Traffic Flow Dynamics and Simulation

SS 2024, Solutions to Work Sheet 1, page 1

Solution to Problem 1.1: Trajectory data

(a) *Local* macroscopic quantities, e.g., the density $\rho(x,t)$ only make sense if (i) the spatiotemporal aggregation region includes at least a few trajectories, (ii) there are no significant changes in the average microscopic state (speed: gradient of trajectories; density: inverse of average distance between trajectories).

For the free traffic, at least, say , 5 trajectories are contained for time intervals above 10 s and/or space intervals of more than 100 m (only one criterion is needed). For congested traffic, we need 50 s and/or 50 m. The second criterion of steady state is also satisfied for the proposed regions, though only marginally for the congested case.

(b) Region for free traffic $[10\,\text{s}, 30\,\text{s}] \times [20\,\text{m}, 80\,\text{m}]$:

- Flow $Q = 12\,\text{Fz}/20\,\text{s} = \underline{\underline{2\,160\,\text{Fz/h}}}$
- Density $\rho = 3\,\text{Fz}/60\,\text{m} = \underline{\underline{50\,\text{Fz/km}}}$
- Speed by trajectory gradient: $V = 60\,\text{m}/5\,\text{s} = 12\,\text{m/s} = \underline{\underline{43.2\,\text{km/h}}}$
- Speed from hydrodynamic relation: $V = \frac{Q}{\rho} = \underline{\underline{43.2\,\text{km/h}}}$. In view of the possible errors, this is consistent. After all, the discrete counting ambiguities alone ("12 or 13 vehicles") introduce a discretisation error of about 8 %

(c) Region for congested traffic $[50\,\text{s}, 60\,\text{s}] \times [40\,\text{m}, 100\,\text{m}]$:

- Flow $Q = 2\,\text{Fz}/10\,\text{s} = \underline{\underline{720\,\text{Fz/h}}}$
- Density $\rho = 6\,\text{Fz}/60\,\text{m} = \underline{\underline{100\,\text{Fz/km}}}$
- Speed by gradients: $V = 20\,\text{m}/10\,\text{s} = 2\,\text{m/s} = \underline{\underline{7.2\,\text{km/h}}}$
- Speed by hydrodynamic relation:

$$V = \frac{Q}{\rho} = \underline{\underline{7.2\,\text{km/h}}}.$$

By chance, both methods give an identical outcome. This is pure "luck". Differences of up to 20 % would be OK in view of the nonperfect stationarity in the congested region and the discretisation (counting) ambiguities

(d) Propagation velocity

$$c \approx -\frac{200\,\text{m}}{(60-22)\,\text{s}} = -\frac{200\,\text{m}}{38\,\text{s}} = -5.3\,\text{m/s} = \underline{\underline{-19\,\text{km/h}}}$$

Because of the negative sign, the propagation is *against* the driving direction.

(e) Actual travel time through the region [0 m, 200 m]: 35 s

Free-flow travel time by the undisturbed trajectories (e.g., the one leaving the region at $t \approx 38\,\text{s}$: 18 s. Hence, the delay is given by

$$\tau_{\text{delay}} = (35 - 18)\,\text{s} = \underline{\underline{17\,\text{s}}}.$$

This is the delay given by radio or navigation systems. However, it is *not* the time one drives through a congestion since this time includes the delay *and* the time needed in case of free traffic. *Therefore, it always feels as though the navigation systems err on the low side although this is not the case*

(f) Lane-changing intensity in $[0\,\text{s}, 80\,\text{s}] \times [20\,\text{m}, 120\,\text{m}]$: n=5 changes, so

$$r \approx \frac{4\,\text{changes}}{80\,\text{s}\ 100\,\text{m}} = 0.0005\,\text{changes/m/s} \approx \underline{\underline{1\,800\,\text{changes/km/h}}}.$$

Solution to Problem 1.2: Trajektoriendaten eines Verkehrsflusses mit Störung

(a) Stop at a red traffic light. The thick black line represents the red phase.

(b) Traffic demand is estimated by the *potential* inflow which is equal to the actual inflow for free traffic (“supply exceeds demand”). Hence, e.g., for $x = -80\,\text{m}$ and times $t < 50\,\text{s}$: 5 lines per 20 s= $\underline{\underline{0.25\,\text{veh/s}}}$ =$\underline{\underline{900\,\text{veh/h}}}$.

(c) Select, e.g., the trajectory beginning at $[-80\,\text{m}, -16\,\text{s}]$ and ending at $[80\,\text{m}, 0\,\text{s}]$

$$v_{\text{in}} = 10\,\text{m/s} = \underline{\underline{36\,\text{km/h}}}.$$

Density: One line per 40 m or $\rho = Q/v$. Both leads to $\rho =$$\underline{\underline{25\,\text{veh/km}}}$.

(d) The “jam density” is, in this case, the density of the queue of waiting vehicles behind the red light: Eight lines per 40 m $\Rightarrow \rho_{\text{jam}} = \underline{\underline{200\,\text{veh/km}}}$.

(e) *Outflow in the steady-state region* to the right and above the blue symbols: It is easiest to just count the number of trajectories crossing a horizontal edge of a box (length 20 s) in this region: $Q_{\text{out}} = 10\,\text{veh}/20\,\text{s} = \underline{\underline{0.5\,\text{veh/s} = 1\,800\,\text{veh/h}}}$

Speed in the outflow region: Lines parallel to inflow trajectories $\Rightarrow$ Geschwindigkeit wie beim freien Upstream-Verkehr, da Linien zu jenen parallel: $\underline{\underline{V_{\text{out}} = V_{\text{in}} = 36\,\text{km/h}}}$

Outflow density by counting the trajectories along the vertical edges (length 40 m) of a box or by the hydrodynamic relation: $\rho_{\text{out}} = \underline{\underline{50\,\text{veh/km}}}$.

(f) Propagation velocities of the upstream and downstream jam fronts either by the gradient of the chain of red and down symbols, respectively, or by the “shock-wave speed equation” (to be derived later):

$$\text{Free} \to \text{jam: } c^{\text{up}} = \frac{\Delta Q}{\Delta \rho} = \frac{-900\text{Fz/h}}{175\text{Fz/km}} = \underline{\underline{-5.14\text{km/h}.}}$$

$$\text{Jam} \to \text{free: } c^{\text{down}} = \frac{\Delta Q}{\Delta \rho} = \frac{1800\text{Fz/h}}{-150\text{Fz/km}} = \underline{\underline{-12\text{km/h}.}}$$

(g) Travel time (T) with delay: $T_{\text{delay}} = 50\,\text{s}$.

Free-flow travel time: $T_{\text{free}} = 180\,\text{m}/10\,\text{m/s} = 18\,\text{s}$.

Hence $\tau_{\text{delay}} = T_{\text{delay}} - T_{\text{free}} = \underline{\underline{32\,\text{s}}}$

(h) Braking distance from the red symbols to the horizontal section of the trajectory: $s_b = 25\,\text{m}$.

Acceleration distance from the stopped phase to the blue symbols: $s_a = 50\,\text{m}$. The speed before the braking and after the acceleration maneuvers is equal and given by $v = 10\,\text{m/s}$ (use SI units!!), so using the kinematic “school formula”:

$$b = \frac{v^2}{2s_b} = \underline{\underline{2\text{m/s}^2.}}, \quad a = \frac{v^2}{2s_a} = \underline{\underline{1\text{m/s}^2.}}$$

Alternatively direct calculation by the definition of acceleration as rate of speed change with Δt the durations of the phases:

$$a = \frac{\Delta V}{\Delta t} = \frac{10\,\mathrm{m/s}}{10\,\mathrm{s}} = \underline{\underline{1\,\mathrm{m/s}^2}}, \quad b = -\frac{\Delta V}{\Delta t} = -\frac{-10\,\mathrm{m/s}}{5\,\mathrm{s}} = \underline{\underline{2\,\mathrm{m/s}^2}}$$

Since $\Delta t = 5\,\mathrm{s}$ and $10\,\mathrm{s}$ for the deceleration and acceleration phases, respectively, are hard to determine from the graphics, the school formula implies less estimation errors, in this case.