Traffic Flow Dynamics and Simulation

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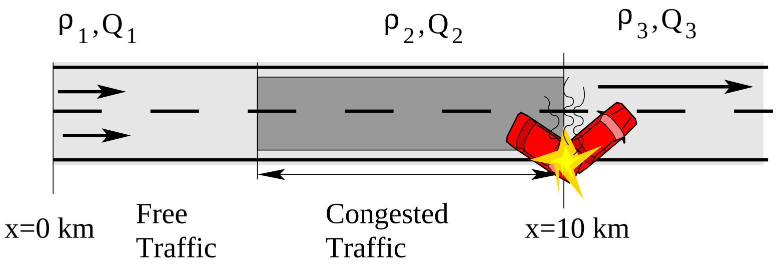
Problem 6.1: Jam propagation on a highway I: Accident

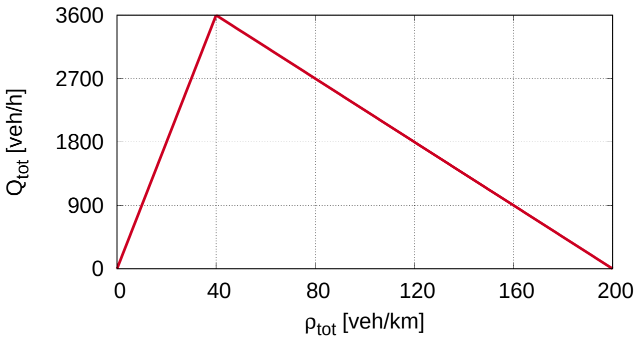
At the highway/freeway depicted below, an accident happens at 17:00 h:



On the remaining left lane, $Q_{\text{out}} = 1800\,\text{veh/h}$ can pass the bottleneck. The traffic demand for the whole direction (both lanes together is given by

$$Q_1 = \begin{cases} 2\,700\,\text{veh/h} & t \le 18:00\,\text{h} \\ 900\,\text{veh/h} & t > 18:00\,\text{h} \end{cases}$$

Assume that all three regions I: upstream, II: congested upstream of the bottleneck, III: outflow are in the local homogeneous steady state ("equilibrium") and the total traffic flow (summed over both lanes) is given as a function of the total density by the fundamental diagram depicted below:



(a) Draw the point $P_3 = (\rho_{3,tot}, Q_{3,tot})$ reflecting the downstream region after the accident into the fundamental diagram of total flow *vs.* total density assuming free traffic

(b) Argue that a congestion will form upstream of the accident-bottleneck and draw the point $P_2 = (\rho_{2,tot}, Q_{2,tot})$ into the fundamental diagram.

(c) Calculate the propagation velocity of the upstream jam front (transition free → congested) for $17:00\,\mathrm{h} \le t \le 18:00\,\mathrm{h}$

Hint The velocity is given by the *shockwave formula* $c_{12} = (Q_2 = Q_1)?(\rho_2 - \rho_1)$, i.e., by the gradient of the line $\overline{P_1 P_2}$.

(d) Argue that the maximum jam length is reached at 18:00 h and calculate its length. Also calculate the *delay* caused by the jam for the *worst case*.

(e) Determine the time where the congestion dissolves.

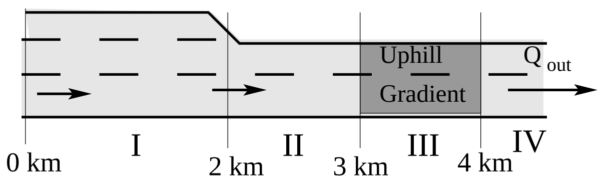
Hint: calculate the new shockwave velocity of the upstream front and determine the time where the front passes $x = 10\,\mathrm{km}$

(f) The first intuition seems to suggest that the jam should dissolve after exactly two hours: after all, the congestion grew for one hour driven by a demand excess of 900 veh/h, so it should shrink at the same rate once the *supply* excess of 900 veh/h becomes effective after one hour. However, the calculation at part (e) leads to a later time. Can you reason at a second look why, indeed, the jam *should* last longer than two hours?

Hint: consider the balance of the vehicle number in the 10 km upstream region at different times

Problem 6.2: Jam propagation II: Uphill grade and lane drop

Consider following highway section containing a three-to-two lane drop and an uphill grade on parts of the two-lane region:



The effective traffic flow is modeled with a triangular fundamental diagram.

$$Q(\rho) = \min\left[V_0\rho, \frac{1}{T}\left(1 - \rho l_{\text{eff}}\right)\right].$$

Because of a high percentage of trucks, the mean free speed in the uphill region III is only $V_{03} = 60\,\mathrm{km/h}$ while $V_0 = 120\,\mathrm{km/h}$ applies elsewhere. Furthermore, assume an increase of the time gap from $T = 1.5\,\mathrm{s}$ to $T_3 = 1.9\,\mathrm{s}$ in the gradient region III, and an effective vehicle length $l_{\text{eff}} = 10\,\mathrm{m}$ everywhere.

(a) Show that the capacity per lane is given by 2 000 vehicles/h outside the uphill region, and 1440 vehicles/h in the uphill region.

(b) Before the onset of the rush hour at 4:00 pm, assume free steady-state traffic everywhere and a total inflow of 2 000 vehicles/h. Calculate the effective and total densities and the speeds in all regions I – IV.

(c) At 4:00 pm, the total traffic demand at $x = 0$ increases abruptly to 3 600 vehicles/h. Does this cause a breakdown? If so, at which time and where?

Hint: Traffic breaks down as soon as somewhare on the road section the traffic demand given by the inflow becomes larger than the local road capacity. If this happens, the bottleneck gets *activated* and a congestion forms with a stationary downstream front at this location

Hint 2: The shockwave formula also applies for abrupt transitions between two free-flow states of different densities. Here, such a transition crosses $x = 0$ at 4:00 pm

(d) Now assume that the breakdown happened at $x = 3$ km and consider two stages of the developing congestion: (i) The upstream jam front is in region I at $x = 1$ km, (ii) the front is in region II at $x = 2.5$ km. Calculate, for both stages, flows and densities of all four regions of the considered road stretch.

Hint: Distinguish the regions I to IV and additionally subdivide the region where the upstream front is located into an upstream free and downstream congested section

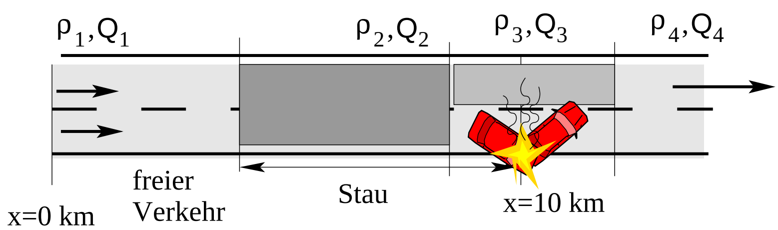
(e) At which velocity does the upstream jam front propagate in stages (i) and (ii) of Part (d) of this problem? Also calculate, for Situation (i), the travel time and the delay for traversing the whole section $0\,\text{km} \le x \le 4\,\text{km}$.

(f) Visualize the different traffic states in the Regions I to IV in the fundamental diagram for the total flow vs. density and the spatiotemporal development of the congestion for $t \ge 16:00$ h.

(g) Calculate the travel and delay times for traversing the first four kilometers if the vehicle passes the upstream jam front at 1 km.

Problem 6.3: Jam propagation III: Temporary partial road block

Consider traffic flow on a two-lane highway between the road kilometers 0 and 10 during and after a closure of one lane at kilometer 10 at 15:00 h. In the considered time period, the traffic demand (inflow at $x = 0$) is constant and given by 3024 vehicles/h. The lane closure is effective for half an hour until 15:30 h.



Solve the following questions using the LWR with the triangular fundamental diagram with the parameters $l_{\text{eff}} = 8\,\text{m}$, $T = 1.5\,\text{s}$, and $V_0 = 28\,\text{m/s}$.

(a) Calculate the total road capacity and the effective capacity (per lane) prior to the lane closure. Does the capacity satisfy the demand? Calculate the traffic density and the traveling time to traverse the 10 km long road stretch.

(b) Show that, after the lane closure is active, the bottleneck capacity of the remaining lane does not satisfy the demand. Calculate the effective and total traffic density of the forming jam. Assume that the drivers symmetrically use both lanes (i.e., consider locations upstream of the transfer zone where people change lanes to the through lane).

(c) Calculate the growth rate of the jam by determining the velocity of its upstream front. *Hint:* Distinguish carefully between total and effective (lane-averaged) densities and flows.

(d) After the lane closure has been lifted, the downstream front of the jam sets into motion. Since it moves faster than the upstream front, the jam eventually dissolves. Calculate the velocity of the moving downstream front and the time for complete dissolution (in seconds since 15:00 h). Also calculate the position of the last vehicle to be obstructed by the jam at obstruction time.

(e) Visualize the spatiotemporal dynamics of the jam by drawing its boundaries in a space-time diagram.

(f) How much time does a vehicle need to traverse the 10 km long road section if it enters at 15:30 h?