Traffic Flow Dynamics and Simulation

SS 2024, Solutions to Work Sheet 4, page 1

Solution to Problem 4.1: A simple fundamental diagram (Greenshields, 1935)

(a) Equation for the fundamental diagramm:

$$Q(\rho) = \rho V(\rho) = \rho V_0 \left(1 - \frac{\rho}{\rho_{\text{max}}}\right)$$

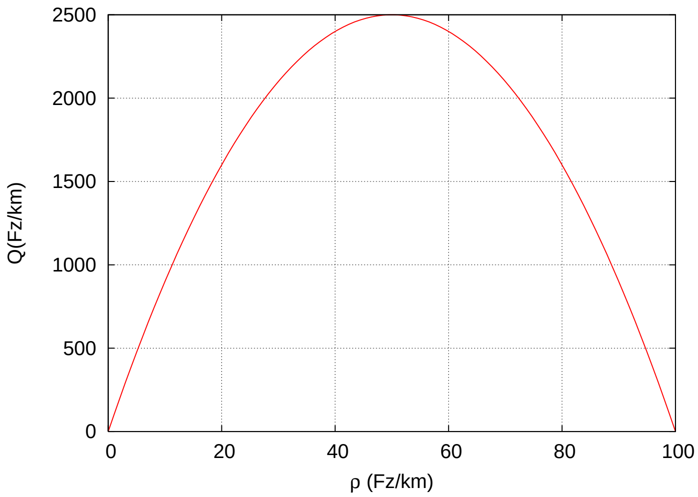
(b) Maximum flow Q_{max} at a critical density ρ_c calculated as usual:

$$Q'(\rho_m) = V_0 - 2\frac{V_0 \rho_m}{\rho_{\text{max}}} = 0, \quad \Rightarrow \quad \underline{\underline{\rho_m = \frac{1}{2}\rho_{\text{max}}}}.$$

Maximum flow (static capacity) itself:

$$Q_{\text{max}} = Q(\rho_m) = \underline{\underline{\frac{\rho_{\text{max}} V_0}{4}}}.$$

(c) Fundamental diagram für $V_0 = 100\,\text{km/h}$ and $\rho_{\text{max}} = 100\,\text{km}$:



(d) Speed-flow diagram: Repolace in $V(\rho) = V_0\,(1 - \rho/\rho_{\text{max}})$ the density $\rho = Q/V$ (hydrodynamic relation):

$$V = V_0 \left(1 - \frac{Q}{V \rho_{\text{max}}}\right)$$

Solving for Q:

$$Q(V) = V \rho_{\text{max}} \left(1 - \frac{V}{V_0}\right).$$

Solution to Problem 4.2: Fundamental diagram estimated from stationary detector data

(a) The **free speed** V_0 can be read off the speed-density diagram (at low densities):

$$V_{\text{A8}}^{\text{free}} = 125\,\text{km/h}, \quad V_{\text{A9}}^{\text{free}} = 110\,\text{km/h}.$$

(Dutch police is very rigorous in enforcing speed limits and uses automated systems to do so. This explains why few people drive faster than 110 km/h.)

The **maximum density** $\rho_{\textbf{max}}$ can be read off from the diagrams directly where the flow data of the flow-density diagrams drop to zero at the right-hand side):

$$\rho_{\text{A8}}^{\text{max}} = 80\,\text{veh/km}, \quad \rho_{\text{A9}}^{\text{max}} = 110\,\text{veh/km}$$

The **capacity** $Q_{\textbf{max}}$ (maximum flow per lane) can be read off similarly:

$$Q_{\text{max, A8}} = 1\,700\,\text{veh/h}\,, \quad Q_{\text{max, A9}} = 2\,400\,\text{veh/h}\,.$$

The **headway-parameter** T can be calculated by evaluating the flow $Q_{\text{cong}}(\rho) = 1/T(1-\rho/\rho_{\text{max}})$ of the congested branch “at capacity” (known density $\rho = \rho_c = Q_{\text{max}}/V_0$ and known flow Q_{max}) and solving for the only unknown quantity T:

$$Q_{\text{max}} = \frac{1}{T}\left(1 - \frac{\rho_c}{\rho_{\text{max}}}\right) \quad \Rightarrow\ T = \frac{1}{Q_{\text{max}}}\left(1 - \frac{\rho_c}{\rho_{\text{max}}}\right) = \frac{1}{Q_{\text{max}}}\left(1 - \frac{Q_{\text{max}}}{V_0\rho_{\text{max}}}\right) \tag{1}$$

and thus

$$T_{\text{A8}} = 1.91\,\text{s}, \quad T_{\text{A9}} = 1.27\,\text{s}.$$

the capacity drops on the two freeways are estimated as

$$\begin{aligned} \Delta Q_{\text{A9}} &= 2\,600\,\text{veh./h} - 2\,400\,\text{veh./h} = 200\,\text{veh./h}, \\ \Delta Q_{\text{A8}} &= 1\,800\,\text{veh./h} - 1\,700\,\text{veh./h} = 100\,\text{veh/h} \end{aligned}$$

(b) In order to determine the fit lines for the congested branch of the fundamental diagram, we need both the density and flow components of the scatter plot. While the flow is unbiased, the density estimate $\hat{\rho} = Q/V$ is systematically underestimated because the arithmetic time mean speed V systematically overestimates the space mean speed needed for an unbiased density estimate. Hence,

- The maximum density is biased towards lower values,
- the critical density is hardly biased (since it shares the free-flow branch where V and $\hat{\rho} = Q/V$ has little bias if at all),
- for the same reasons, the V_0 estimate is unbiased,
- the capacity Q_{max} and the capacity drop ΔQ_{max} is unbiased (since only the y coordinate matters; follows also from the two previous statements),

- the time-gap parameter T is biased towards lower values but much less than the maximum density since the contribution $Q_{\text{max}}/(V_0\rho_{\text{max}}$ in the above T estimate generally is $\ll 1$,
- the implied wave velocity $w = Q'_{\text{cong}}(\rho) = -\frac{1}{\rho_{\text{max}}T}$ is biased towards higher values because of the biases of both ρ_{max} and T.

(c) Using several detector cross sections at a distance Δx_{det} recording traffic waves, the wave velocity $w = \Delta x_{\text{det}}/\Delta t_{\text{det}}$ (with Δt_{det}, e.g., the time shift between the recorded speed minima, cf. Lecture 03) can be estimated directly in an unbiased way. By substituting $1/(\rho_{\text{max}}T)$ by $-w$ and T for Q_{max}, cf. Eq. (1), we can write the congested branch in an unbiased way in terms of w:

$$Q_{\text{cong}}(\rho) = Q_{\text{max}} + c(\rho - \rho_c) = Q_{\text{max}} + c\left(\rho - \frac{Q_{\text{max}}}{V_0}\right)$$

This allows for unbiased estimates:

$$\rho_{\text{max}} = Q_{\text{max}}\left(\frac{1}{V_0} - \frac{1}{c}\right), \quad T = -\frac{1}{\rho_{\text{max}}w}$$

An explicit calculated example for this is given in the next tutorial.

(d) At a complete standstill, no vehicle passes a stationary detector, so the estimate is stuck in a "0/0" situation, i.e., it is not possible: stationary detectors cannot distinguish between an empty road and standstill traffic (when the occupamcy is recorded as well, a detector may record an occupancy of 1 but this is usually interpreted as a detector fault)