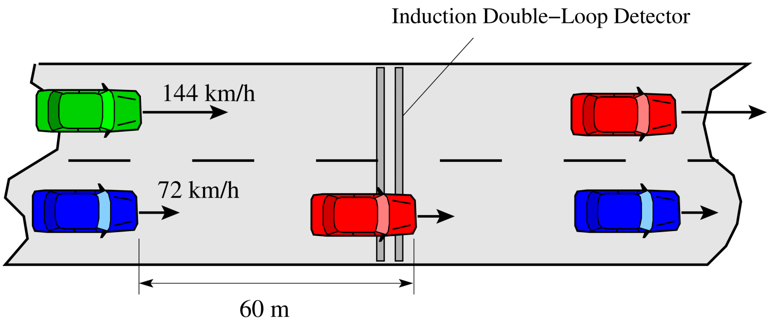
Lecture 03: Cross-Sectional Data Analysis

3.1. Estimating Spatial Quantities from SDD

Following example shows how biased the arithmetic mean speed and "density=flow/speed" can be when naively estimating spatial quantities:



? determine Q^{tot}, V and $\rho^{\text{tot}} = Q^{\text{tot}}/V$ for the total cross-section of two lanes and compare with the "true" density and spatial mean speed

Solution: left: $V_l = 40\,\text{m/s}$, $Q_l = 1/3\,\text{veh/s}$; right: $V_r = 20\,\text{m/s}$, $Q_r = 1/3\,\text{veh/s}$;
total: $Q^{\text{tot}} = Q_l + Q_r = 2/3\,\text{veh/s}$, $V = 1/2(V_l + V_r) = 30\,\text{m/s}$, $\rho^{\text{tot}} = Q^{\text{tot}}/V = 1\,\text{veh}/(45\,\text{m})$,
true value: 3 veh/120 m=1 veh/(40 m)

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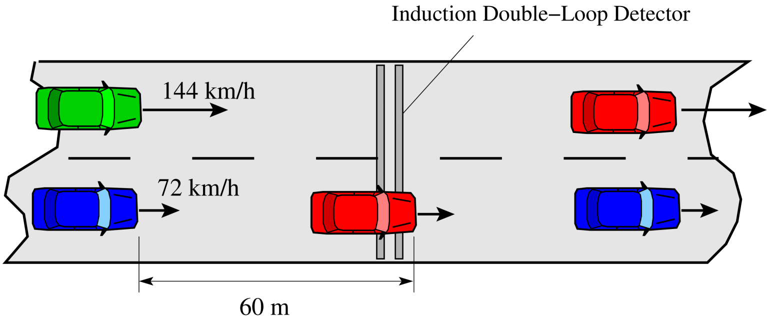

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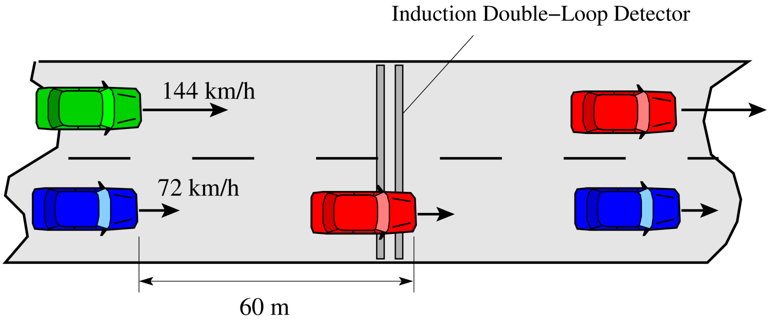

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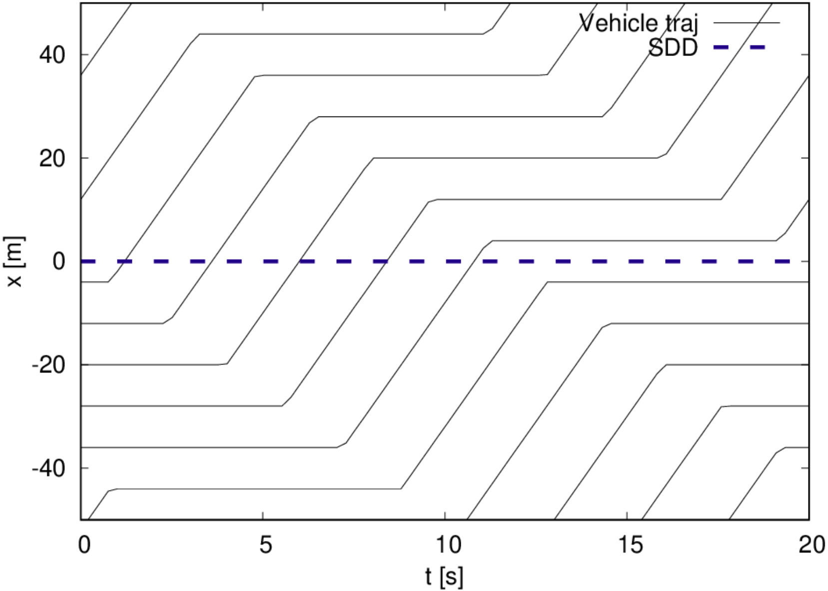

Determine flow and speed over an aggregation interval $\Delta t_{\text{aggr}} = 20\,\text{s}$

$Q = 0.25\,\text{veh/s}$,

$V = 10\,\text{m/s}$

Compare flow divided by speed with the true spatial density aggregated over 100 m

$Q/V = 25\,\text{veh/km}$,

$\rho^{\text{true}} = 70\,\text{veh/km}$

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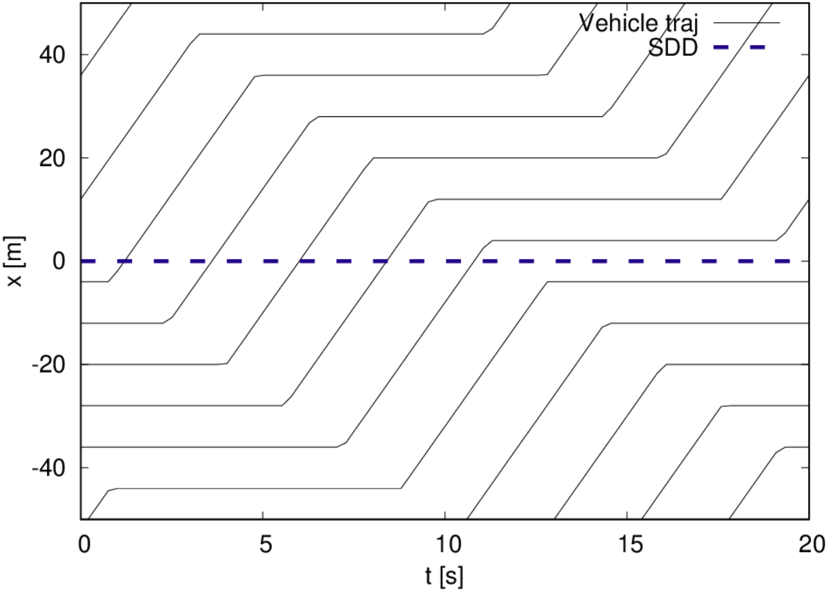

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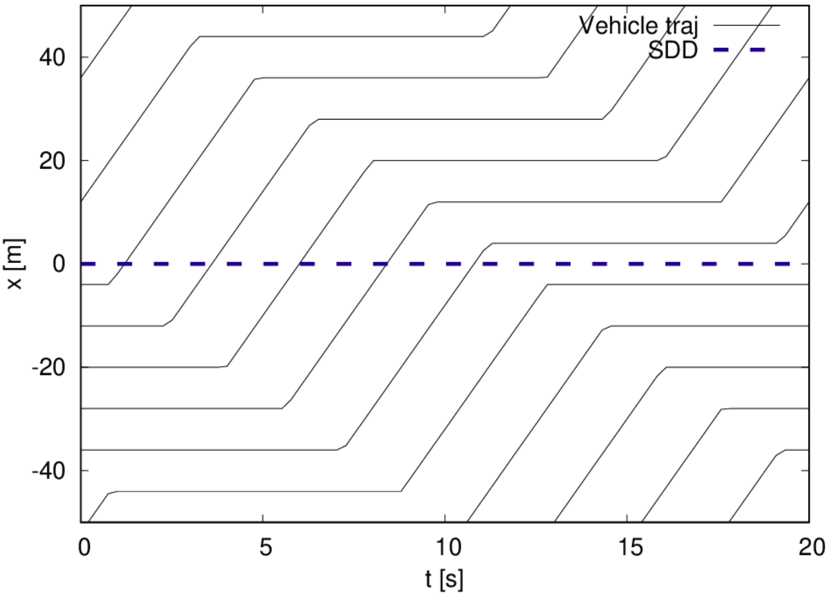

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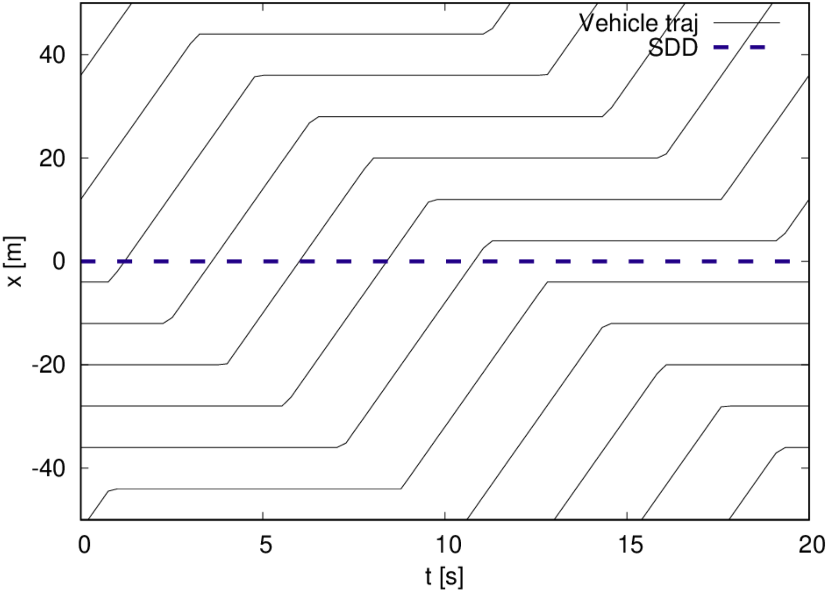

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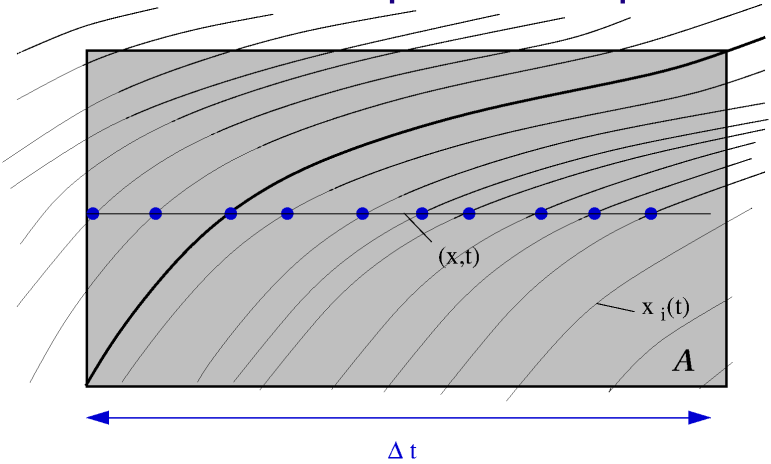

- Local average or time mean of quantity y (stationary detectors)

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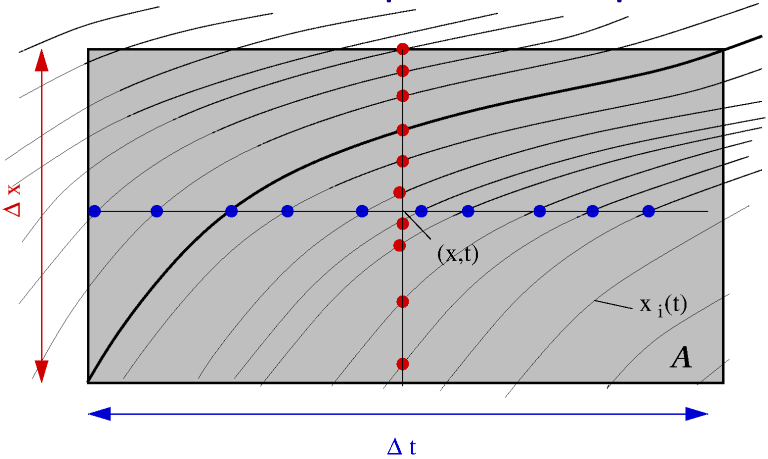

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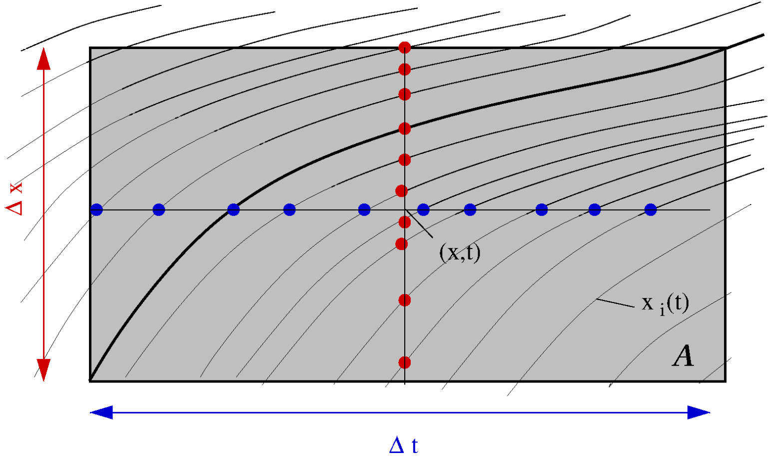
- Instantaneous average or space mean of quantity y (snapshot)

$$Y_s = E(y_i|t \text{ fixed}) := E_s(y_i) = 1/N_{\Delta x} \sum_i y_i(x_i, t)$$

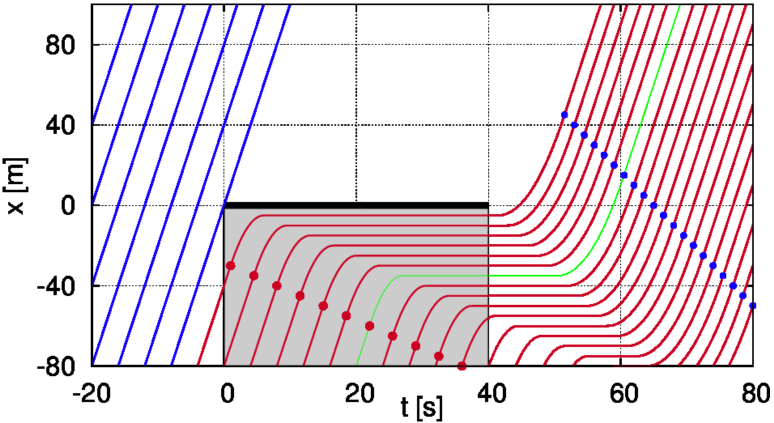
Definition of macroscopic traffic flow quantities II: Edie's definitions



Spatiotemporal mean (Edie's definition) of density, flow, speed:

$$\rho_{\text{Edie}} = t^{\text{tot}}/A, \quad Q_{\text{Edie}} = x^{\text{tot}}/A, \quad V_{\text{Edie}} = Q_{\text{Edie}}/\rho_{\text{Edie}}$$

Problems



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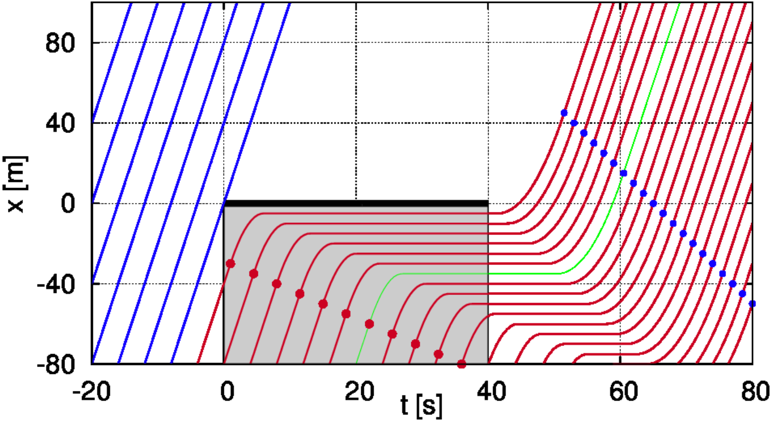

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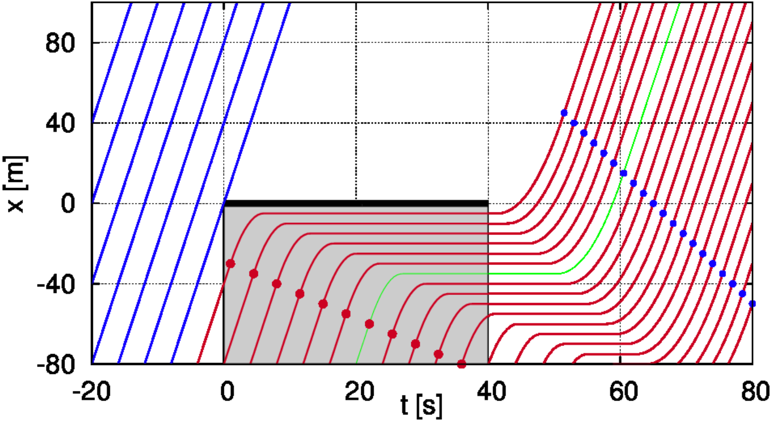

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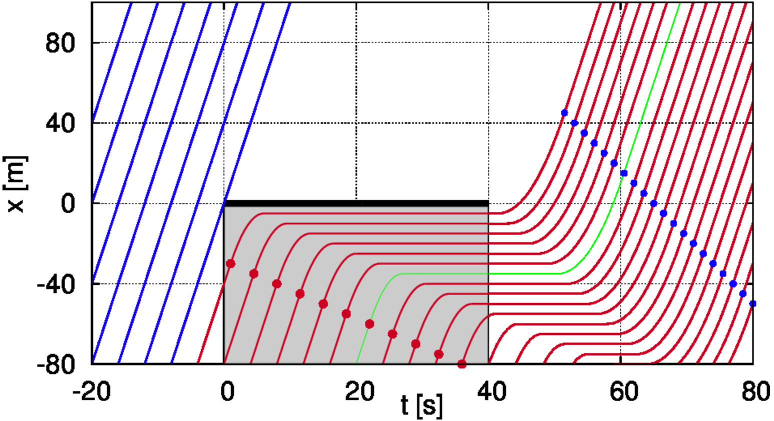

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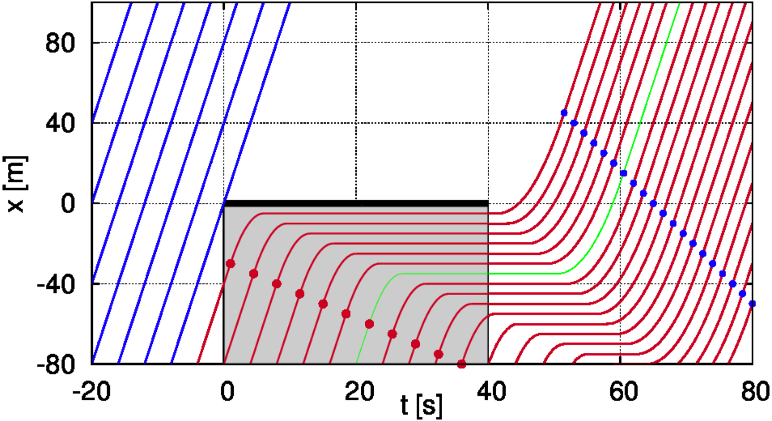

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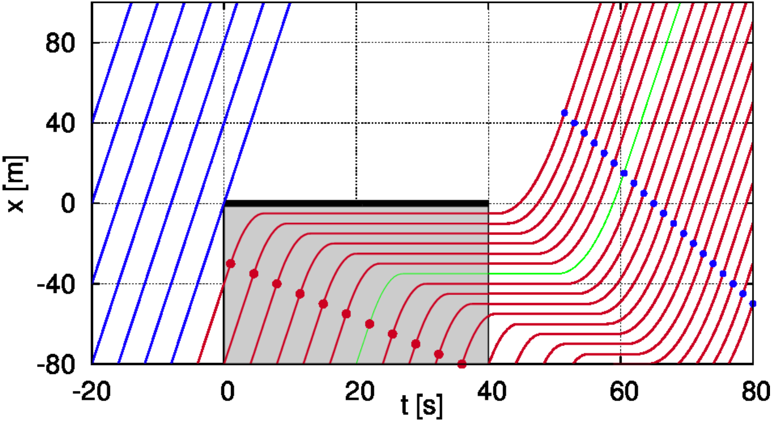

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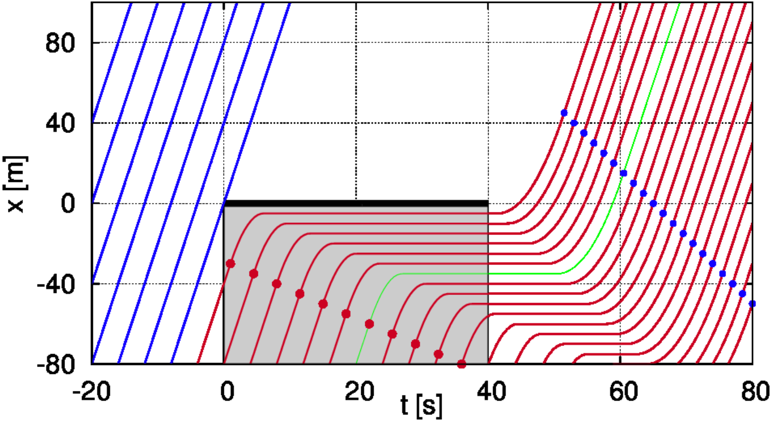

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Assume a steady state at a spatial or **instantaneous speed density function $f(v)$**. Then,

- the *partial density* of a speed layer is given by $\mathrm{d}\rho = \rho f(v)\,\mathrm{d}v$. $f(v)$ and $w(v)$ blackboard
- Since the number of stationary detector recordings (time mean!) is proportional to the flow, the temporal or **local speed density function $w(v)$** relevant for detector measurements is proportional to the *partial flow* $\mathrm{d}Q = v\,\mathrm{d}\rho$:

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- For the time-mean speed V as a function of the space-mean speed V_s, we obtain

$$V = \int v w(v)\,\mathrm{d}v = \frac{1}{V_s}\int v^2 f(v)\,\mathrm{d}v = \frac{E_s(v_i^2)}{V_s}$$

- With the general relation $E(X^2) = \mathrm{Var}(X) + (E(X))^2$ also valid for spatial averages $E_s(.)$, we have $E_s(v_i^2) = \mathrm{Var}_s(v_i) + V_s^2$, so

$$V = \frac{\mathrm{Var}_s(v_i) + V_s^2}{V_s} = V_s + \frac{\mathrm{Var}_s(v_i)}{V_s} \qquad \text{Leutzbach relation}$$

Estimating space mean speed by harmonic averages

- *Both time and space means can be applied to any function y_i of recorded single-vehicle data* such as $y_i = v_i$ or $y_i = 1/v_i$:
  - temporal arithmetic average: $V = E(v_i)$
  - temporal harmonic average: $V_H = 1/E(1/v_i)$
  - spatial arithmetic average: $V_s = E_s(v_i)$
- Derivation of the Leutzbach relation → any expected time average $E(y_i)$ of data y_i can be written in terms of the spatial (!) speed distribution function $f(v)$ via the weighting $w(v)\,\mathrm{d}v = vf(v)/V_S\,\mathrm{d}v$ as

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$$\frac{1}{V_H} = E(1/v_i) = \frac{1}{V_s}\int f(v)\,\mathrm{d}v = \frac{1}{V_s}$$

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The harmonic time mean speed is an unbiased estimator of the space mean speed provided stationarity (in the statistical sense, i.e., $f(v)$ is unchanged over averaging space and time).

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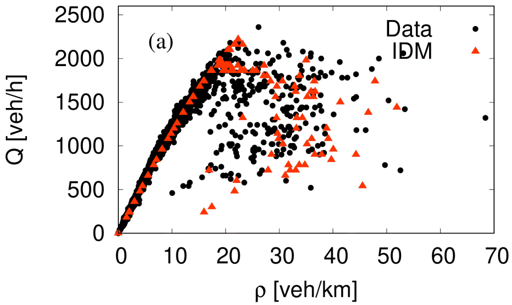
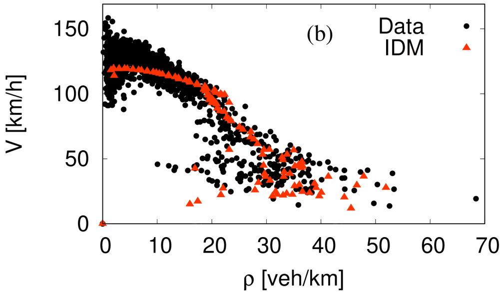
? Show that the expected true density $\hat{\rho}$ is generally underestimated by $\rho = Q/V$. In which situations this bias becomes pronounced? Hint: what is the expected sign of this covariance?

Comparison with a model reveals systematic bias





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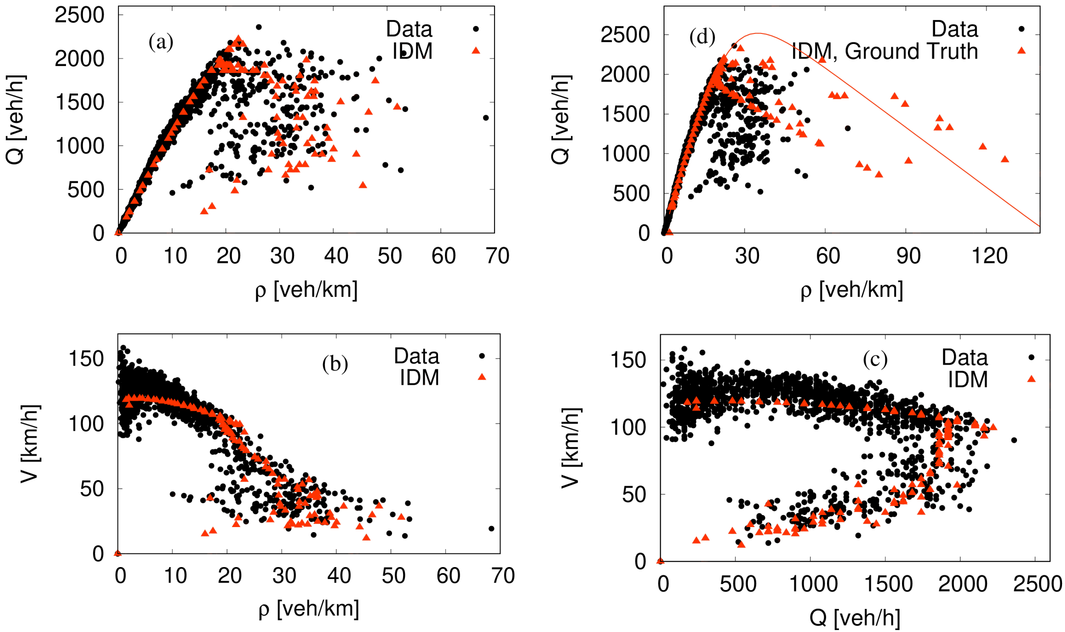
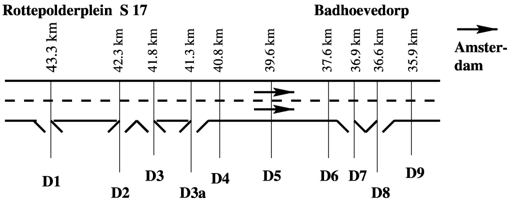


3.2. Analysis I: Local Flow Characteristics



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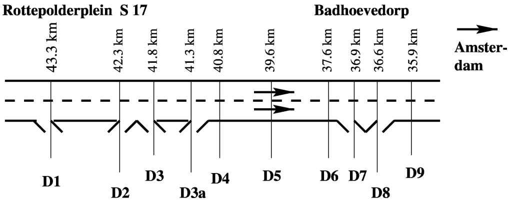

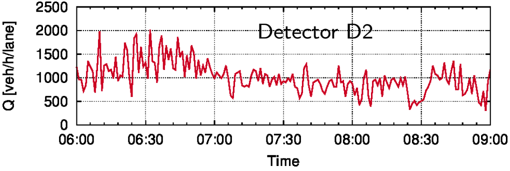

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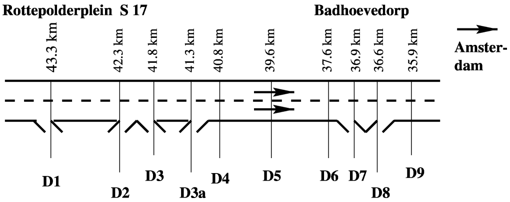

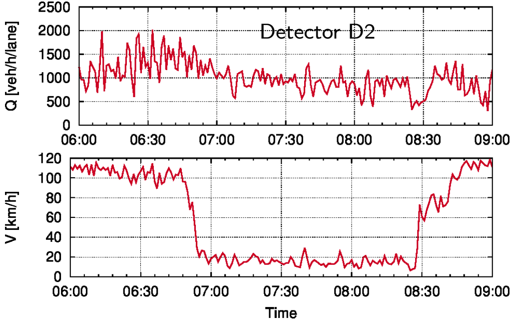

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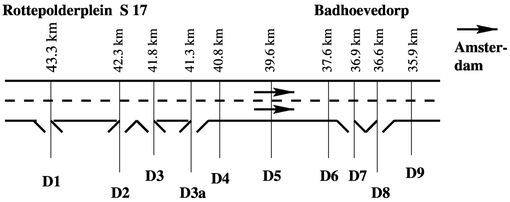

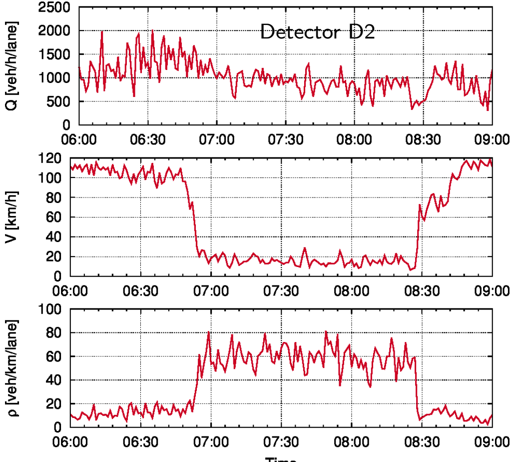

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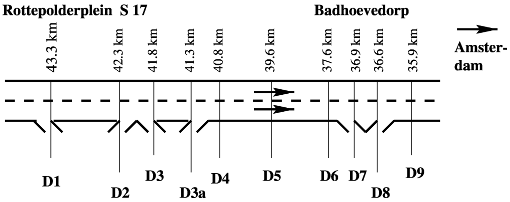
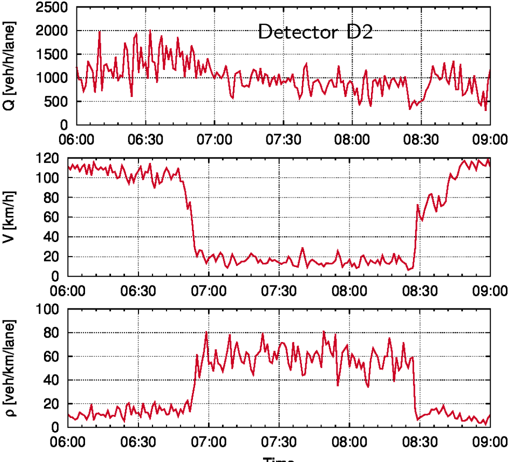
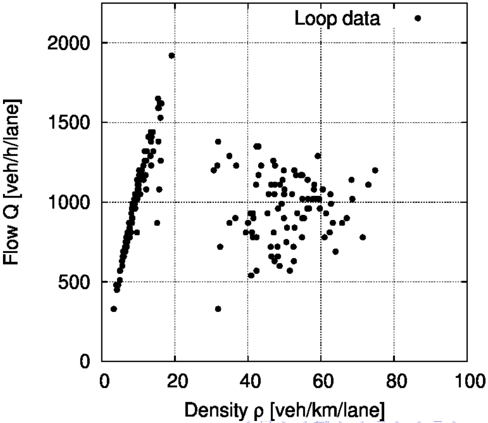
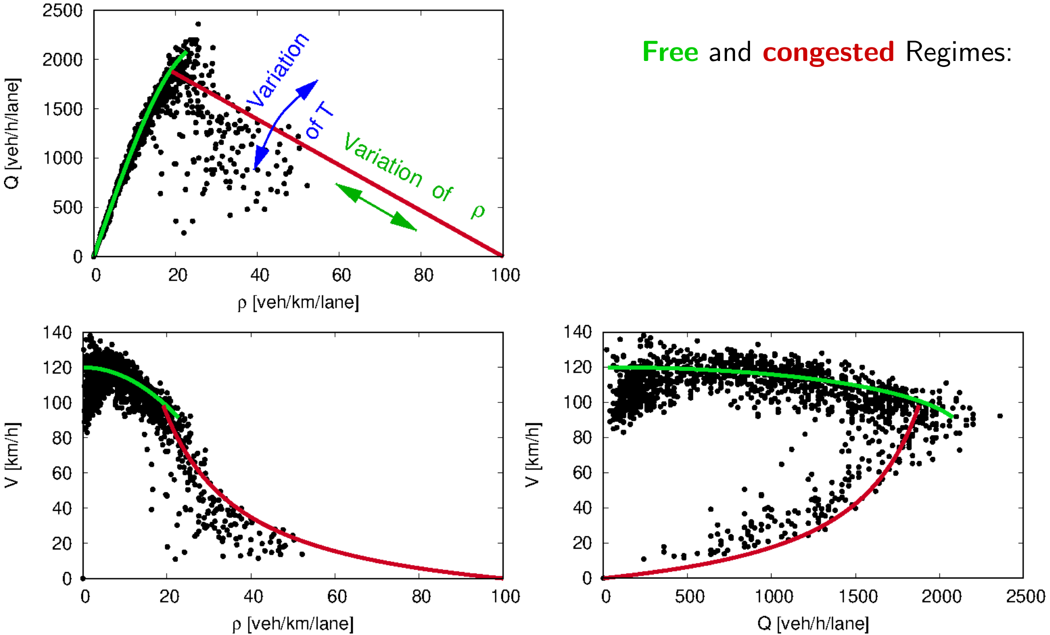






Flow-density-speed data and fundamental diagram

Free and **congested** Regimes:



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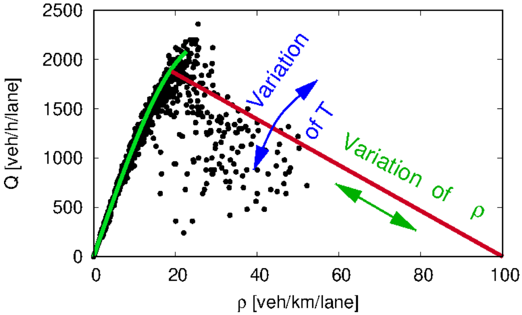
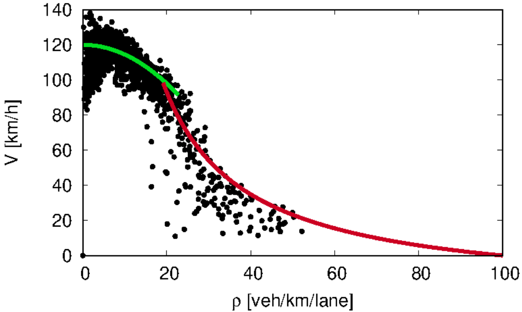
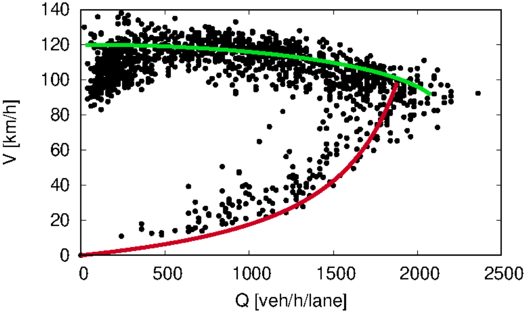


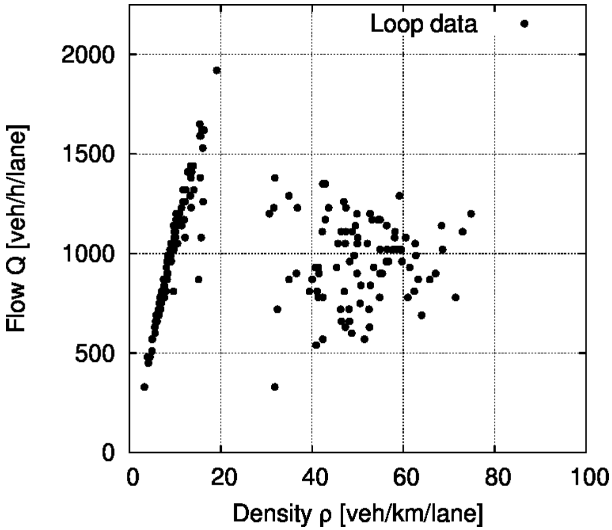
Free and **congested** Regimes:

Why is the red congested line of the FD not a regression line of the congested data points?





Why is the fundamental diagram so “fundamental”



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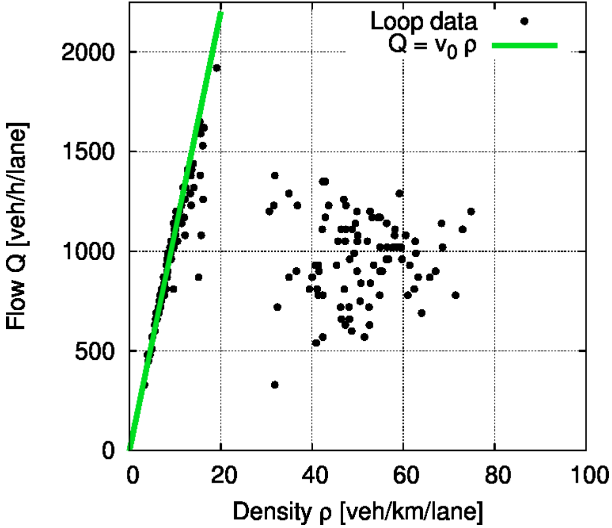

- **free traffic**:
  $Q(\rho) = V_0 \rho$

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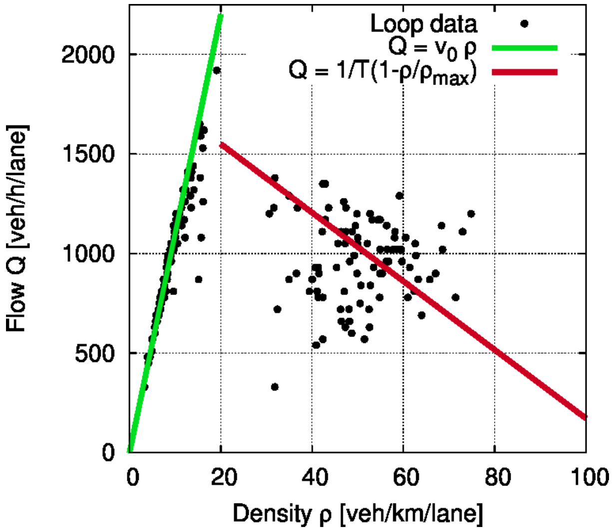

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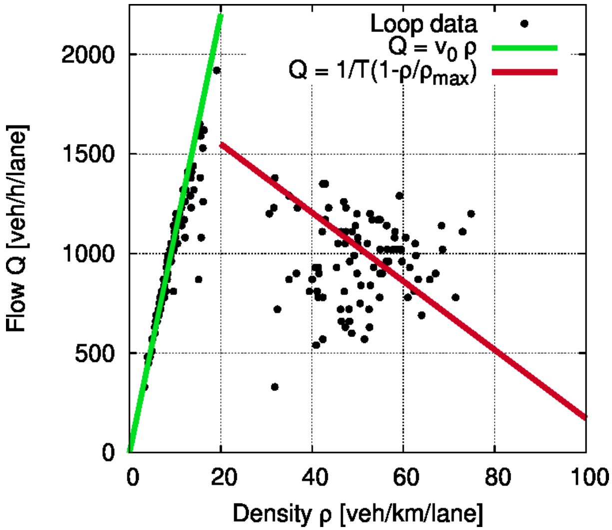

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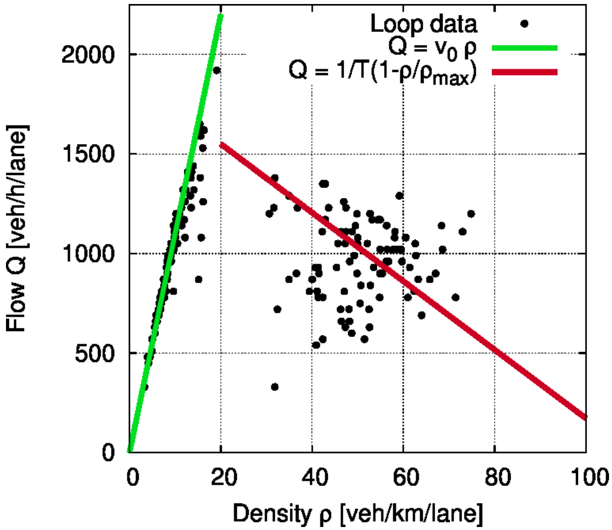

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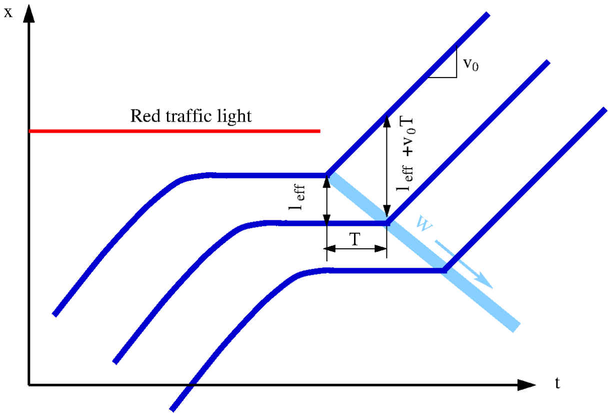
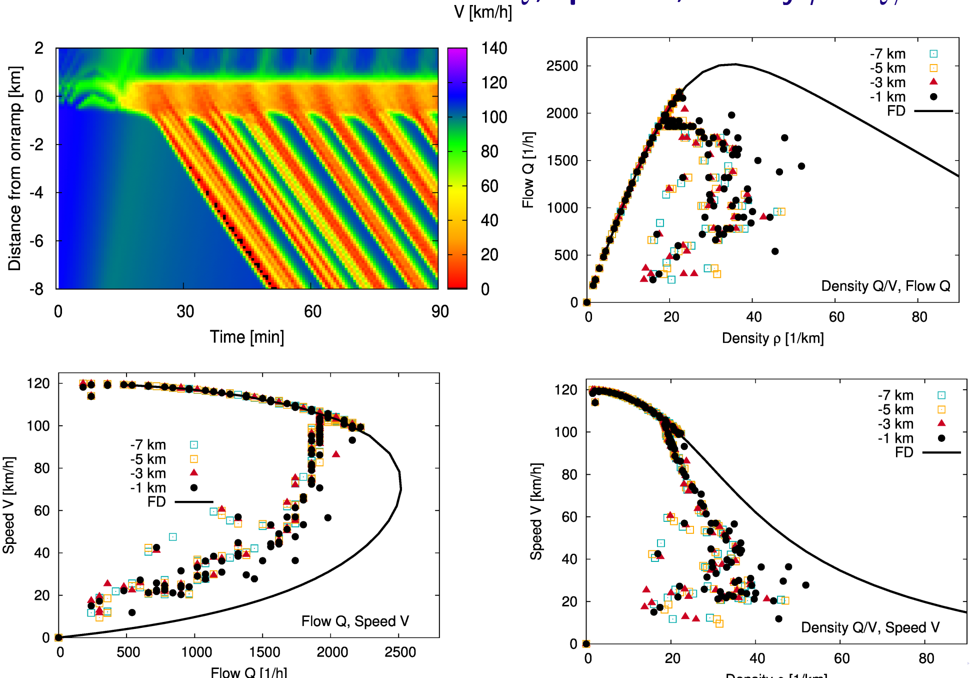
- Intersection with Q_f:
  $\Rightarrow$ estimate for capacity
  $Q_{\text{max}} = V_0/(V_0 T + l_{\text{eff}})$

Derivation of the wave speed

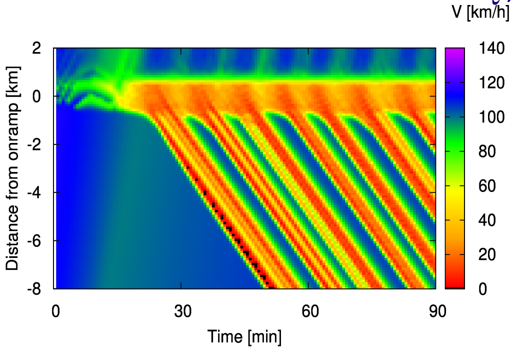
- The sequential starting of vehicles once a traffic light turns green has nothing to do with reaction time but that a moving vehicle needs more space headway $(l_{\text{eff}} + vT)$ than a standing one (l_{eff})
- Extrem case: Zero reaction time, infinite aceleration to the desired speed v_0 one the space headway $\Delta x = l_{\text{eff}} + v_o T$
- Reasoning also valid for the general congested case ($\Rightarrow$ Newell's model)
- Wave speed equal to gradient of the congested part of the FD $\Rightarrow$ later

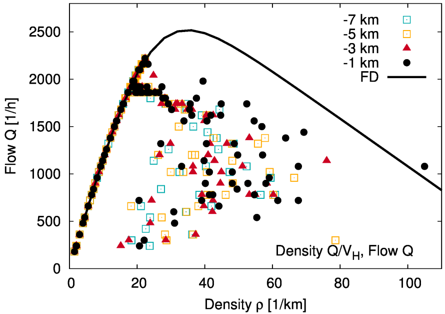


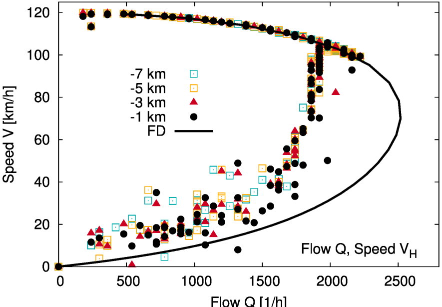
Bias check I: flow Q, speed V, density $\rho = Q/V$

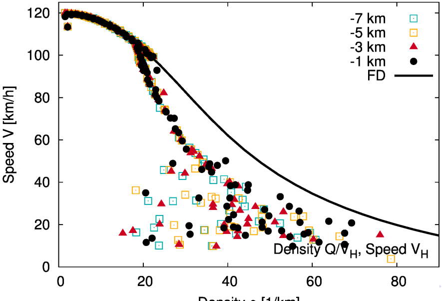


Bias II: flow Q, speed V_H, density Q/V_H

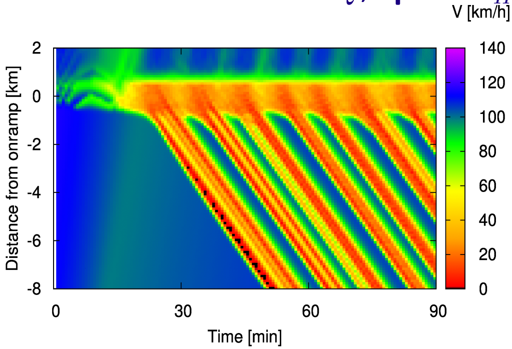


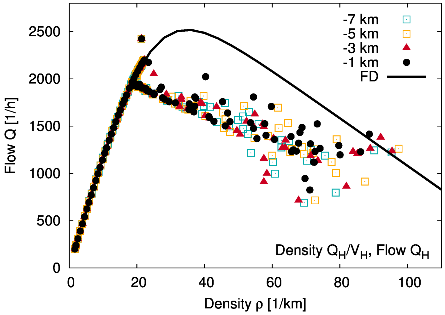


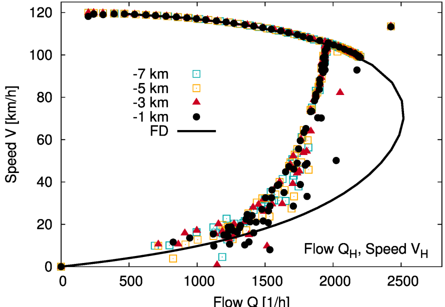


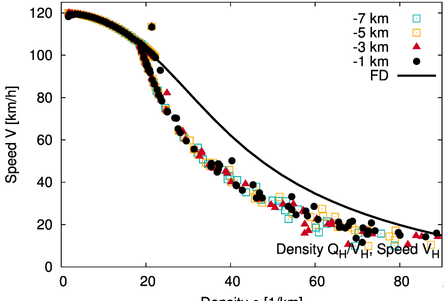


Bias III: flow Q, speed V_H, density $\rho = Q_H/V_H = E(1/\Delta t_i)/V_H$

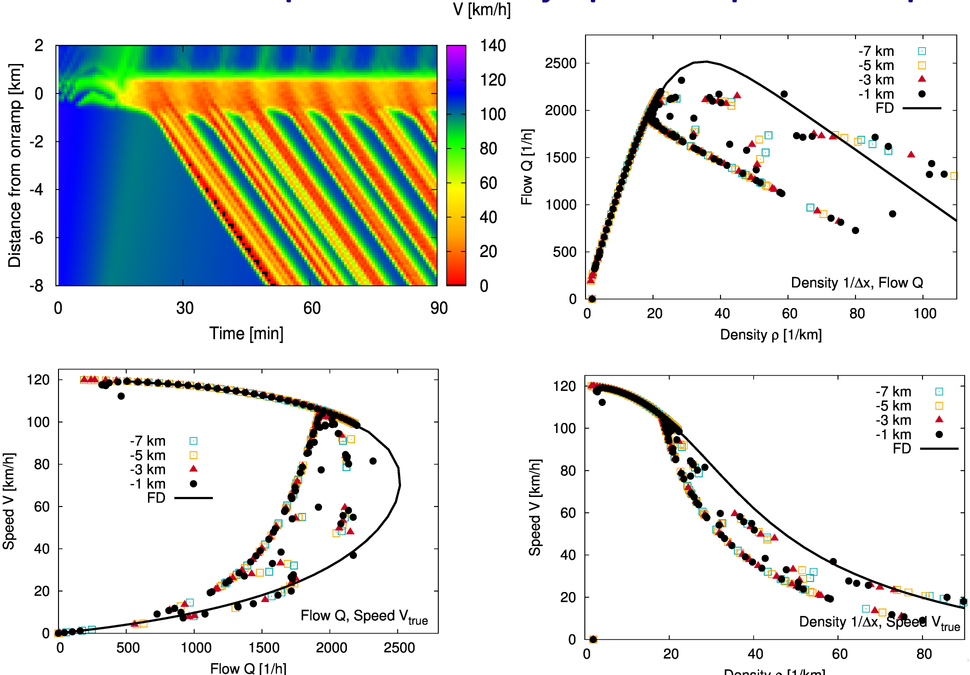




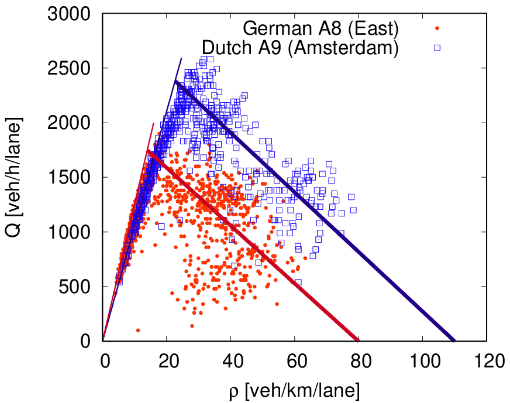


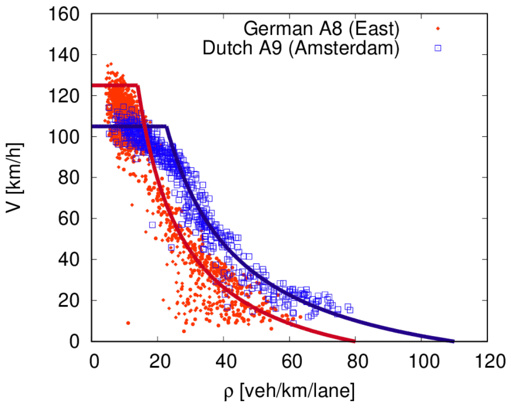


Real spatial local density, spatio-temporal local speed



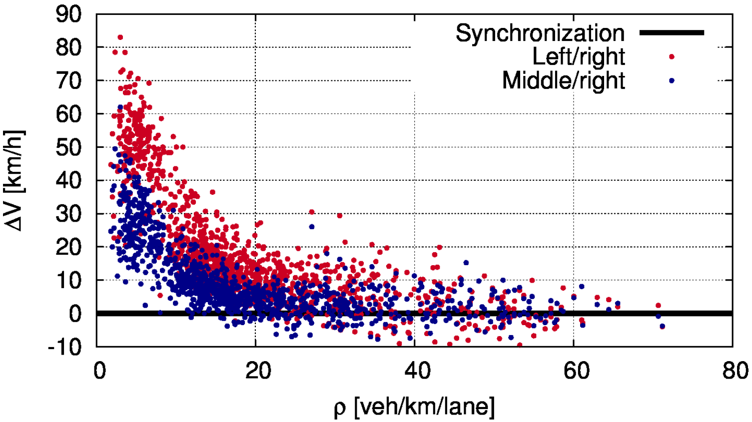
Regional and infrastructural differences





- German A8-East near Munich: Higher maximum speed and lower capacity compared to the Dutch A9 near Amsterdam

Speed synchronisation across lanes



- Low densities → little interactions → nearly everybody can drive at his/her desired speed chosing the suitable lane (“fast”, “middle”, or “slow”) → Δv large;
- densities near capacity: still no congestion but much interaction → small Δv values;
- jammed region: $\Delta v \approx 0$ (“in a jam, everybody is equal”)

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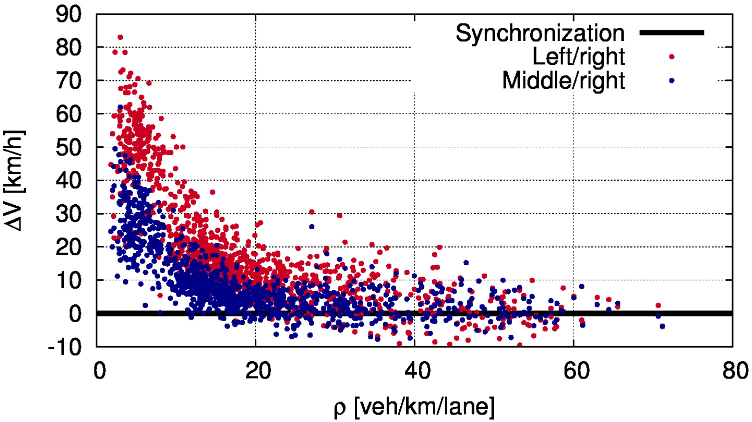

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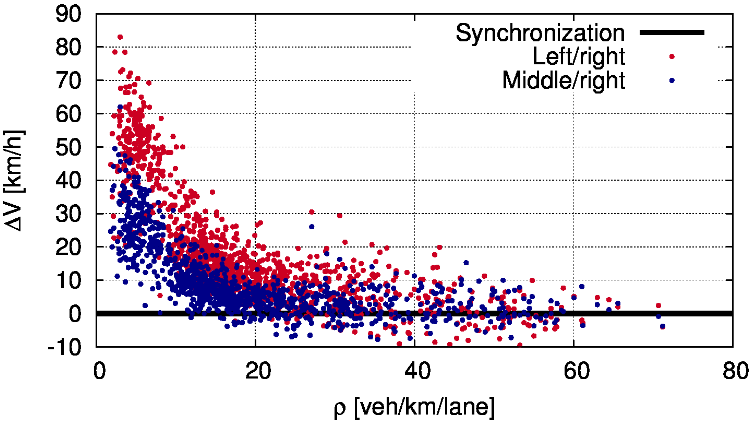

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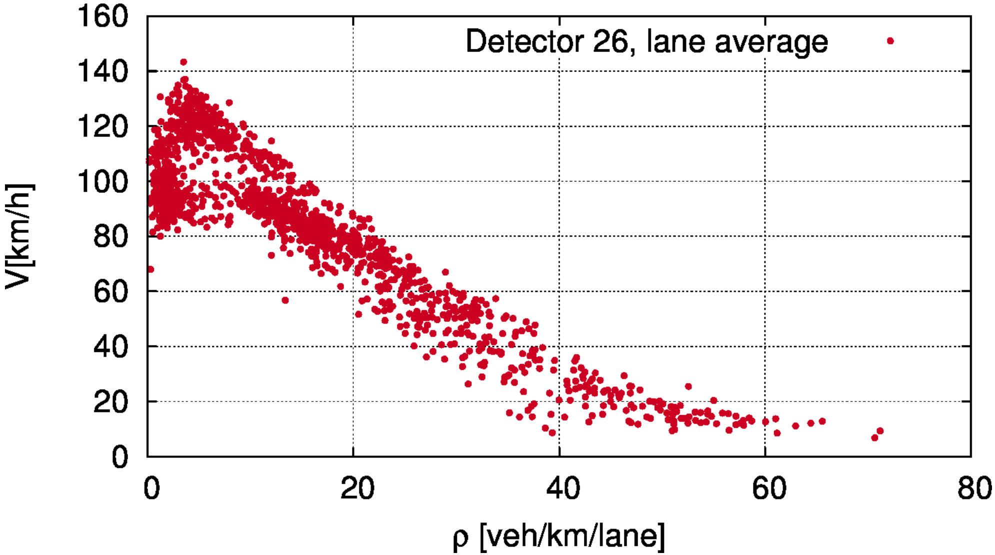
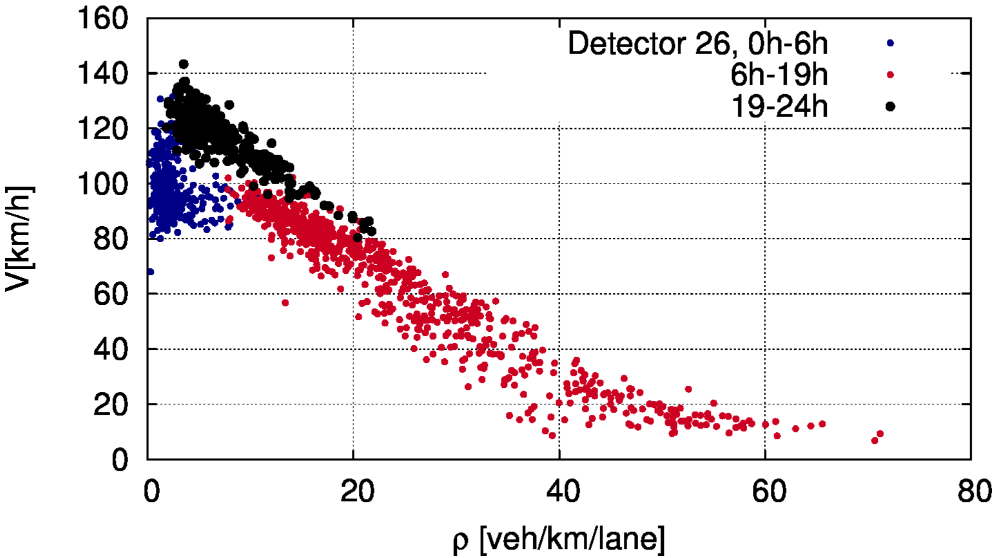
Horror Vacui



? Is there a "fear of the empty Autobahn" (*Horror Vacui*)?

Horror Vacui explained: Simpson's Effect/Paradox



! Different weather and/or traffic composition and/or speed limits in the three time intervals → **Simpson's Effect**

Problem: Simpson's effect in local flow characteristics

? Explain Simpson's effect for exam ratings: Languages: females 80 %, avg grade 2.5, males 20 %, avg grade 2.0; STEM: females 20 %, avg grade 3.5, males 20 %, avg grade 3.0.

! In each department, males have a better average. Still, mixing the departments together, the women are better (average 2.7) than the men (2.8).

? In traffic flow data, Simpson's effect is relevant if the time variable is eliminated such as in speed-density scatter plots. Why?

! because (i) the vehicle and driver composition, visibility/road conditions and possibly traffic regulations change during the daytime, (ii) these changes are correlated with flow, density, and speed $\Rightarrow$ sampling of heterogeneous data with heterogeneities correlated to the variables of interest $\Rightarrow$ Simpson's effect.

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? Summarize all discussed biases affecting

(i) flow, time-mean speed V, space-mean speed V_{S}, density ρ estimated by vehicle count and arithmetic speed mean as obtained from SDD,

(ii) speed-density and speed-flow scatter plots obtained from SD data

! Time series:

- Flow $Q = n_{\text{veh}}/\Delta t_{\text{aggr}}$ and time-mean speed V: none since SDD imply time means
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- True density ρ_{real} according to Edie's definition: Underestimated by $\rho = Q/V$; unbiased if estimated by Q/V_{H} and stationarity applies; partial correction for nonstationarity by $Q_{\text{H}}/V_{\text{H}}$ or if $\text{Cov}(v_i, \Delta t_i)$ can be estimated

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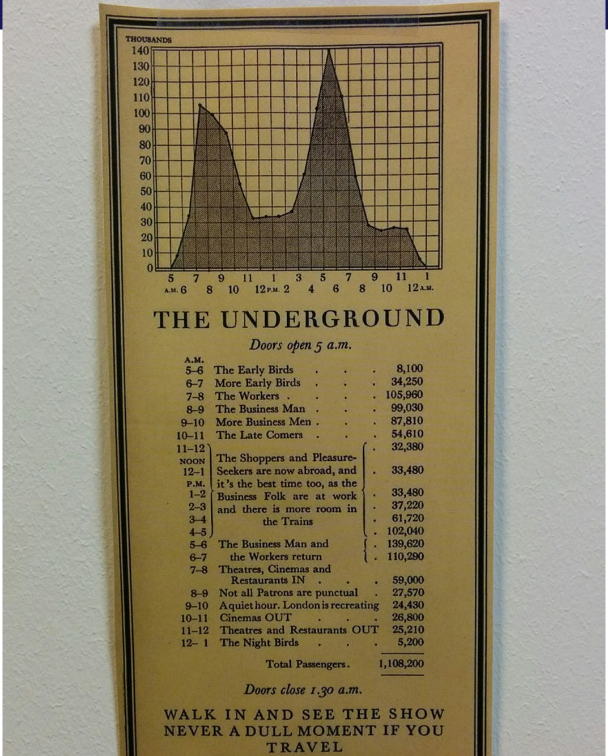
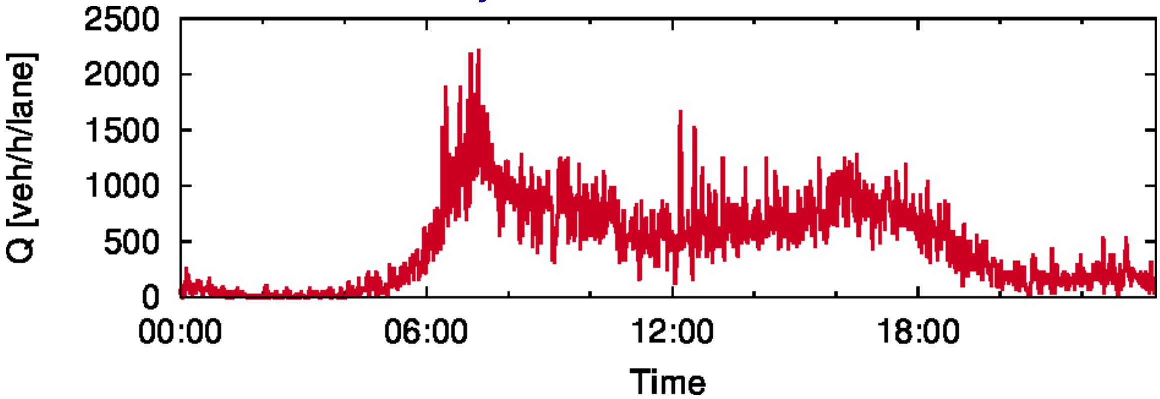
3.3. SDD
Time Series



Daily time series of flow



- Only single-loop detectors needed
- Unless there is congestion, this data reflects the traffic demand (why this restriction?)
- Application mainly in transportation/traffic planning and traffic politics ⇒ DTV
- Traffic flow application: **historic data base** to improve traffic state estimation/short-term prediction for dynamic navigation

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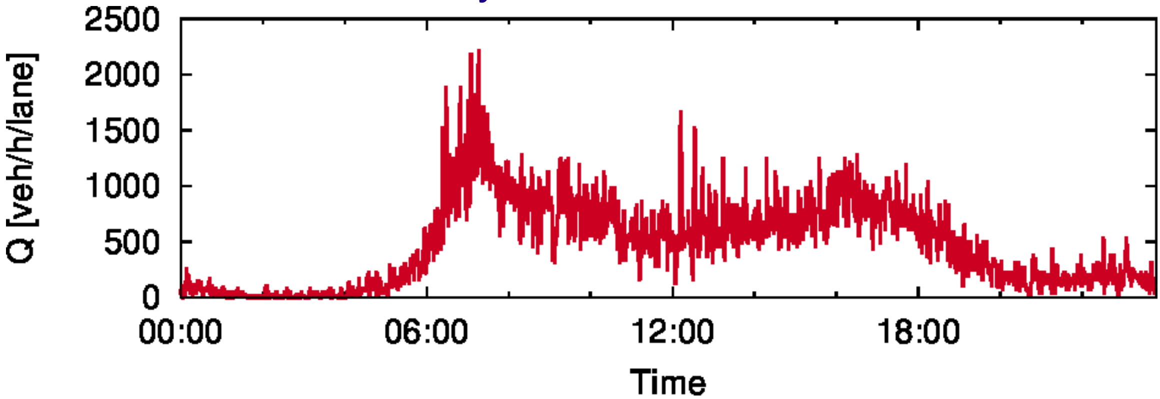

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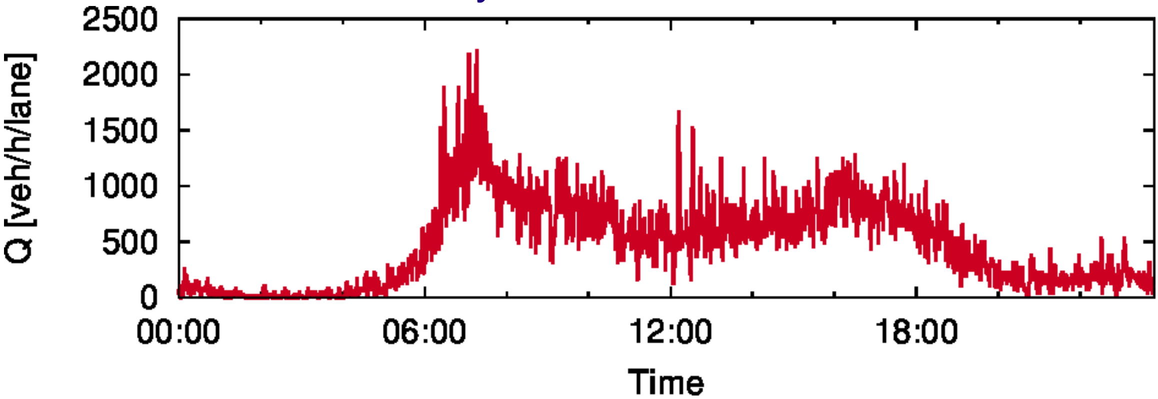

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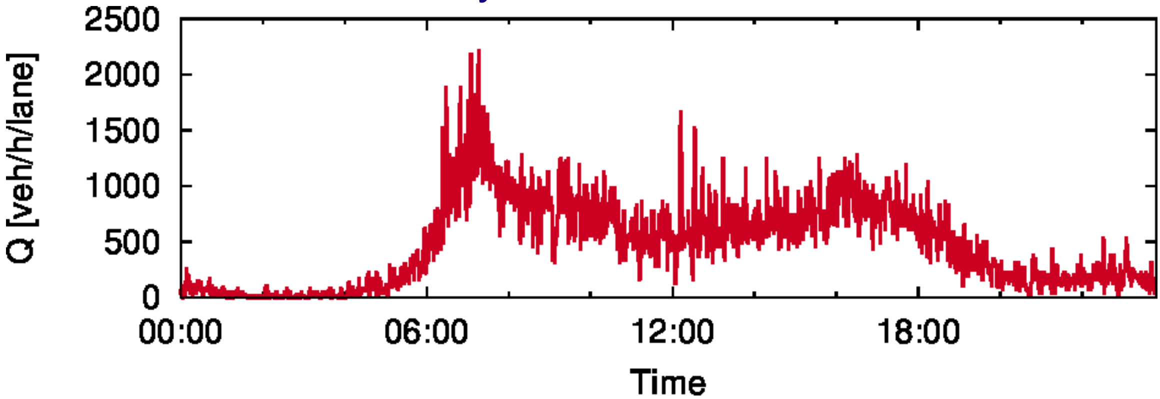

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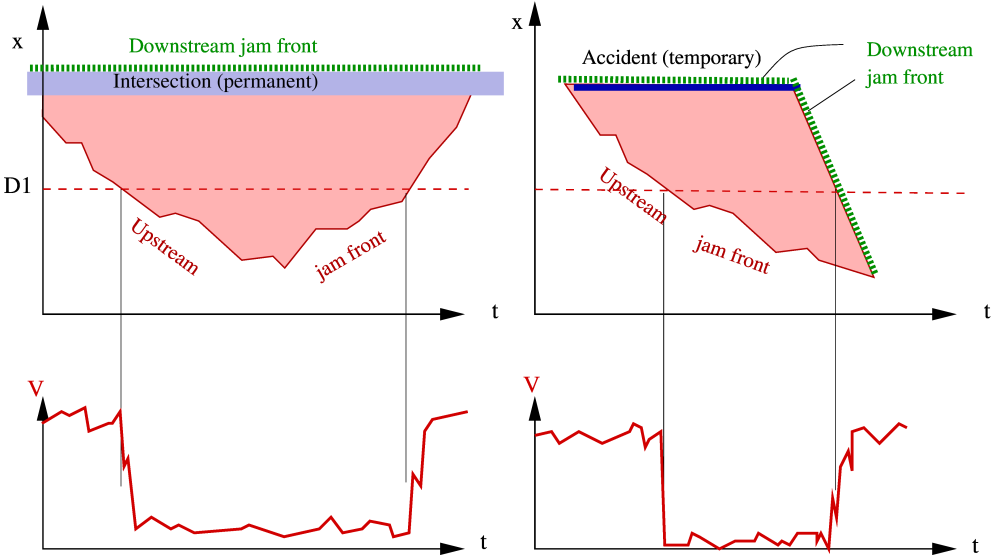
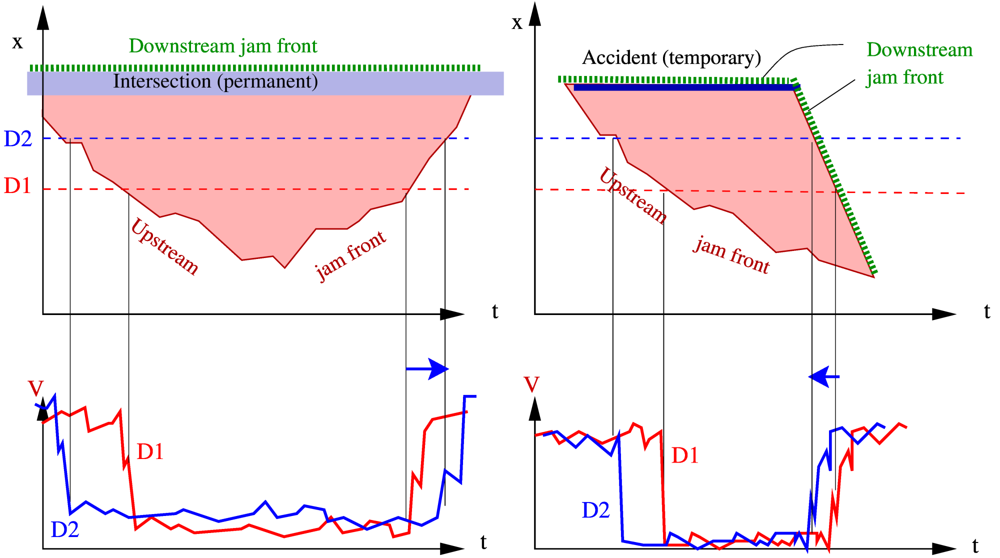
3.4. Analysis III: Spatio-Temporal State



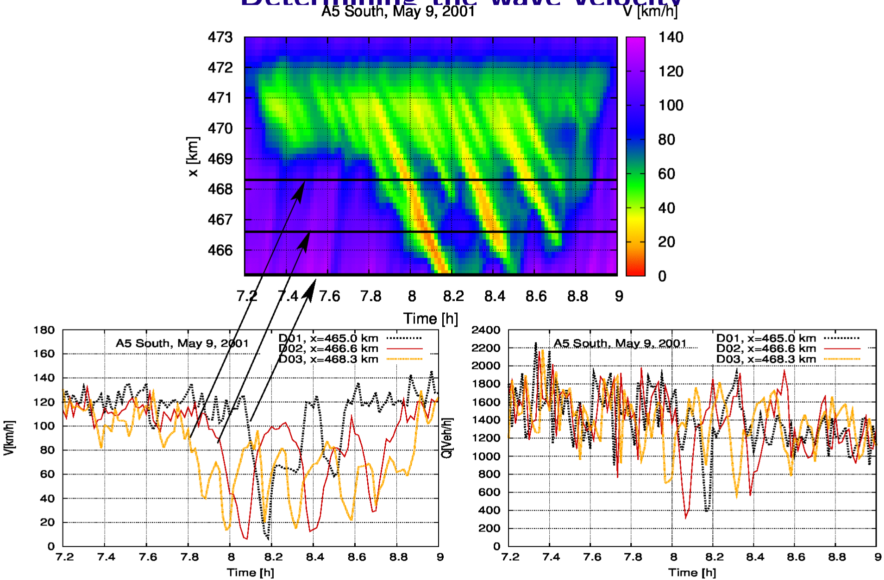
Analysis of a single detector station cannot resolve the upstream-downstream ambiguity when traffic gets free again at the cross-section

Resolution by two or more cross-sections

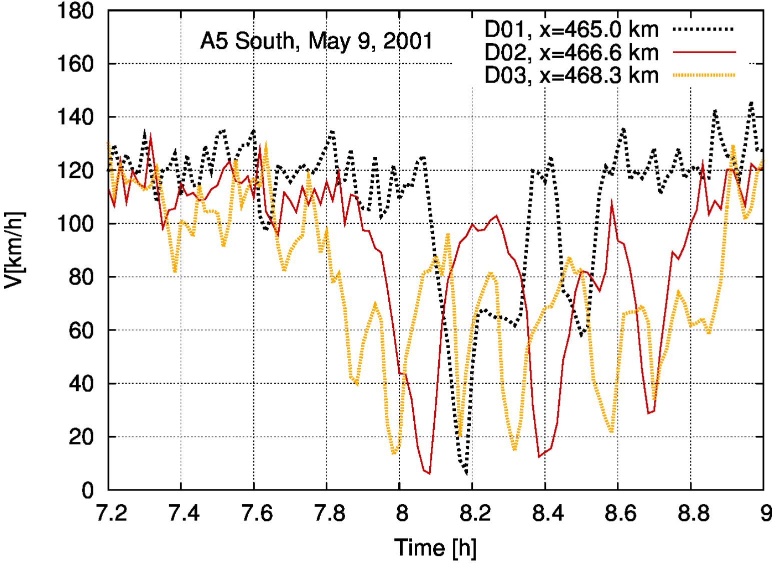


Ambiguity resolved!

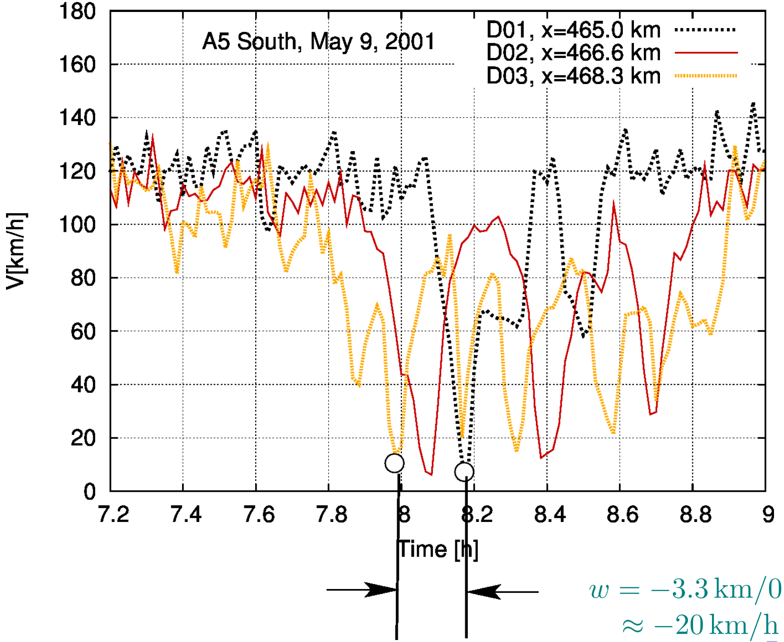
Determining the wave velocity



Problem: Determine the wave velocity by speed time series

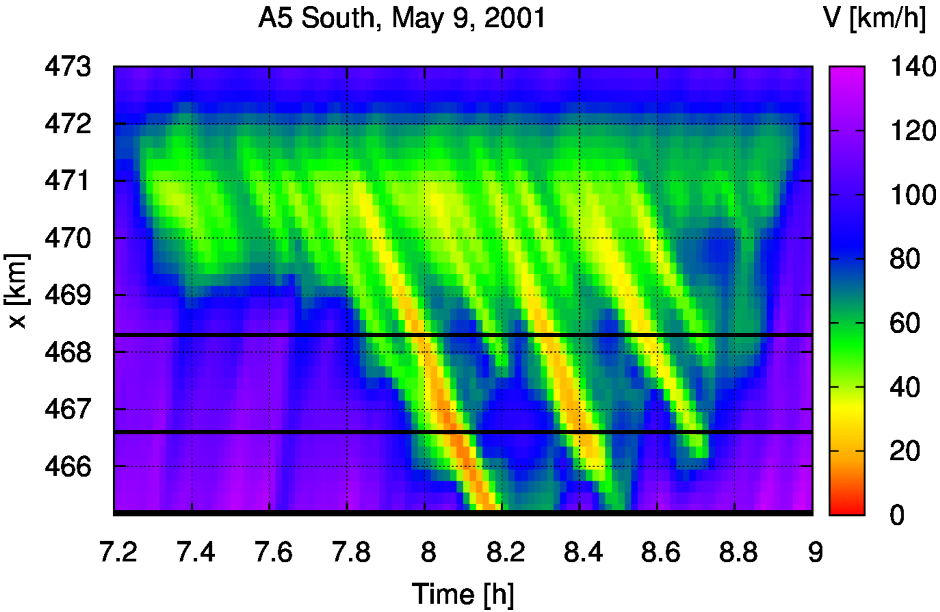


Solution

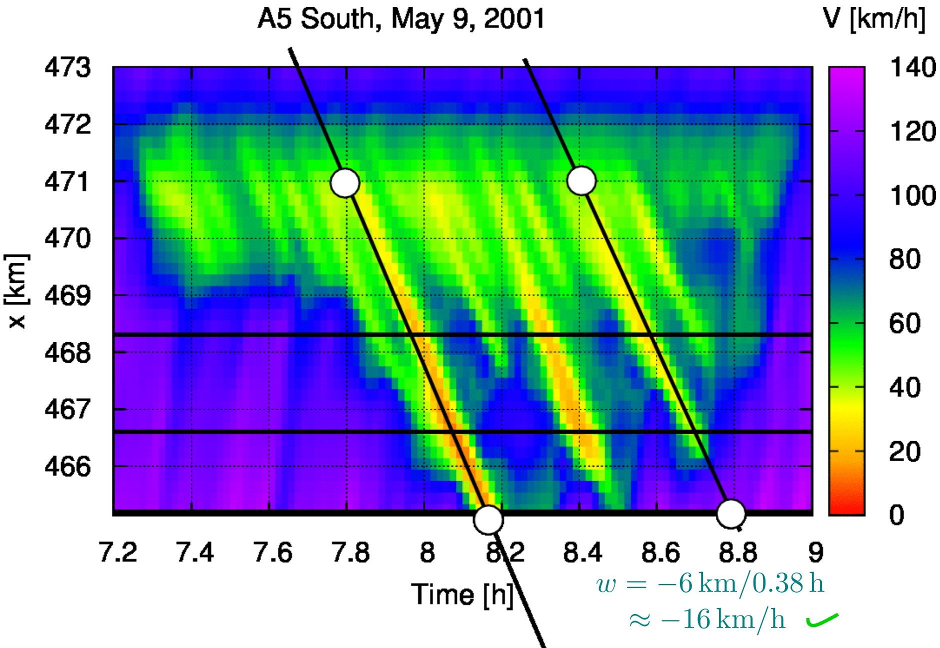


$$w = -3.3\,\mathrm{km}/0.2\,\mathrm{h}$$
$$\approx -20\,\mathrm{km/h}$$

Check results by approximate ground truth ⇒ 2.9

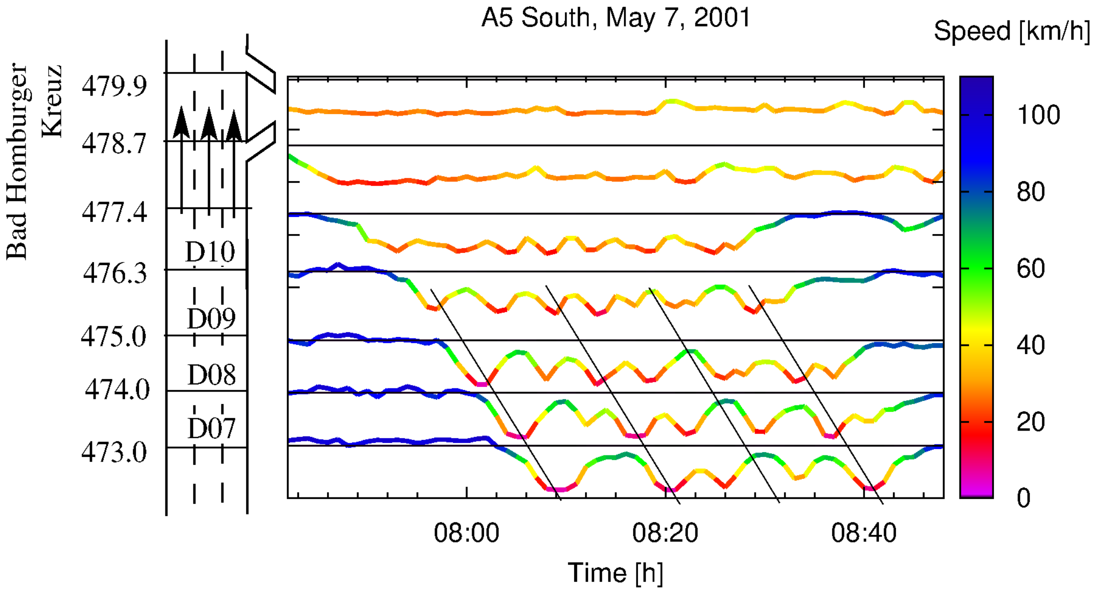


Solution

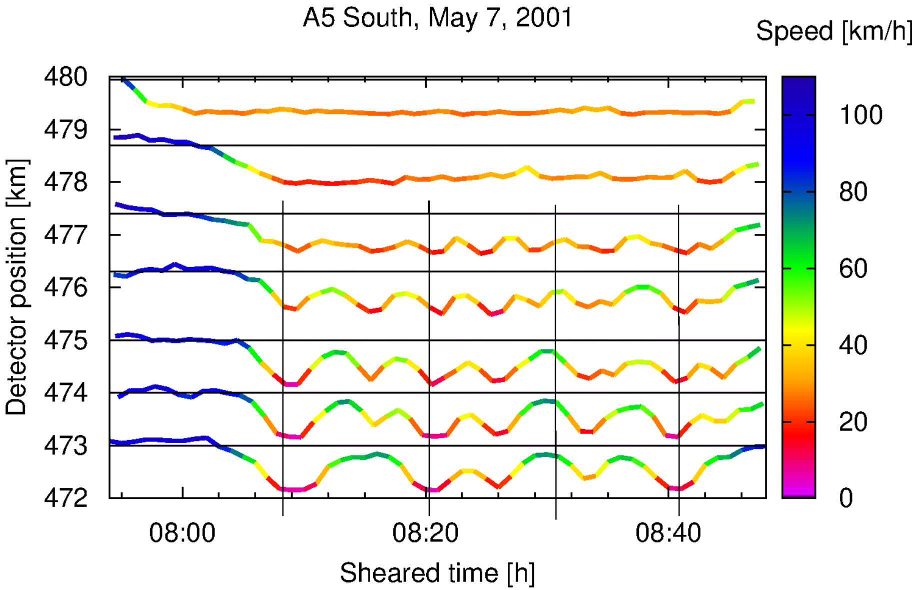


$$w = -6\,\mathrm{km}/0.38\,\mathrm{h}$$
$$\approx -16\,\mathrm{km/h}$$

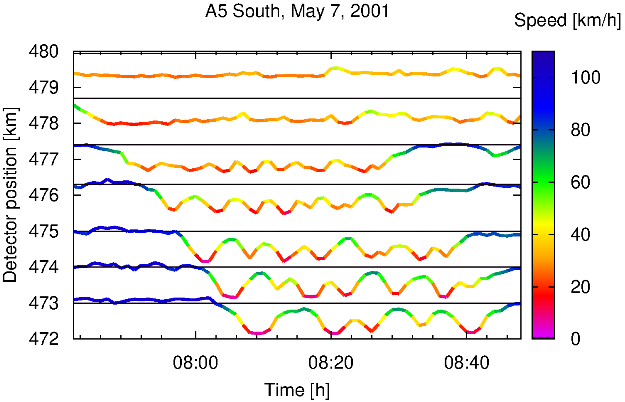
Determining the wave speed w statistically



Rectification by skewed time

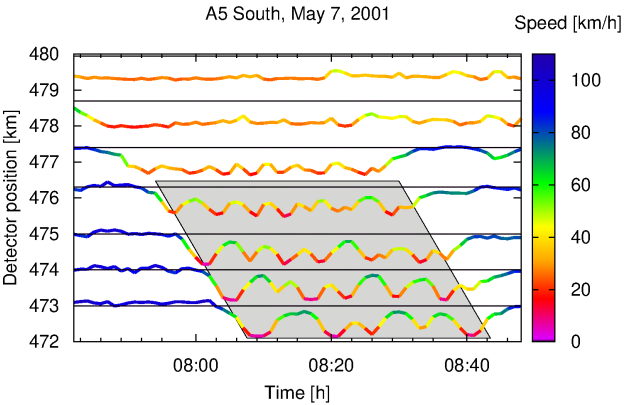


Statistical procedure for determining w



Statistical procedure for determining w

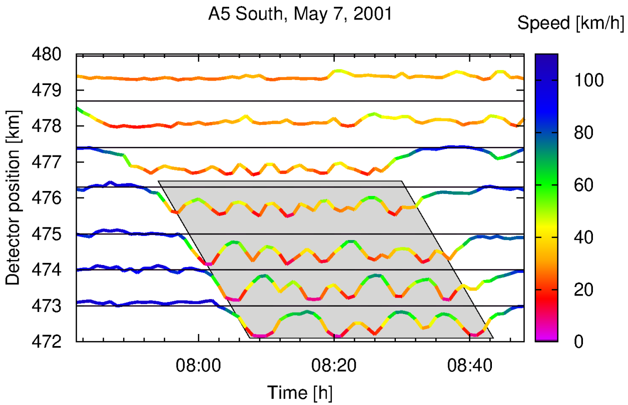
- Determine the oscillation area



Statistical procedure for determining w

- Determine the oscillation area
- Estimate inside the oscillation area the **speed cross correlation functions (CCF)** between the detectors k and l

$$r_{kl}(\tau) = \frac{E\Big((V_k(t) - E(V_k))(V_l(t+\tau) - E(V_l))\Big)}{\sqrt{\text{Var}(V_k)\text{Var}(V_l)}}$$



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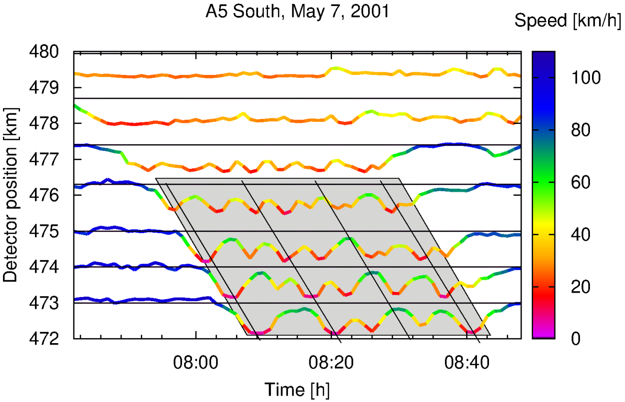

- Determine the time shifts τ_{kl} for the maxima of the CCF and the individual wave speed estimates $w_{kl} = \Delta x_{kl}/\tau_{kl}$ with $\Delta x_{kl} = x_l - x_k$ the distance between the respective detector stations

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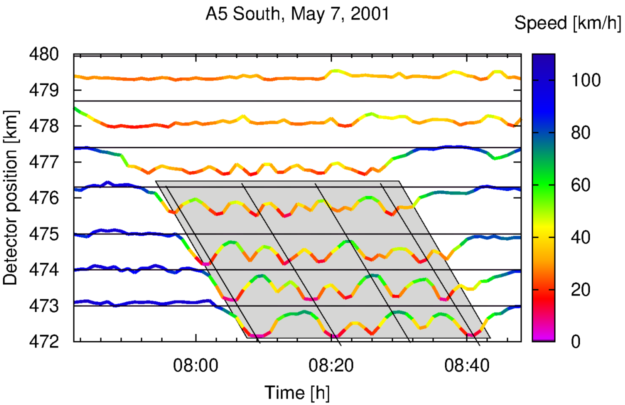

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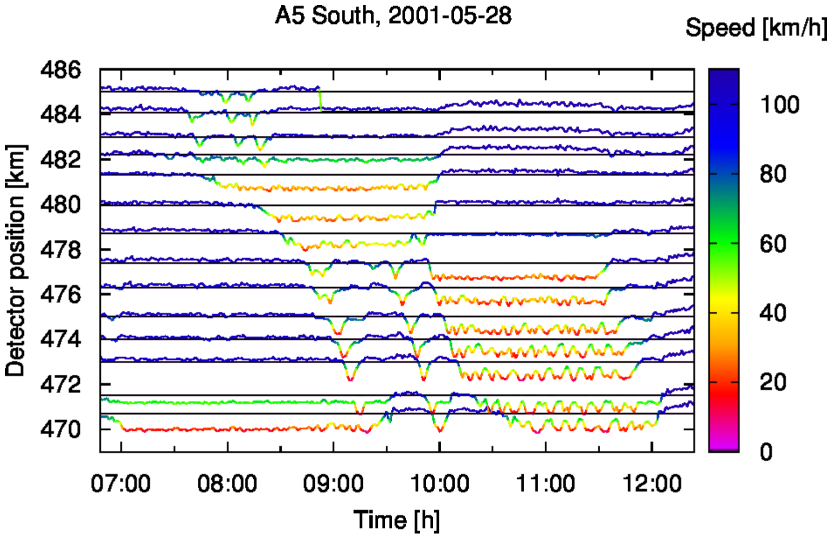
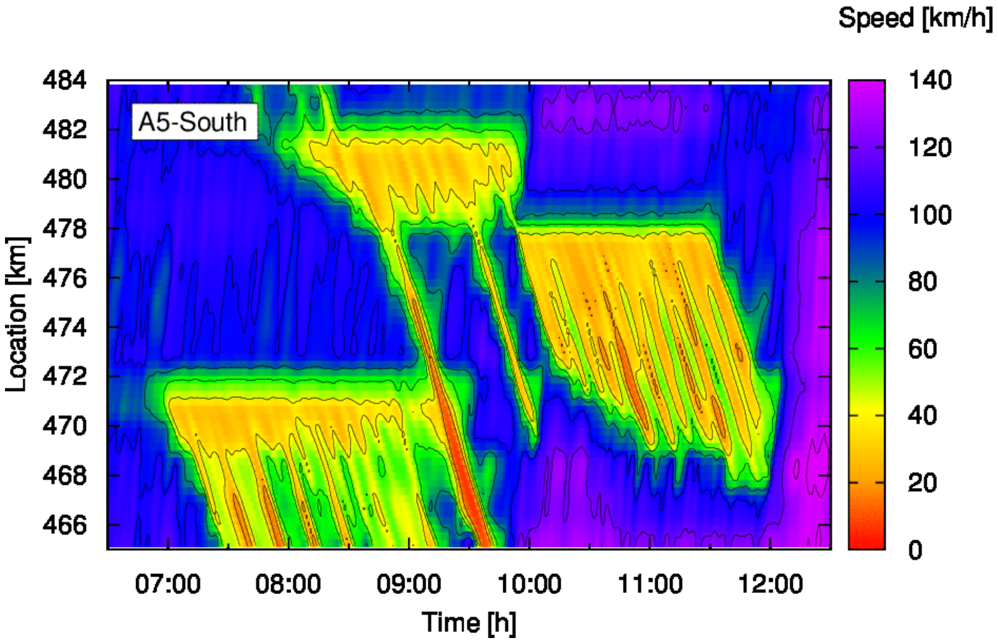
- The estimate W is the (weighted) mean of the w_{kl}

Another example

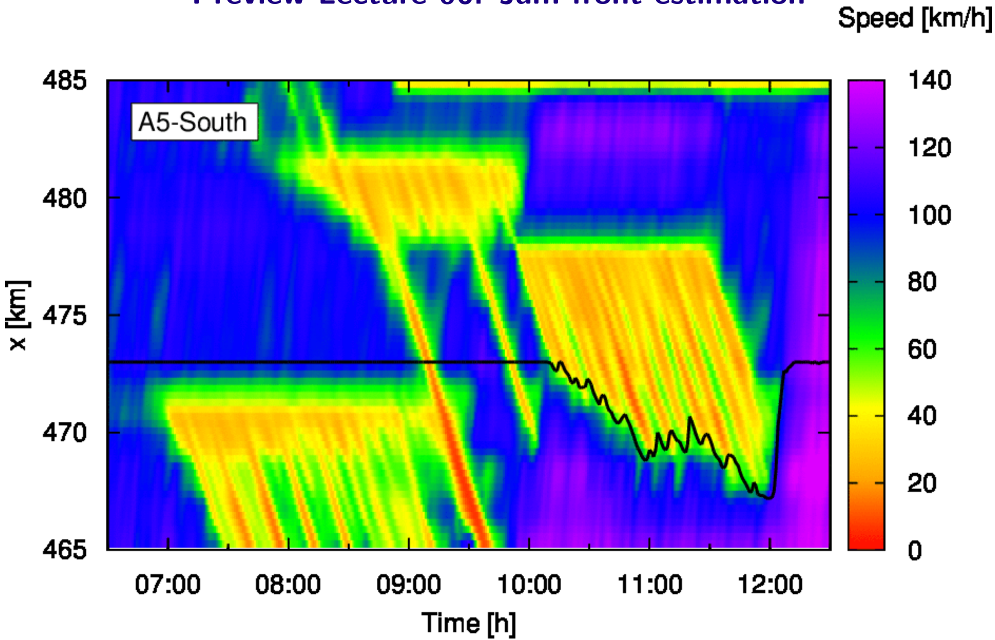


? Discuss what you see on this graphics!

Preview Lecture 04: state reconstruction



Preview Lecture 06: Jam-front estimation



You will learn how to do that three lessons ahead!