Traffic Flow Dynamics and Simulation

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Problem 8.1: Rules of thumb for the safe gap and braking distance

(a) In Continental European countries, one learns in driving schools the following rule: „The safe gap should be at least half the reading of the speedometer“. Translate this rule into a safe time gap rule.

(b) A common US rule for the safe gap is the following: „Leave one car length for every ten miles per hour of speed“. Another rule says „Leave a time gap of two seconds“. Compare these two rules assuming a typical car length of 15 ft. For which car length are both rules equivalent?

(c) A rule of thumb for the braking distance says „Speed squared and divided by 100“. If speed is measured in km/h, what braking deceleration is implied by this rule?

Problem 8.2: A simple model for emergency braking maneuvers

Critical situations requiring emergency braking maneuvers can be described by following microscopic model:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \begin{cases} 0 & \text{if } t < T_r, \\ -b_{\text{max}} & \text{otherwise.} \end{cases}$$

(a) Give an intuitive meaning of the parameters T_r and b_{max}.

(b) Calculate the braking distance and the overall stopping distance for initial speeds of 50 km/h and 70 km/h assuming $b_{\text{max}} = 8\,\text{m/s}^2$ and $T_r = 1\,\text{s}$.
Hint: The overall stopping distance is composed of the *braking distance*, i.e., the vehicle displacement during the actual braking phase, and the *reaction distance* the vehicle travels during the reaction time of the driver.

(c) *This is a task from the theoretical driver's license MC test sheets* Imagine a situation where a child suddenly runs into the road from a hidden position behind a vehicle. A driver driving according to the above model just manages to stop if his or her initial speed is 50 km/h. At what speed would this driver collide with the child if the initial speed is 70 km/h and the situation is otherwise unchanged? (please tick)

◯ 20 km/h ◯ 40 km/h ◯ 50 km/h ◯ 60 km/h

Note: Assume the reaction time $T_r = 1\,\text{s}$ and maximum braking deceleration $b_{\text{max}} = 8\,\text{m/s}^2$ from part (b)

Problem 8.3: OVM acceleration on an empty road

Consider a single vehicle on an empty road whose acceleration is described by the *Optimal Velocity Model*

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v_0 - v}{\tau}$$

assuming the initial conditions $x(0) = 0$, and $v(0) = 0$.

(a) At which time does the vehicle reach its maximum acceleration? What is its value?

(b) Determine the parameter τ if the desired speed is given by 120 km/h and the maximum acceleration by $2\,\mathrm{m/s^2}$.

(c) Solve this simple differential equation by the ansatz

$$v(t) = a + be^{-t/\tau}$$

(the exponent $-t/\tau$ comes from the “homogeneous” differential equation $\frac{\mathrm{d}v}{\mathrm{d}t} = -v/\tau$ and general “cooking-book” rules in solving differential equations)

(d) Give the acceleration profile

(e) At which time does the vehicle reach a speed of 100 km/h?