Traffic Flow Dynamics and Simulation

SS 2024, Solutions to Work Sheet 2, page 1

Solution to Problem 2.1: Use of FC data

FC powered by GNSS provide space-time data points and (anonymized) IDs of the equipped vehicles at a certain sampling rate. As specified here, the FC data has following properties and use cases:

- *Precision:* As specified in the problem statement, the GNSS uncertainty of the order of 10 m
- *Temporal resolution:* As specified in the problem statement, 30 s
- *Spatial resolution:* With a sampling rate of two per minute, the vehicle can cover 1 km and more between a data point in free traffic. Fortunately, in congestions, the points are naturally closer
- *Generating single trajectories:* We have points in space (uncertainty 10 m) and time (precise, but only every 30 s) that can be connected to create *trajectories*.
- *Travel time and speed:* From the trajectories, we can directly obtain the travel time and infer the instantaneous speed by taking the gradients.
- *Existence and location of congestions, spatial resolution:* Low speeds on a highway or freeway, e.g., 5 m/s=18 km/h, usually indicate a *traffic jam*. Since the data provide spatiotemporal positions of the vehicles, we can deduce the location of congested zones, including their upstream and downstream boundaries. At the time of a FC "floating through", the resolution is of the order 5 m/s * 30 s=150 m
- *Temporal resolution of jam fronts* This resolution of 150 m is only valid whenever an equipped vehicle "floats" through. The temporal resolution depends on the penetration rate of equipped vehicles with activated communication. Example: Jam, density 3*50 veh/km=150 veh/km, flow 3*1200 veh/h=1 veh/s, penetration rate 2 % ⇒ partial density 3 veh/km, partial flow 1/50 veh/s ⇒ average total resolution 300 m in space and 50 s in time
- *Lane information:* GNSS measurements are only accurate to the order of 10 m and careful map-matching/error checking is necessary to exclude, for example, stopped vehicles on the shoulder or at a rest area, or vehicles on a parallel road (we will treat this in the problem *map matching* later on). Therefore, GNSS data do *not* reveal lane information, nor information on *lane changes*.
- *Extensive quantities:* Since the percentage of equipped vehicles with connected devices is low, variable, and unknown, we can *not* deduce extensive quantities (values increase with the vehicle number such as traffic density and flow) from this type of data.

To wrap it up:

(1) trajectories of single vehicles ✓

(2) location and time of lane changes (✗)

(3) traffic density (vehicles per kilometer) ✗

(4) traffic flow (vehicles per hour) ✗

(5) vehicle speed ✓

(6) length and position of traffic jams ✓

(7) travel time ✓

Trucks and taxis: In spite of their non-representative driving properties, data from trucks and taxis *are* useful. In congestions, information is most useful and, just in this situation, fast and slow vehicles are equal. One of the few examples of an inverse "Murphy's Law". Another example is that, by definition, traffic, and hence FCD, are most dense in congested situations.

Note on usefulness of FCD: FCDs are particularly useful because they are recorded everywhere where vehicles are driving, e.g., on deviations of blocked freeway sections where, typically, no stationary detectors are installed. So they are indispensable for dynamic (traffic-dependent) routing.

Solution to Problem 2.2: Data aggregation at a cross-section

(a) *Flow and speed:* With an aggregation interval $\Delta t = 30\,\text{s}$ and $n_1 = 6$, $n_2 = 4$ measured vehicles on lanes 1 and 2, respectively, the flow and time mean speed on the two lanes are

$$Q_1 = \frac{n_1}{\Delta t} = 0.2\,\text{veh/s} = 720\,\text{veh/h}, \quad Q_2 = \frac{n_2}{\Delta t} = 0.133\,\text{veh/s} = 480\,\text{veh/h},$$

$$V_1 = \frac{1}{n_1}\sum_i v_{1i} = 25.8\,\text{m/s}, \qquad V_2 = \frac{1}{n_2}\sum_i v_{2i} = 34.0\,\text{m/s}.$$

(b) *Density:* In Lecture 03, we will learn that estimating the traffic density by the ratio flow divided by time-averaged speed only is unbiased if there is no correlation between the individual speeds and time headways, i.e., the covariance $\text{Cov}(v_i, \Delta t_i) = 0$.:

$$\rho_1 = \frac{Q_1}{V_1} = 7.74\,\text{veh/km}, \quad \rho_2 = \frac{Q_2}{V_2} = 3.92\,\text{veh/km}\,.$$

(c) *Both lanes combined:* Density and flow are *extensive quantities* increasing with the number of vehicles. Therefore, building the total quantities by simple summation over the lanes makes sense:

$$\rho_\text{tot} = \rho_1 + \rho_2 = 11.66\,\text{veh/km}, \quad Q_\text{tot} = Q_1 + Q_2 = 1\,200\,\text{veh/h}\,.$$

Since speed is an *intensive* quantity (it does not increase with the vehicle number), summation over lanes makes no sense. Instead, we define the effective aggregated speed by requiring the hydrodynamic relation to be valid for total flow and total density as well:

$$V = \frac{Q_\text{tot}}{\rho_\text{tot}} = \frac{\rho_1 V_1 + \rho_2 V_2}{\rho_\text{tot}} = \frac{Q_\text{tot}}{Q_1/V_1 + Q_2/V_2} = 28.5\,\text{m/s} = 102.9\,\text{km/h}.$$

By its derivation from the hydrodynamic relation, this effective speed is the space mean speed rather than the time mean speed measured directly by the detectors. We notice that the effective speed is simultaneously the *arithmetic mean* weighted with the densities, and the *harmonic mean* weighted with the flows. However, the weighting with the densities requires that the density estimates itself are known without bias. This is the case here but not generally. Since flows can always be estimated without systematic errors from stationary detectors, the harmonic mean weighted with the flows is preferable.

(d) *Fraction of trucks:* Two out of six (33 %) are in the right lane, none in the left, two out of ten (20 %) total. Notice again that the given percentages are the fraction of trucks *passing* a fixed location (*time mean*). In the same situation, we expect the fraction of trucks observed by a „snapshot“ of a road section at a fixed time (*space mean*) to be higher, at least if trucks are generally slower than cars.

Solution to Problem 2.3: Determining macroscopic quantities from single-vehicle data

The distance headway $\Delta x_i = 60\,\text{m}$ is constant on both lanes. All vehicles are of the same length $l = 5\,\text{m}$ and all vehicles on a given lane l (left) or r (right) have the same speed $v_i^{\text{l}} = 144\,\text{km/h} = 40\,\text{m/s}$ and $v_i^{\text{r}} = 72\,\text{km/h} = 20\,\text{m/s}$, respectively.

(a) **Time gap / headway:**

The headways $\Delta t_i = \Delta x_i / v_i$ are

$$\Delta t_i^{\text{l}} = \frac{60\,\text{m}}{40\,\text{m/s}} = 1.5\,\text{s}, \quad \Delta t_i^{\text{r}} = \frac{60\,\text{m}}{20\,\text{m/s}} = 3.0\,\text{s}\,.$$

The time gaps T_i are equal to the headway minus the time needed to cover a distance equal to the length of the leading vehicle, $T_i = \Delta t_i - \frac{l_{i-1}}{v_{i-1}}$. Since all vehicle lengths are equal, this results in

$$T_i^{\text{l}} = \frac{60\,\text{m} - 5\,\text{m}}{40\,\text{m/s}} = 1.375\,\text{s}, \quad T_i^{\text{r}} = \frac{60\,\text{m} - 5\,\text{m}}{20\,\text{m/s}} = 2.75\,\text{s}\,.$$

(b) **Macroscopic quantities:**

Quantities separately for each lane: We assume an aggregation time interval $\Delta t = 60\,\text{s}$. However, due to the stationary situation considered here, any other aggregation interval will lead to the same results. Directly from the definitions of flow, occupancy, and time-mean speed, we obtain for each lane

$$Q^{\text{l}} = \frac{1}{\Delta t_i^{\text{l}}} = \frac{1}{1.5\,\text{s}} = 2\,400\,\text{veh/h}, \qquad Q^{\text{r}} = \frac{1}{\Delta t_i^{\text{r}}} = \frac{1}{3\,\text{s}} = 1\,200\,\text{veh/h}\,.$$
$$O^{\text{l}} = \frac{0.125}{1.5} = 0.083 = 8.3\,\%, \qquad O^{\text{r}} = \frac{0.25}{3.0} = 0.083 = 8.3\,\%\,.$$
$$V^{\text{l}} = 144\,\text{km/h}, \qquad V^{\text{r}} = 72\,\text{km/h}.$$

Due to the homogeneous traffic situation, the arithmetic and harmonic time-mean speed are the same and directly given by the speed of the individual vehicles.

Totals and averages of both lanes: As already discussed in Problem **??** summing over the lanes to obtain a total quantity makes only sense for extensive quantities (Q, ρ) but not for the intensive ones (V, O).

Flow:

$$Q_{\text{tot}} = \frac{\Delta N}{\Delta t} = \frac{\Delta N^{\text{l}} + \Delta N^{\text{r}}}{\Delta t} = 3\,600\,\text{veh/h}, \quad Q = \frac{Q_{\text{tot}}}{2} = 1\,800\,\text{veh/h}.$$

Occupancy:

$$O = O^{\text{l}} = O^{\text{r}} = 0.083.$$

Arithmetic time mean speed:

$$V = \frac{1}{\Delta N} \sum_i v_i = \frac{40 \cdot 40\,\mathrm{m/s} + 20 \cdot 20\,\mathrm{m/s}}{60} = 120\,\mathrm{km/h}.$$

Harmonic time mean speed:

$$V_\mathrm{H} = \frac{\Delta N}{\sum 1/v_i} = \frac{60}{\frac{40}{40\,\mathrm{m/s}} + \frac{20}{20\,\mathrm{m/s}}} = 108\,\mathrm{km/h}.$$

We observe that the arithmetic mean is larger than the harmonic mean.

(c) **Which mean?**

In traffic flow, there are four sensible ways to average, consisting of the four combinations of (i) one of two physical ways (time mean and space mean), (ii) one of two mathematical ways (arithmetic and harmonic).

- *Time mean* means averaging at a fixed location over some time interval as done by stationary detectors.
- *Space mean* means averaging at a fixed time over some space interval (road section), e.g., when making a snapshot of the traffic flow.

For the *space mean*, we have

$$V = \frac{\rho_1 V_1 + \rho_2 V_2}{\rho_\mathrm{tot}} = \frac{Q_\mathrm{tot}}{Q_1/V_1 + Q_2/V_2}$$

while, for the time mean, we simply have

$$V = \frac{Q_1 V_1 + Q_2 V_2}{Q_\mathrm{tot}}.$$

The time mean is generally larger than the space mean because, at the same partial densities, the class of faster vehicles passes the cross-section more often within the aggregation interval than the vehicles of the slower class do. The arithmetic average is generally larger than the harmonic average which can be shown for any data. Only for the trivial case of identical data, both averages agree.

Here, $\rho_1 = \rho_2$ but $Q_1 \neq Q_2$, so the simple (not weighted) arithmetic average over lanes applies for the space mean speed.

(d) **Speed variance** The speed variance within each lane is zero. Therefore, the total variance of the speeds in the left and right lane is the same as the inter-lane variance sought after:

$$\begin{aligned}
\sigma_V^2 &= \left\langle (v_i - \langle v_i \rangle)^2 \right\rangle \\
&= \frac{1}{60} \left(40[40 - 33.3]^2 + 20[20 - 33.3]^2 \right) \\
&= 88.9\,\mathrm{m}^2/\mathrm{s}^2\,.
\end{aligned}$$