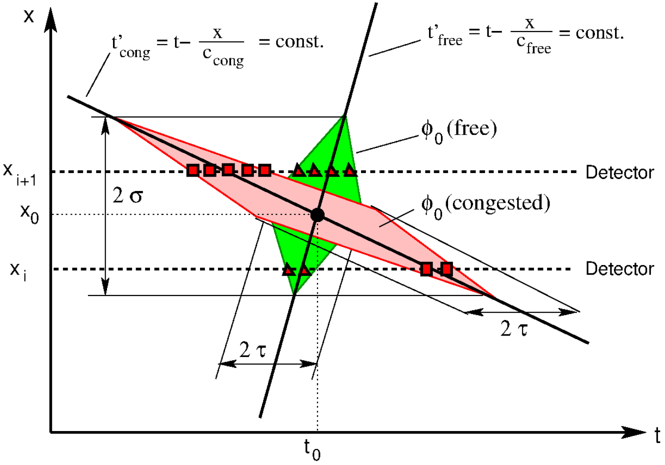
- **Given:** data points $\{(y_i, x_i, t_i)\}$ of quantity Y at the spatiotemporal points (x_i, t_i)
- **Wanted:** Estimate $y(x, t)$ everywhere
- **Isotropic solution:**

  $y(x, t) = \sum_i w_i y_i$ with
  $w_i \propto \phi_0(x - x_i, t - t_i)$ and

$$\phi_0(x, t) = \exp\left[-\left(\frac{|x|}{\sigma} + \frac{|t|}{\tau}\right)\right]$$

Adaptive Smoothing Method
2. anisotropic smoothing

Use smoothing kernels with skewed time axis representing the wave velocities



- "Free" filter with c_{free} near v_0:

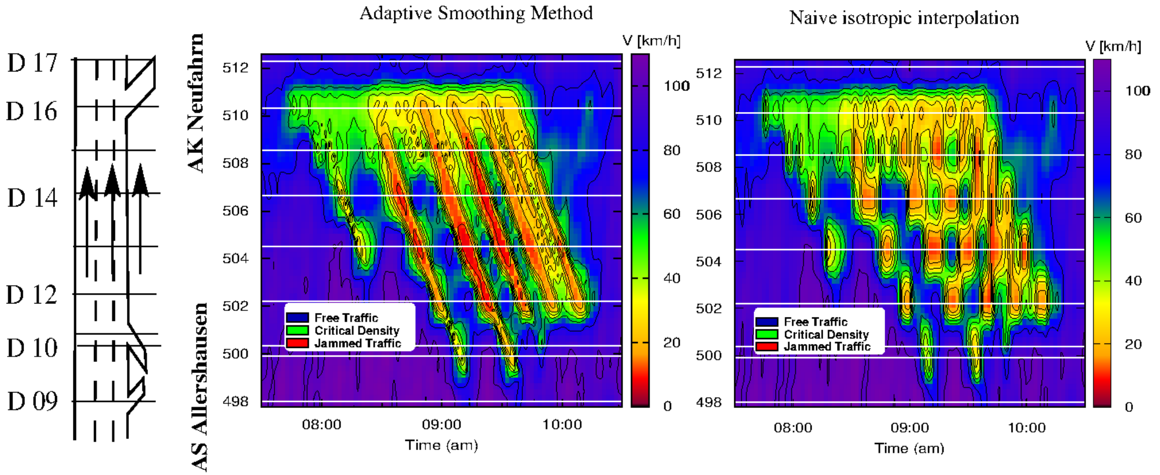
$$w_i \propto \phi_0\left(x - x_i, t - t_i - \frac{x - x_i}{c_{\text{free}}}\right)$$

- "Congested" filter with $c_{\text{cong}} \approx -15\,\mathrm{km/h}$:

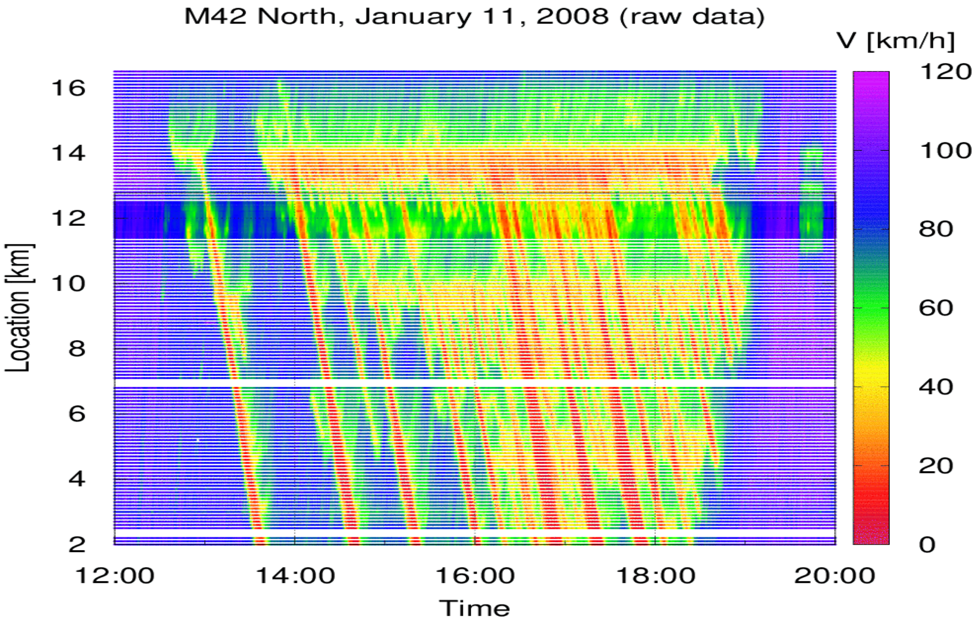
$$w_i \propto \phi_0\left(x - x_i, t - t_i - \frac{x - x_i}{c_{\text{cong}}}\right)$$

- Weighting of the filters according to the "congested" predictor

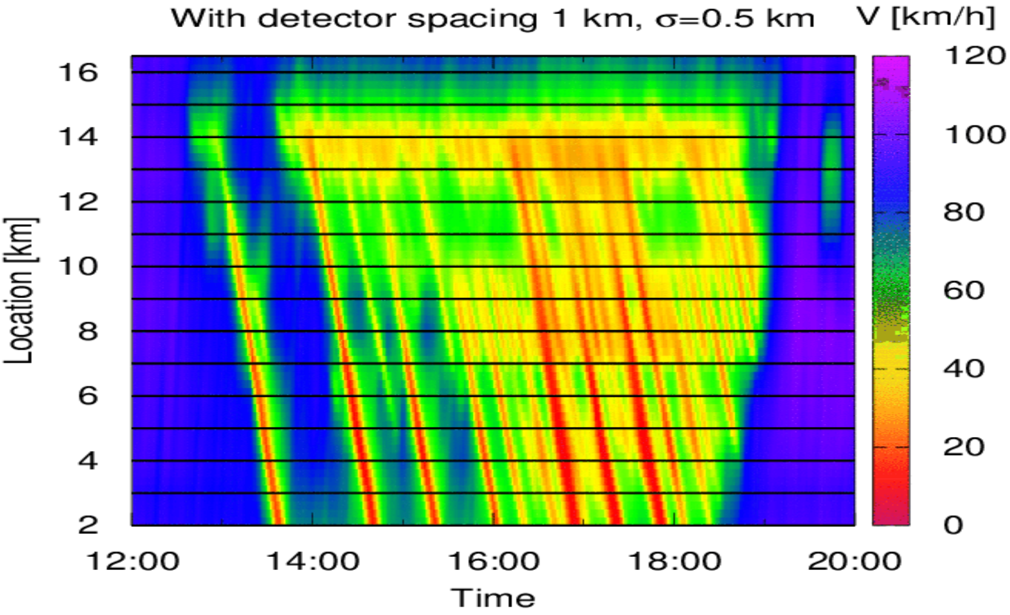
ASM vs. conventional smoothing



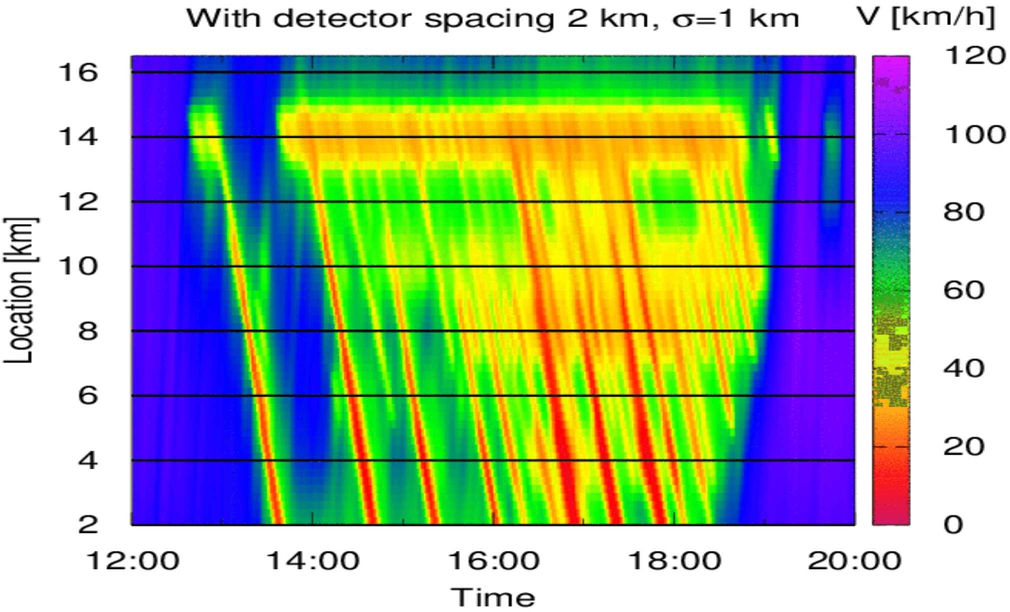
Validation of the Adaptive Smoothing Method: reference



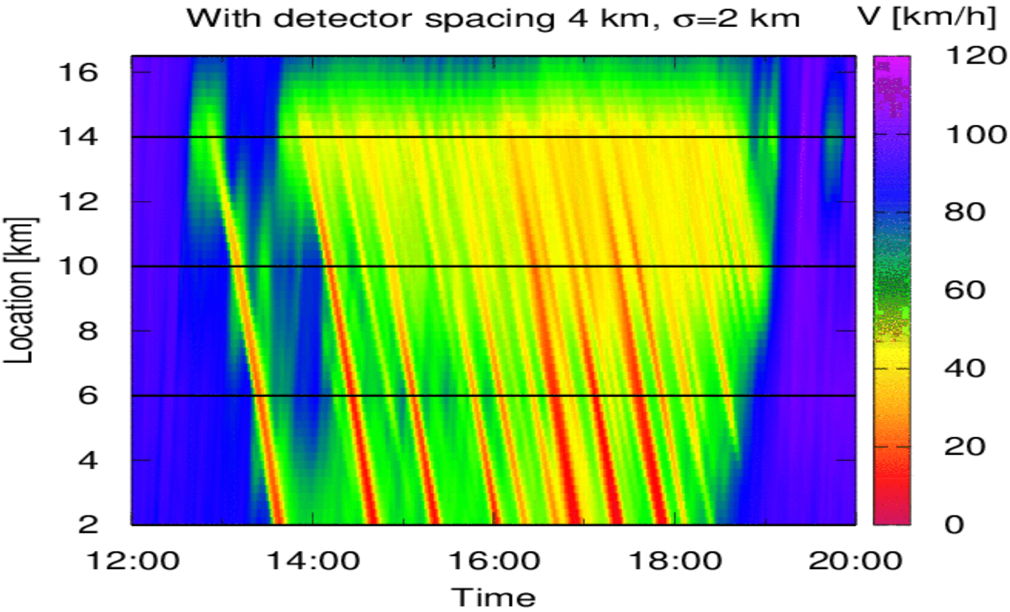
Validation I: detector distance 1 km



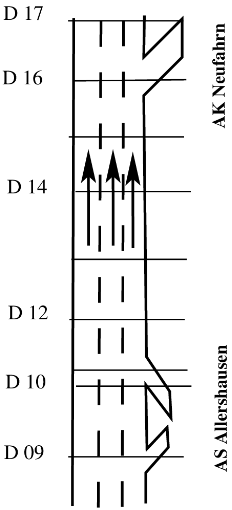
Validation II: detector distance 2 km

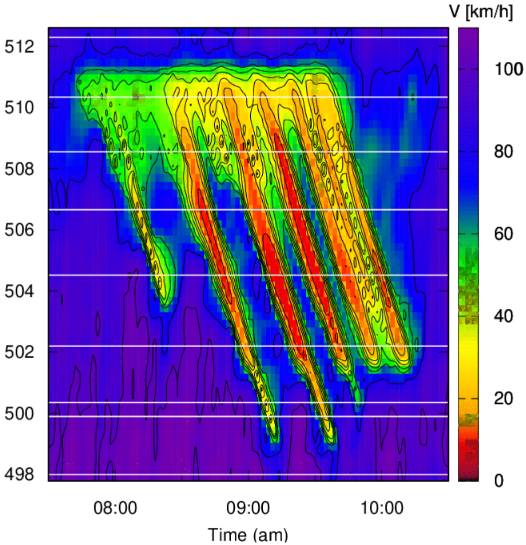


Validation III: detector distance 4 km



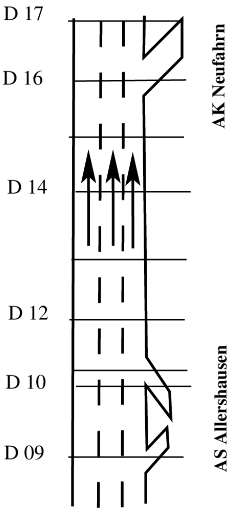
Robustness of the ASM: Sensitivity analysis Reference



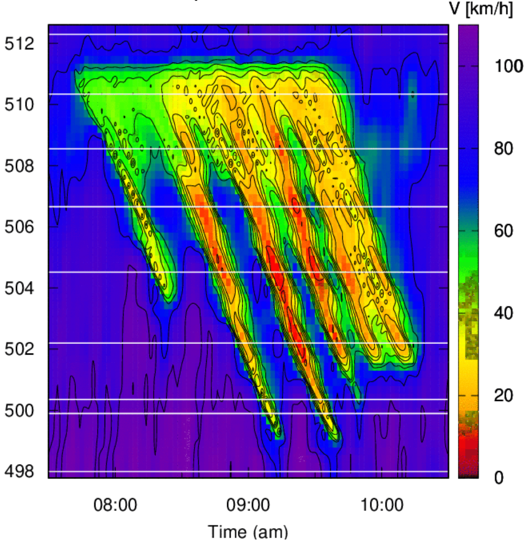


ASM parameters: $\sigma = 600\,\mathrm{m}$, $\tau = 40\,\mathrm{s}$, $c_{\text{free}} = 50\,\mathrm{km/h}$,
$w = c_{\text{cong}} = -15\,\mathrm{km/h}$, $vc1 = 50\,\mathrm{km/h}$, $vc2 = 60\,\mathrm{km/h}$

Robustness of the ASM: Sensitivity analysis I

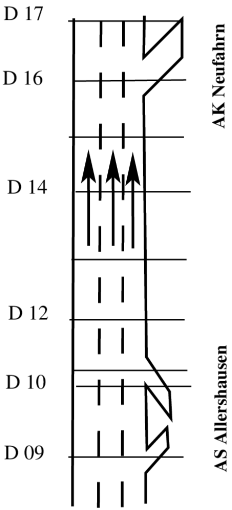


Wave speed w too slow

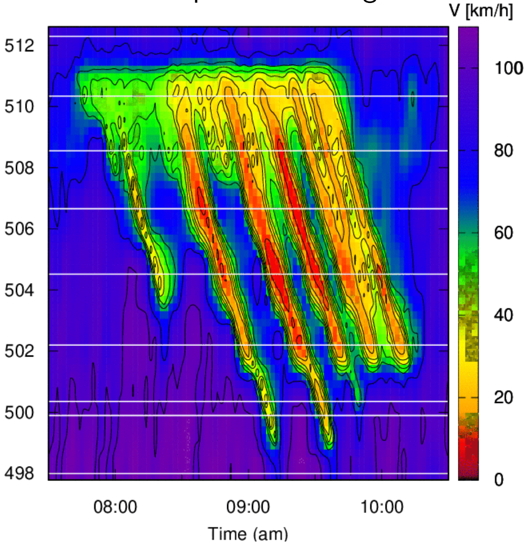


wave speed $w = -10\,\mathrm{km/h}$ instead of $w = -15\,\mathrm{km/h}$

Robustness of the ASM: Sensitivity analysis I



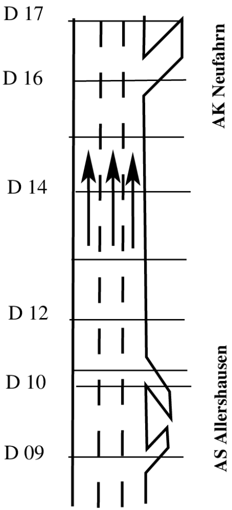
Wave speed w too high

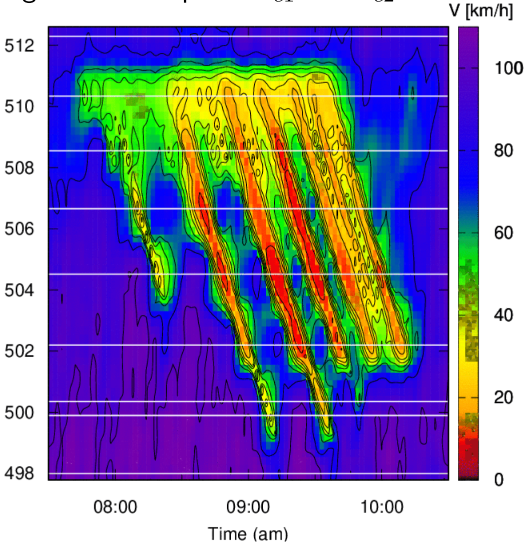


wave speed $w = -20\,\mathrm{km/h}$ instead of $w = -15\,\mathrm{km/h}$

Robustness of the ASM: Sensitivity analysis I

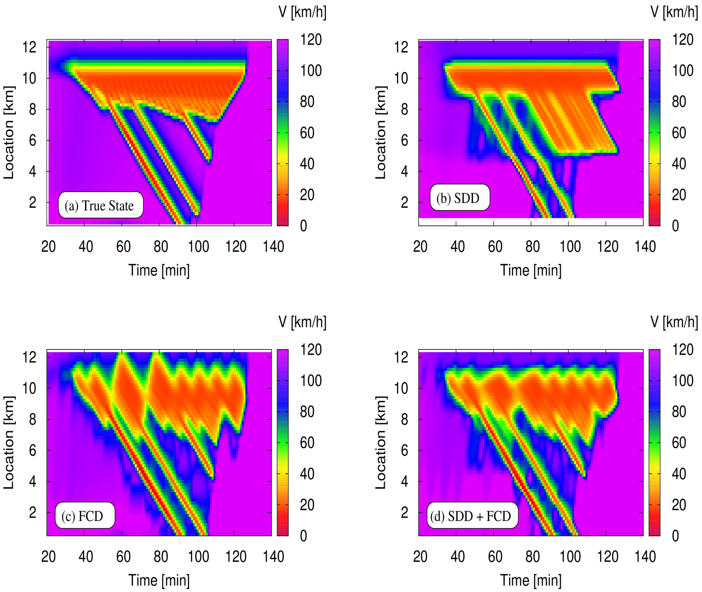
Wrong transition speeds v_{c1} and v_{c2}

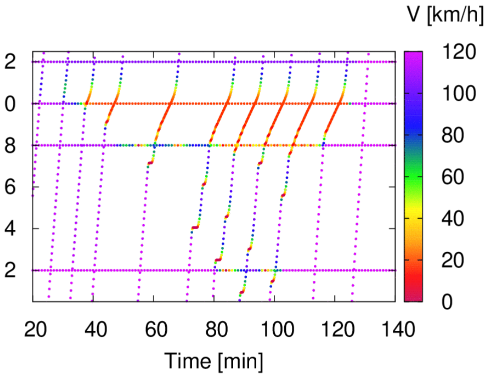




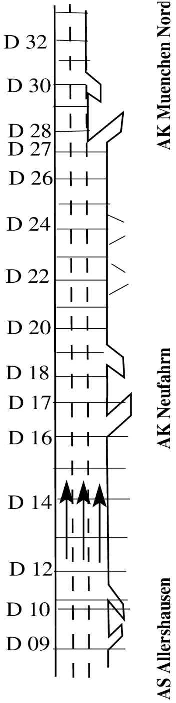
Transit speeds $v_{c1} = 30\,\mathrm{km/h}$ instead of $50\,\mathrm{km/h}$, $v_{c2} = 50\,\mathrm{km/h}$ instead of $60\,\mathrm{km/h}$

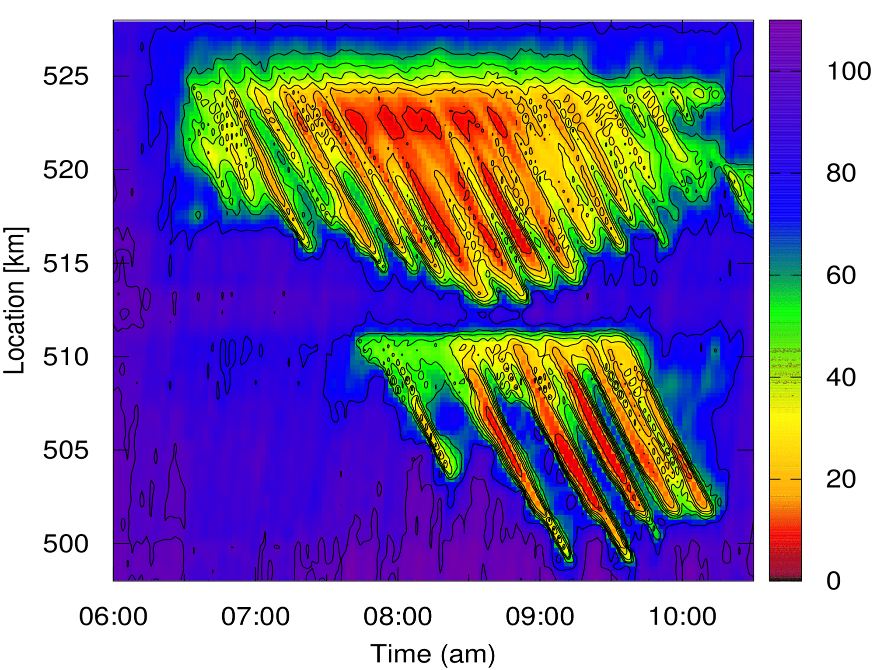
Applying the ASM to SDD, FCD, and both



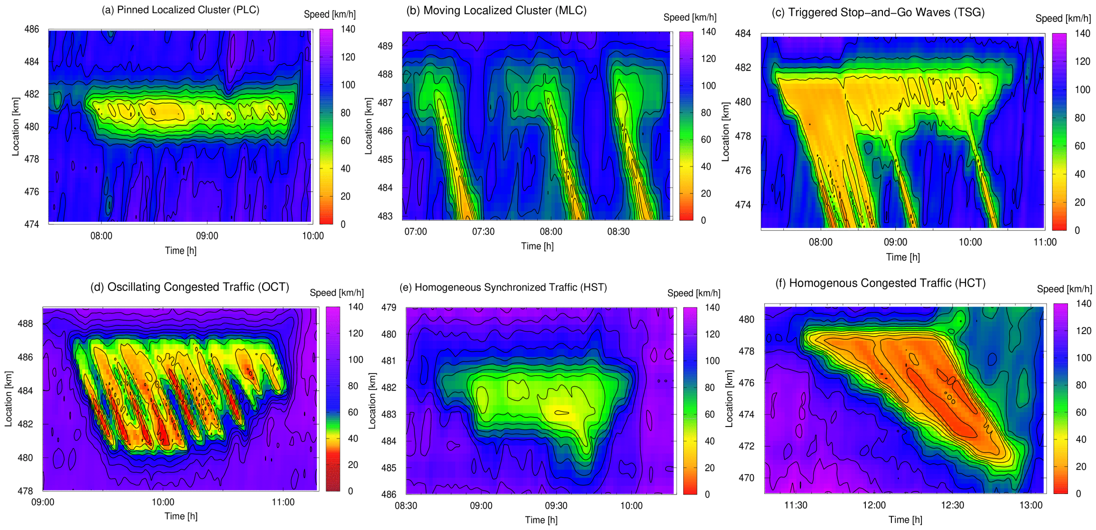


Application A9 Munich: the full congested region





Application: understanding the dynamics of congestions



$\Rightarrow$ Models!